Advantages of Per Unit System :- U-5 (1)

1. Manufacturers usually specify the impedance values of equipments in per unit of equipment's rating. If any data is not available, it is easier to assume its per unit value than its numerical values.

2. In an large interconnected power system with various voltage levels and various capacity equipments it has been found quite convinient to work with per unit (p.u.) system of quantities for analysis purposes rather than in absolute values of quantities.

3. Per unit data representation yields information (important) about relative magnitudes.

4. The chance of confusion between line and phase quantities in a three phase balanced system is greatly reduced.

5. The manufacturers usually provide the impedance values in per unit

6. The computational effort in power system is very much reduced with the use of per unit quantities.

7. Power systems contain a large number of transformers. The ohmic values of impedance as referred to secondary is different from the values as referred to primary. However, if base values are selected properly the p.u. impedance is the same on the two sides of the transformer

8. The per unit impedance referred to either side of three phase transformer is the same regardless of the three phase connections whether they are Y-Y, Δ-Δ (or) Δ-Y.

Per unit Quantities :-

Analysis of power systems employing actual values such as Electrical units of voltampere (VA), voltage, current and impedance donot adapt themselves easily to computations. However, these quantities are often expressed as per unit of a base (or) Reference value specified for each. Electrical power engineers often prefer to express impedances, currents, voltages and power in per unit (PU) values rather than their actual units.

* Conversion of PU quantities from one Base to another base :-

The per unit impedance of generator (or) a transformer, as supplied by the manufacturer is generally based on the rating of the generator (or) the transformer itself, while the selected base kV and selected base MVA for a power system are different from the ratings of the components. Therefore, it is important to change the pu values computed on the rated base quantities to the common system base quantities

*Per Unit Representation :-

The per unit value of any quantity is defined as the ratio of actual quantity to its base quantity.

$$\therefore \text{Per unit value} = \frac{\text{Actual Value}}{\text{Base Value}}$$

Both the actual value and the Base value are in the same units so that the per unit value is dimensionless. When base values are specified properly for the various parts of power system connected by transformers, the p.u. values of impedances determined in their own part of the system remain the same when viewed from another part.

If base MVA and base KV ratings are given for a 1ϕ system then base current and base impedance can be formulated as

$$\text{Base Current KA} = \frac{\text{Base MVA}}{\text{Base KV}}$$

If once the base voltage and base current are known then the base impedance can be found out as,

$$\text{Base impedance} = \frac{\text{Base KV}}{\text{Base KA}}$$

$$= \frac{\text{Base KV}}{\frac{\text{Base MVA}}{\text{Base KV}}} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

Now, the perunit value of the impedance will be

$$Z_{pu} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$= \frac{\text{Actual Impedance}}{\text{Base Impedance}}$$

Changing the base of Per unit quantities:-

If the values are already in pu values referred to their own ratings, then to convert to selected base values, let us derive the formula,

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$\therefore Z_{pu(given)} = \frac{Z_{actual}}{(\text{Base } KV_{given})^2} \times \text{Base MVA}_{given} \quad \text{——} (1)$$

Similarly, when referred to new base values,

$$Z_{pu\,(new)} = \frac{Z_{actual}}{(\text{Base } KV_{new})^2} \times \text{Base MVA}_{(new)} \quad \text{——} (2)$$

Now, by dividing (2) by (1) we get,

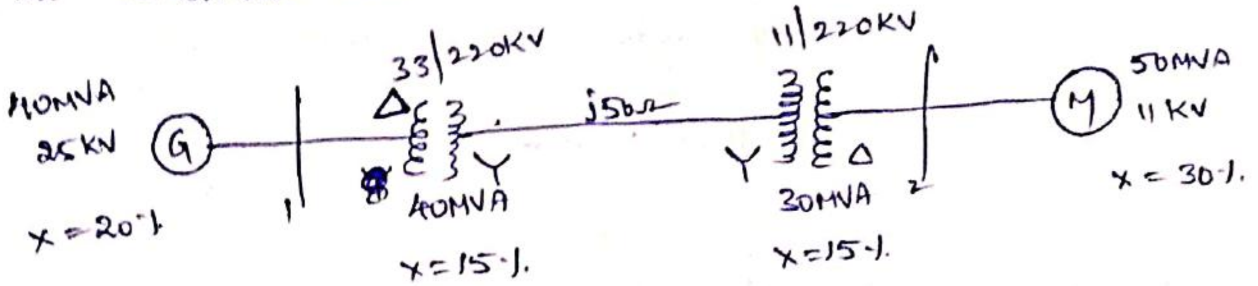
$$\frac{Z_{pu\,(new)}}{Z_{pu\,(old)}} = \frac{\dfrac{\cancel{Z_{actual}}}{(\text{Base } KV_{new})^2} \times \text{Base MVA}_{new}}{\dfrac{\cancel{Z_{actual}}}{(\text{Base } KV_{old})^2} \times \text{Base MVA}_{old}}$$

$$= \frac{(\text{Base } KV_{old})^2}{(\text{Base } KV_{new})^2} \times \frac{\text{Base MVA}_{new}}{\text{Base MVA}_{old}}$$

$$\therefore \boxed{Z_{pu\,(new)} = Z_{pu\,(old)} \times \left(\frac{\text{Base } KV_{old}}{\text{Base } KV_{new}}\right)^2 \times \frac{\text{Base MVA}_{new}}{\text{Base MVA}_{old}}}$$

Problems:-

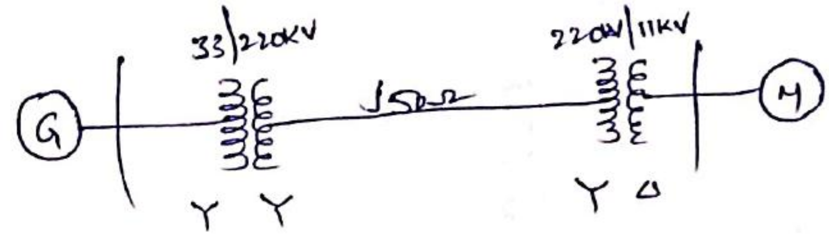
① Draw the per unit impedance diagram for a power system. Neglect the Resistance and use a base of 100MVA, 220 KV in 50Ω line given, 11Δ / 220KV Y



Sol. Δ-Y transformer is generally a STEP-UP Transformer and

Y-Δ transformer is generally a STEP-DOWN Transformer

∴



~~At generator,~~

$$\cancel{Z_{pu(new)} = Z_{pu(old)} \times \frac{(KV_{base\ old})^2}{(KV_{base\ new})^2} \times \frac{MVA_{new}}{MVA_{old}}}$$

$$\cancel{= 20\% \times \left(\frac{25K}{33K}\right)^2 \times \left(\frac{100}{40}\right) = 0.2869} = 0.287\ pu.$$

NOTE:-

[Here, Base values are given at Transmission line, so the problem solution should be done from the transmission line. If Base values are given at Generator circuit then solution to be done from the Generator side and vice-versa.]

Generator
A ~~Transformer~~-1 :-

Base kV at generator circuit $= 220 \times \frac{33}{220}$

$= 33\,kV.$

Now, per unit Reactance of Generator $= 20\% \times \frac{100}{40} \times \left(\frac{25}{33}\right)^2$

$= 0.2867$ pu.

for Motor circuit :-

Base kV at Motor side $= 220 \times \frac{11}{220}$

$= 11\,kV.$

per unit Reactance at Motor $= 0.3 \times \frac{100}{50} \times \left(\frac{11}{11}\right)^2$

$= 0.6$

Transformer-1

per unit Reactance at $X_1^{mer} \Rightarrow 0.15 \times \frac{100}{40} \times \left(\frac{33}{33}\right)^2$

$= 0.375$

Transformer-2

per unit Reactance of $X_{mer}^{2} \Rightarrow 0.15 \times \frac{100}{30} \times \left(\frac{11}{11}\right)^2$

$= 0.5$ pu.

2a

a) Transmission line :-

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

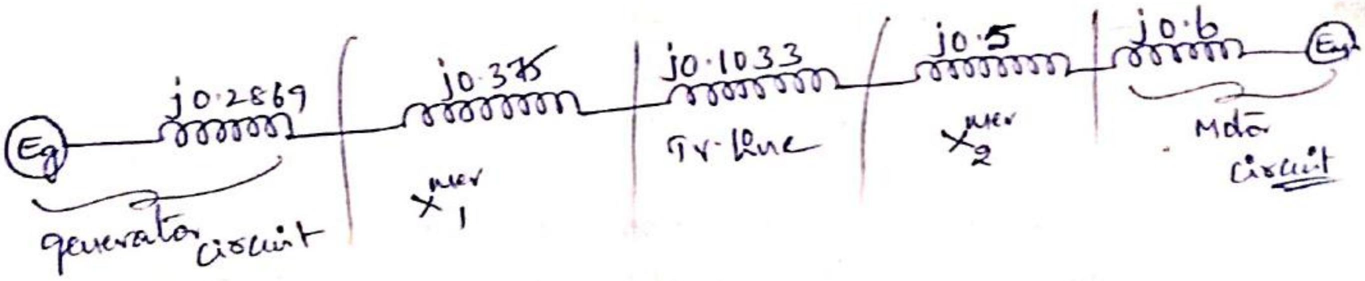
$$= \frac{(220)^2}{100} = 484\ \Omega$$

$$\text{Now, p.u. impedance of Tr. line} = \frac{\text{Actual value}}{\text{Base value}}$$

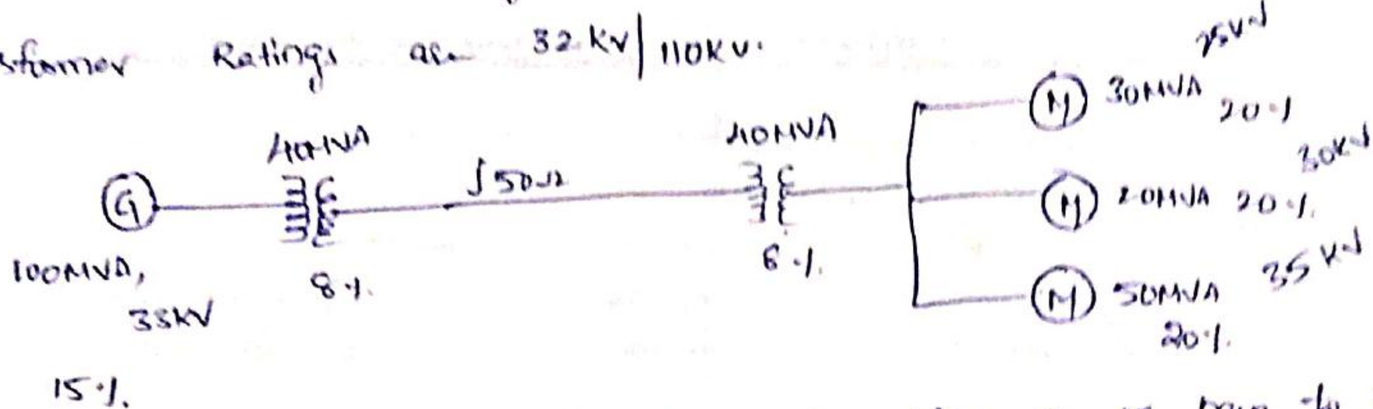
$$= \frac{j50}{484}$$

$$= j0.1033\ \text{pu}$$

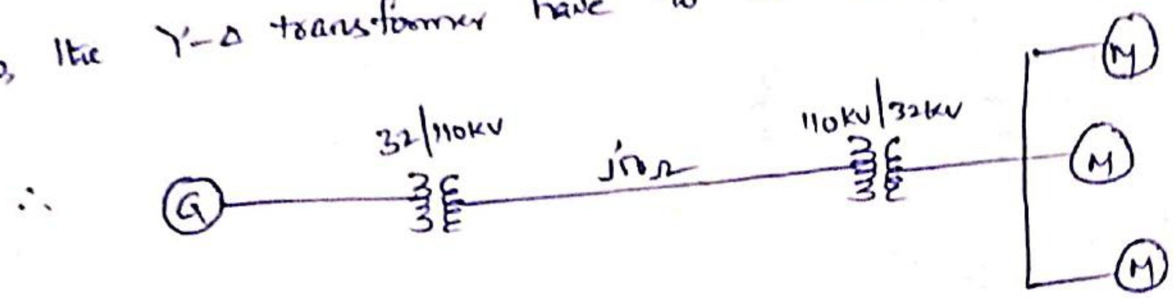
Now, the Reactance diagram will be,



(2) Draw the impedance diagram for the given system and the (3) transformer Ratings are 32 KV | 110 KV.



Sol: The voltage from the generator is of low value so we have to step-up the value with help of transformer. So the 1st transformer will be of Δ-Y type and to supply for motors the total voltage has to step-down. So, the Y-Δ transformer have to be used.



∴ at generator

$$Z_{pu} = Z_{pu}(\text{old}) \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.15 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{100}{100}\right) = 0.15 \text{ pu.}$$

Transformer -1

$$Z_{pu} = 8\% \times \left(\frac{32}{33}\right)^2 \times \left(\frac{100}{40}\right) = 0.188 \text{ pu.}$$

Transmission line:

For the Transmission line the Base KV will be calculated as,

Base KV = Old Base KV × Transformer-1 ratio

$$= 33 \times \frac{110}{32} = 113.43 \text{ KV.}$$

Now, Base impedance $= \dfrac{(\text{Base kV})^2}{\text{Base MVA}}$

$$= \frac{(113.43)^2}{100}$$

$$= 128.66$$

Now, P.u. of Transmission line $= \dfrac{\text{Actual}}{\text{Base value}} = \dfrac{j50}{128.66}$

$$= j0.388 \text{ pu.}$$

At transformer-2

$$Z_{pu(new)} = Z_{pu}(old) \times \left(\frac{kV_{base\,old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 8\% \times \left(\frac{110kV}{113.43kV}\right)^2 \times \left(\frac{100}{40}\right)$$

$$= 0.188 \text{ p.u.}$$

for motor ① :-

Base kV = previous Base kV × Trans Ratio of X_{mer}

$$= 113.43 \times \frac{32}{110}$$

$$= 33kV.$$

$$\therefore Z_{pu\,(new)} = Z_{pu}(old) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.2 \times \left(\frac{25kV}{33kV}\right)^2 \times \left(\frac{100}{30}\right)$$

$$= \cancel{0.388}\ 0.3826 \text{ pu.}$$

for Motor-2:-

Base MVA = 100, Base KV = 33KV

$$\therefore \quad Z_{pu}(new) = Z_{pu}(old) \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.2 \times \left(\frac{30}{33}\right)^2 \times \left(\frac{100}{20}\right) = 0.826 \text{ pu.}$$

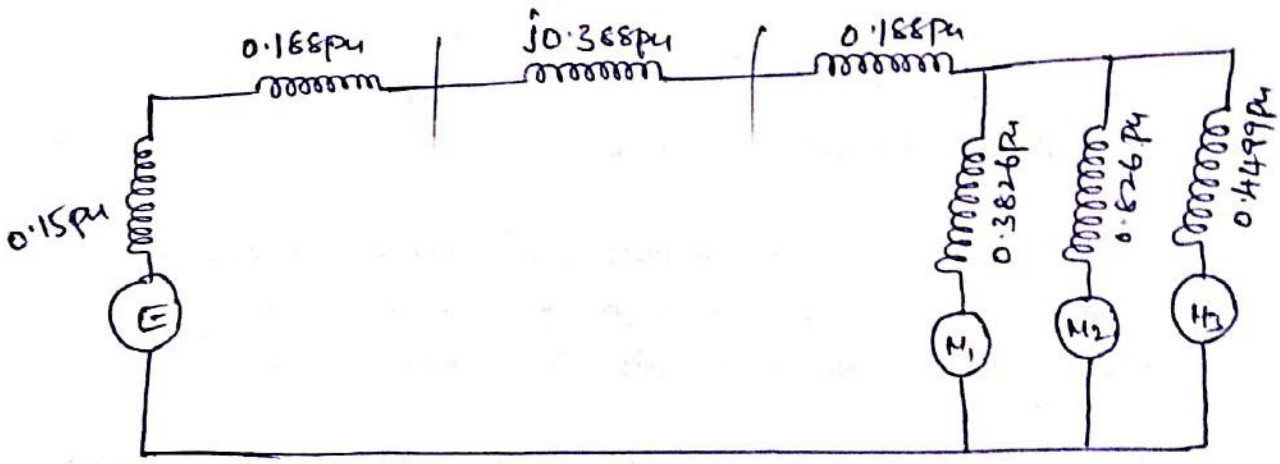
for Motor-(3):-

$$Z_{pu}(new) = Z_{pu}(old) \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

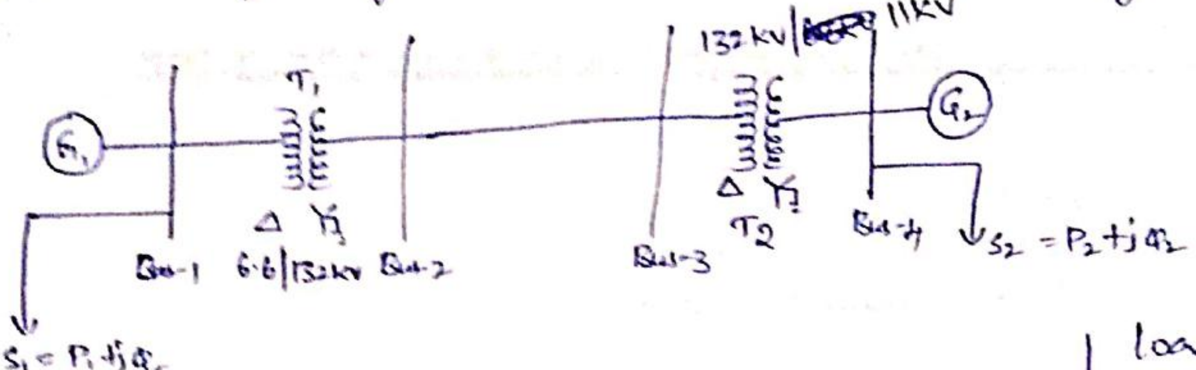
$$= 0.2 \times \left(\frac{35}{33}\right)^2 \times \left(\frac{100}{50}\right)$$

$$= 0.4499 \text{ pu.}$$

Now, Reactance diagram will be,



(3) For power system given below draw the Reactance diagram. (7)



Generator G_1 :- 20 MVA, 6.6 KV, $X_1 = 0.10$ pu.

Generator G_2 = 25 MVA, 11 KV. $X_2 = 0.2$ pu

Transformer T_1 = 25 MVA, 6.6/132 KV, $X_1^{\text{mer}} = 0.08$ pu

" T_2 = 30 MVA, 11/132 KV, $X_2^{\text{mer}} = 0.10$ pu

Transmission line $X = (30 + j120)$ of KV = 132 KV.

load.
S_1 = 10 MVA for 0.8 Pf lagging
S_2 = 25 MVA at 0.9 Pf leading

Assume Base MVA = 50 MVA of the System.

Sol. Assuming Generator-1 as Reference and Base KV = 6.6 KV. while the Base MVA is 50 MVA as from the problem.

Generator-1 :-
$$Z_{pu(new)} = Z_{pu}(old) \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$
$$= 0.1 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{50}{20}\right)$$
$$= 0.25 \text{ pu}$$

Transformer-1 :-
$$Z_{pu}(new) = Z_{pu}(old) \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$
$$= 0.08 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{50}{25}\right)$$
$$= 0.16$$

X^{mer} primary side నుండి pu value కనుక్కొన్నా, లేదా secondary side నుండి కనుక్కొన్నా value ($X_{pu}(total)$) ఒకటే. ఇది pu value Representation లోని Advantage

~~Transformer~~ Transmission line :-

Base kV for Tr. line = Previous Base kV × Trans. Ratio of X^{mer}_1

$$= 6.6 \times \frac{132}{6.6}$$

$$= 132 \text{ kV}.$$

$$Z_{pu} \text{ for Transmission line} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$\therefore \text{Now, Base Value} = \frac{(\text{Base kV})^2}{\text{Base MVA}} = \frac{(132)^2}{50}$$

$$= 348.48\ \Omega$$

$$\text{Now, } Z_{pu} \text{ for Transmission line} = \frac{(30 + j120)}{348.48}$$

$$\therefore Z_{pu} = 0.086 + j0.344$$

Transformer-2 :-

Now, Calculating at primary side of X^{mer}_2.

$$\therefore Z_{pu}(\text{new}) = Z_{pu}(\text{old}) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.1 \times \left(\frac{132}{132}\right)^2 \times \left(\frac{50}{30}\right)$$

$$= 0.1666 = 0.1667$$

Generator-2 :-

Base kV for Generator-2 = Previous Base kV × Trans Ratio of X^{mer}_2

$$\Rightarrow \text{Base kV} = 132 \times \frac{11}{132}$$

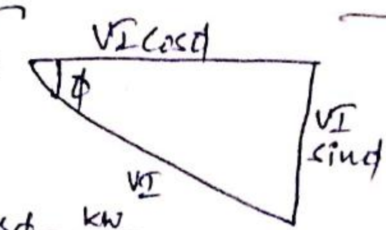
$$= 11 \text{ kV}.$$

Now, $Z_{pu\,(new)} = Z_{pu\,(old)} \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$

$= 0.2 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{25}\right)$

$= 0.4$ pu.

$\because$



$\cos\phi = \frac{KW}{KVA}$

$P = KW = \cos\phi\ KVA$

$Q = KVAR = \sin\phi\ KVA$

$\because S = P + jQ = KVA(\cos\phi + j\sin\phi)$

At load -1 :-

If, $S_{pu} = P_{pu} + jQ_{pu}$

where $P_{pu} = \frac{P}{S_{base}}$ and $Q_{pu} = \frac{Q}{S_{base}}$

$\cos\phi = 0.8$

$\sin\phi = 0.6$

Now, load $S_1 = 10 \times (0.8 + j0.6)$ (since lagging).

$= (8 + j6)$. MVA

Now, $S_{1\,pu} = \frac{\text{Actual value}}{\text{Base value}} = \frac{(8+j6)}{50}$

$= (0.16 + j0.12)$ pu.

At load-2

load $S_2 = 25\,(0.9 - j0.436)$

$= 22.5 - j10.897$

$\because \cos\phi = 0.9$ (lead).

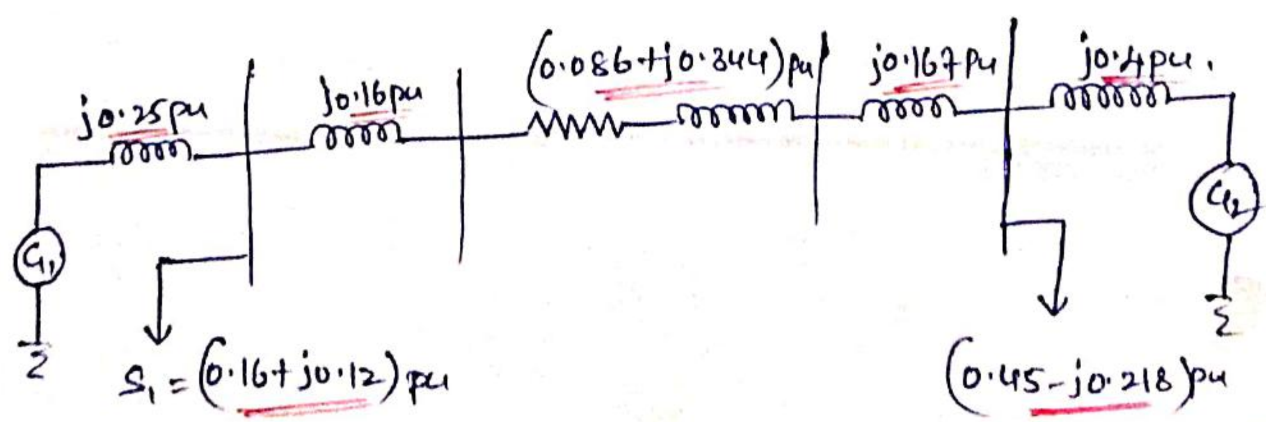
$\sin\phi = 0.4359$.

'–' sign since it is leading

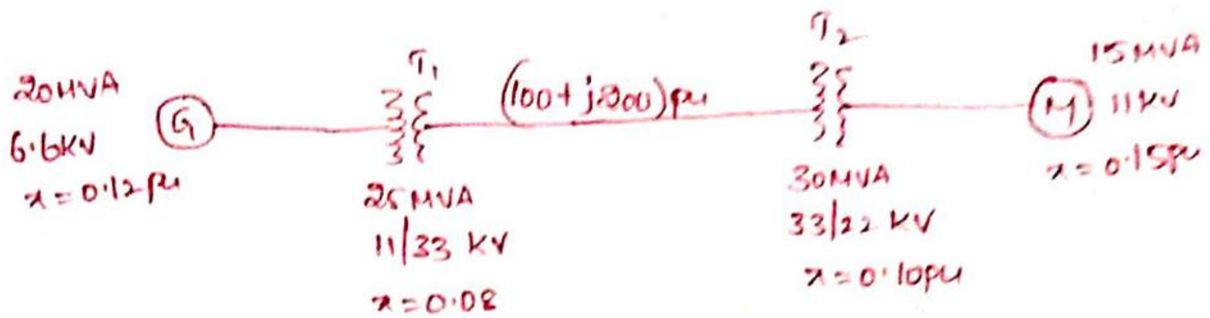
$S_{2\,pu} = \frac{22.5 - j10.897}{50}$

$= 0.45 - j0.218$ pu

Now, Reactance diagram will be,



4) In the power system shown in fig. below, Convert all quantities to pu values on base of 25 MVA. Assume a base voltage of 33kV for Transmission line. (10)



Sol. given Base MVA = 25 MVA and Base kV = 33 for Transmission line. So, the Problem Solvation to be start from the Trans. line to generator side and to the Motor side

At generator:

Base KV for generator section

= previous KV × Transformation Ratio of X_1^{mer}

Now, the Previous KV will be the Trans. line Base KV

$\Rightarrow$ $33 \times \frac{11}{33} = 11$ kV. [∵ Starting from line to Gen. then primary is 33KV & 11KV is sec]

$$\therefore Z_{pu} = Z_{pu(old)} \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.12 \times \left(\frac{6.6}{11}\right)^2 \times \left(\frac{25}{20}\right)$$

$$= 0.054 \text{ pu.}$$

Transformer-1

Calculating at primary end (L.V. side).

Generator ఏ primary end section ఉన్నట్లయితే Generator ఏ X_1^{new} primary
అయితే అప్పుడు Base kV ఏ primary voltage consider చేయాలి.
ఉన్నట్లు Generator నుండి X_1^{new} కి connect అవుతుంది కాబట్టి primary voltage
will be, 11kV.

$$\therefore \quad Z_{pu}(new) = Z_{pu}(old) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.08 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{25}{25}\right)$$

$$= 0.08 \text{ pu.}$$

At Transformer-2

Calculating at H.V. side

ఉన్నట్లు High voltage side అంటే 33kV, అంటే 33kV Tr-line కి connect
అవుతుంది కాబట్టి Tr-line Base kV consider చేస్తారు.

$$\therefore \quad Z_{pu}(new) = Z_{pu}(old) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)^2$$

$$= 0.1 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{25}{30}\right) \left(\frac{25}{30}\right)$$

$$= 0.0833 \text{ pu.}$$

ఏ voltage side లో ఉంటుందో ఆ side కి connect అయితే Base kV consider చేయాలి.
for Eg: X_1^{new} kV_{old} = 11kV అయినప్పుడు, Base kV should be taken between Generator ఏ X_1^{new} కి kV_{new} గా తీసుకోవాలి. (or), Tr-line Base kV new kV గా తీసుకుంటే X_1^{new} high voltage 33kV old kV గా తీసుకోవాలి. కాబట్టి అలా తీసుకుంటే pu value change కాదు. ఇదే pu. Representation యొక్క great advantage

Motor :

Base kV for Motor = Previous Base kV × Transformation Ratio of X_2^{new}

$$= 33 \times \frac{22}{33}$$

$$= 22\text{ kV}$$

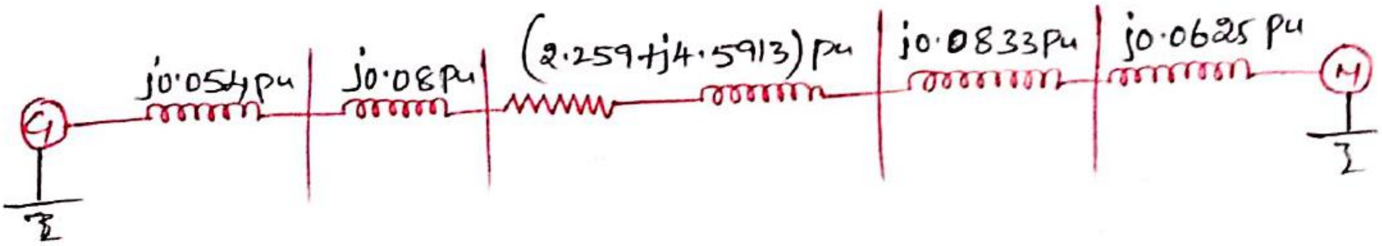
∴ Tr line నుండి Generator Motor కి ఆ direction లో because problem Base kV at Tr line ఇచ్చారు కాబట్టి. ఇక్కడ Primary voltage 33kV & 22kV Secondary voltage తీసుకుంటాం

$$\therefore Z_{pu(new)} = Z_{pu(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.15 \times \left(\frac{11}{22}\right)^2 \times \left(\frac{25}{15}\right)$$

$$= 0.0625\text{ pu.}$$

∴ Reactance diagram will be,



At Transmission line :

$$\text{Base impedance} = \frac{(\text{Base kV})^2}{\text{Base MVA}} = \frac{(33)^2}{25}$$

$$= 43.56$$

∴ pu value for Transmission line

$$\Rightarrow Z_{pu} = \frac{\text{Actual value}}{\text{Base value}}$$

$$= \frac{100 + j200}{43.56}$$

$$= 2.2956 + j\,4.5913.$$

(5) Draw the impedance diagram for the power system shown in fig. (13)

Use a base of 50 KVA, 138KV in the 40Ω line.

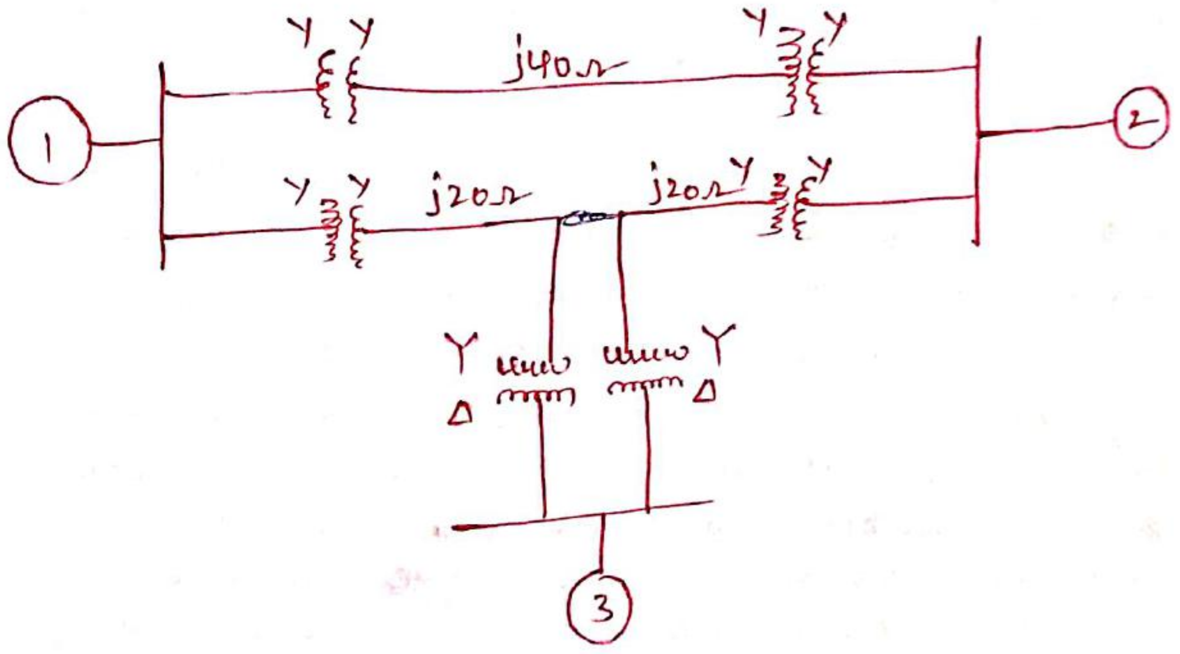
Generator-1 20MVA, 18KV, X = 20%.

Generator-2 :- 20MVA, 18KV, X = 20%.

Synchronous Motor-3:- 30MVA, 13.8KV, X = 20%.

Y-Y transformers:- 20MVA, 138/20 KV X = 10%.

Y-Δ- X^mers :- 15MVA, 138Y/13.8Δ KV, X = 10%.



Sol. Now from the Problem assume Base KVA$_{new}$ = 50, Base KV$_{new}$ = 138K in transmission line of 40Ω.

At Generator 1:-

Base KV at gen.1 = Previous KV$_{base}$ × X^{mer}-1 Ratio

$= 138 \times \frac{20}{138}$

= 20KV.

Now, X_{pu} at generator-1 $\Rightarrow X_{pu}(old) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$

$$= 0.2 \times \left(\frac{18}{20}\right)^2 \times \left(\frac{50k}{20M}\right)$$

$$= 0.000405 \text{ pu.}$$

At Y-Y X^{mer} :-

$$X_{pu}(\cancel{old})_{new} = X_{pu}(old) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.1 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50K}{20M}\right) = 0.00025 \text{ pu.}$$

Now, this value is same at all the Y-Y transformers. Since in same rated transformers are same pu. Representation. the pu values at

At transmission line :-

$$X_{pu} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$\text{Base Value} = \frac{(\text{Base kV})^2}{\text{Base MVA}} \quad (\because \text{A/c to problem})$$

$$= \frac{(138K)^2}{50K} = 0.38088 \times 10^6 \ \Omega$$

Now, X_{pu} at tr. line $= \frac{j40\Omega}{0.38088 \times 10^6 \Omega}$

$$= 0.000105 \text{ pu.}$$

The Reactance at $j40\Omega$ line $= 0.000105$ pu.

At generator-2 :-

Base KV at gen.2 = previous Base KV × $X^{mer}-2$ Ratio

$$= 138 \times \frac{20}{138} = 20\,KV.$$

$$\therefore X_{pu(new)} = 0.2 \times \left(\frac{\cancel{18}}{20}\right)^2 \times \left(\frac{50K}{20M}\right)$$

$$= 0.0004505\ pu.$$

At transmission line $-j20\Omega$:-

Base KV in 20Ω line = previous Base KV × Xmer Ratio

$$= \cancel{20} \times \frac{138}{\cancel{20}} = 138\ KV.$$

$$\therefore Z_{Base} = \frac{(Base\ kV)^2}{Base\ MVA} = \frac{(138K)^2}{50K} = 0.38088 \times 10^6$$

$$\therefore Z_{pu}\ at\ j20\Omega\ line = \frac{\cancel{0.38088 \times 10^6}}{\cancel{50}}\ \frac{j20}{0.38088 \times 10^6}$$

$$= j0.0000585\ \Omega$$

At Motor-3

Base KV = Previous KV × Xmer Ratio

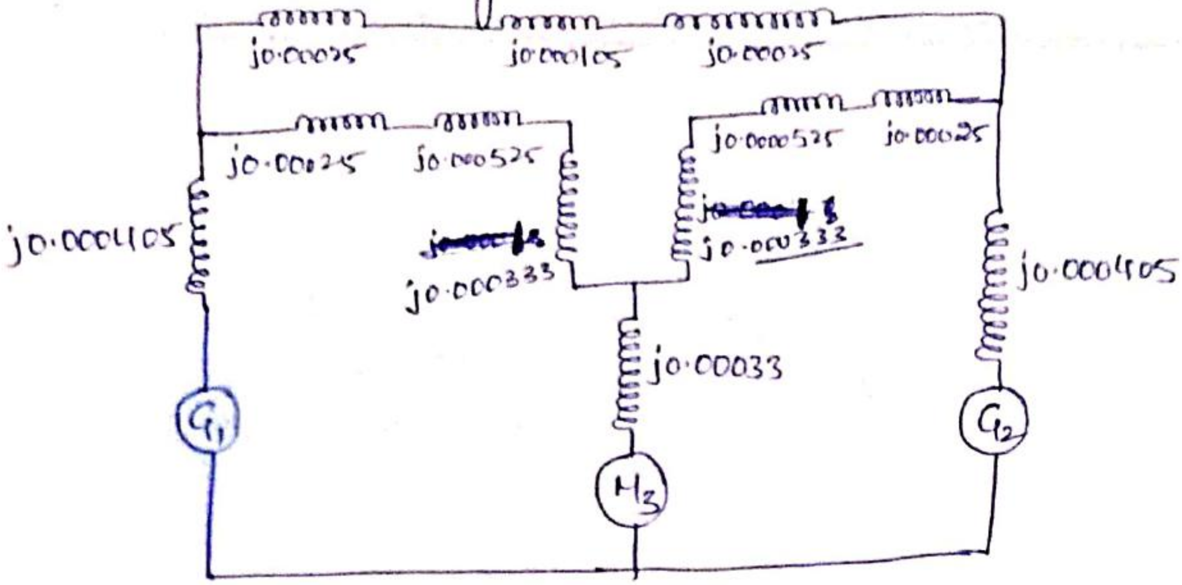
$$= \cancel{138} \times \frac{13.8}{\cancel{138}} = 13.8\ KV.$$

$$\therefore Z_{pu} = Z_{pu}(old) \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.2 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{50K}{30M}\right)$$

$$= 0.000333\ pu.$$

∴ The Reactance diagram will be,



---

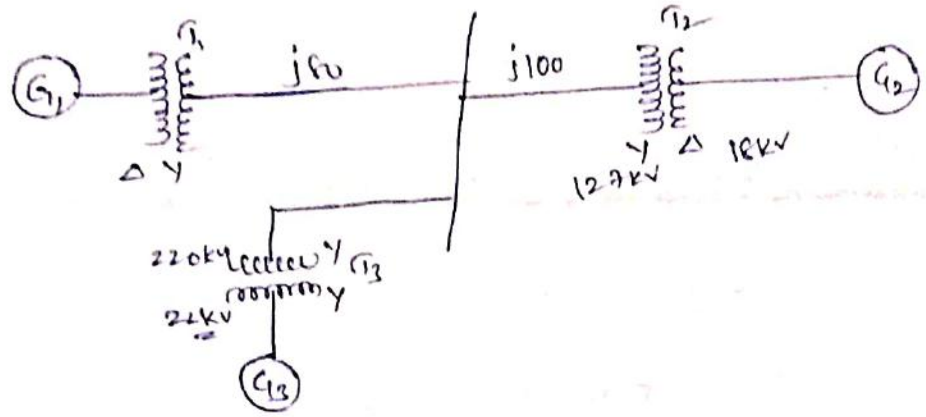
(7)

At Y-Δ transformer

$$X_{pu(new)} = X_{pu(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \frac{MVA_{new}}{MVA_{old}}$$

$$= 0.1 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{50k}{15M}\right)$$

$$= 0.000333 \text{ pu}$$



$G_1 = 20MVA,\ 13.8kV,\ X = 0.2pu$ $G_3 = 30MVA,\ 20kV,\ X = 0.2pu$

$G_2 = 30MVA,\ 18kV,\ X = 0.2pu$

$T_1 = 25MVA,\ 20kV/220kV,\ X = 0.2pu$ T_2 = Single phase units each rated 15MVA, $12.7/18kV$, $X = 10\%$

$T_3 = 35MVA,\ 220Y/22Y,\ X = 10\%$

Sol Transformer-2 is 1ϕ xmer. So, Multiply with $\sqrt{3}$ for Star side and for Δ side the voltage rating is as it Connected Voltage rating.

is,...

If a 1ϕ xmer is Y-Y Connected xmer then we have to Multiply with $\sqrt{3}$ for both Y-Connected side voltage ratings.

Δ ⇒ $V_{line} = V_{phase}$

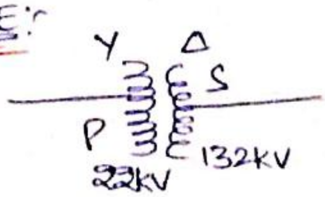
Y → $V_{line} = \sqrt{3} \times V_{phase}$

MVA Ratings:-

MVA rating of 3ϕ operation

= 3 × MVA rating of 1ϕ transformer

NOTE:-



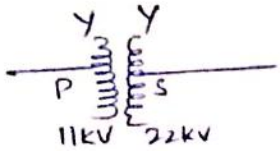
← 1ϕ xmer then, for 3ϕ transformer we have to Multiply with $\sqrt{3}$ for Y Connected winding side Voltage → $22kV \times \sqrt{3}$

and 'Δ' side as it is.

Now, the transformation Ratio for above example is,

$$T.R = \frac{132\,kV}{\sqrt{3}\times 22\,kV}$$

Similarly,

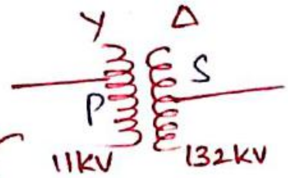


then we have to Multiply on both winding side voltages with $\sqrt{3}$

Now, the Transformation Ratio $= \dfrac{22\,kV\times\sqrt{3}}{11\,kV\times\sqrt{3}}$

ఏదైనా 1ϕ x^{mer} లో mention చేసినట్లయితే Y-Connected side voltage rating ని $\sqrt{3}$ తో Multiply చేయాలి. Δ-Connected side voltage rating అలాగే ఉంచాలి. That means we are converting 1ϕ x^{mer} to 3ϕ x^{mer}.

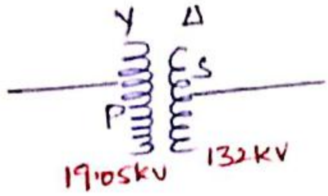
Eg:-



1ϕ x^{mer}

$\Rightarrow$ $T.Ratio = \dfrac{132\,kV}{\sqrt{3}\times 11\,kV} = \dfrac{132\,kV}{19.05\,kV}$

Now 1ϕ converted to 3ϕ x^{mer} –



***

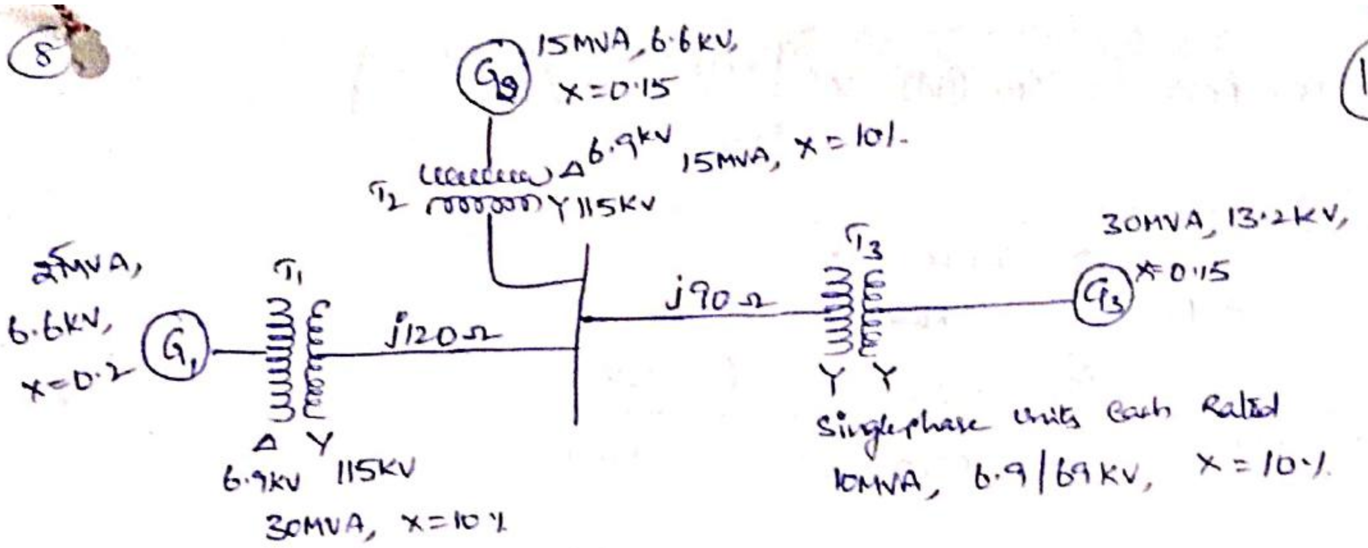
Now the old values at the x^{mer} will be 19.05 kV & 132 kV

But not 11kV and 132 kV.

* Y-Connected $\Rightarrow$ $V_{line} = \sqrt{3}\,V_{phase}$
$\Rightarrow I_{line} = I_{phase}$

Δ-Connected $\Rightarrow$ $V_{line} = V_{phase}$
$I_{line} = \sqrt{3}\,I_{phase}$

(8) (19a)



Draw an impedance diagram of 3 generator power system with the interconnection by means of 3 transformers and a transmission line with two sections. Choose Base MVA = 30 MVA

Sol. Assume the base values at the Generator-1 side

Base MVA = 30MVA. Base kV = 6.6 kV

At Generator-1 :-

$$X_{pu\,(new)} = X_{pu\,(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.2 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{30}{25}\right)$$

$$= 0.24 \text{ pu.}$$

At transmission line j120 Ω :-

Base kV = Previous Base kV × Transformer Ratio

$$= 6.6 \times \frac{115}{6.9}$$

$$= 110 \text{ kV}$$

$$\therefore X_{pu}(new) = X_{pu}(old) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$$

$$\therefore Z_{base} = \frac{(Base\ kV)^2}{Base\ MVA} = \frac{(110kV)^2}{30\ MVA}$$

$$= 403.33\ \Omega$$

$$\therefore Z_{pu} = \frac{j120}{403.33} = j0.297\ pu.$$

Similarly,

At Transmission line $j90\Omega$:-

The same Base MVA & Base kV will exists at the two sections of Transmission lines.

$$\therefore Z_{pu} = \frac{Z_{actual}}{Z_{base}}$$

$$Z_{base} = \frac{(Base\ kV)^2}{Base\ MVA} = \frac{(110)^2}{30}$$

$$= 403.33\ \Omega$$

$$\therefore Z_{pu} = \frac{j90}{403.33} = j0.2231\ pu.$$

At Transformer - 2 :-

The Base kV = 110kV will be same for both the sections as well.

$$\therefore X_{pu}(new) = X_{pu}(old) \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.1 \times \left(\frac{115}{110}\right)^2 \times \left(\frac{30}{15}\right) = 0.2185\ pu.$$

a.) Generator – 2 :- (1991)

The Base kV at Gen-2 Side = Previous kV × T/F Ratio

$$= 110\,kV \times \frac{6.9}{115}$$

$$= 6.6\ kV.$$

$$X_{pu\,(new)} = X_{pu\,(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \frac{MVA_{new}}{MVA_{old}}$$

$$= 0.15 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{30}{15}\right)$$

$$= 0.3\ pu.$$

At Transformer-3 :-

As it is 1ϕ Transformer $\frac{6.9\ Y}{69.\ Y}$

$$\Rightarrow \frac{6.9 \times \sqrt{3}}{69 \times \sqrt{3}} = \frac{11.95}{119.51}$$

Now, these 11.95 / 119.51 are the old values.

$$\therefore X_{pu} = X_{pu\,(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.1 \times \left(\frac{119.51}{110}\right)^2 \times \left(\frac{30}{30}\right)$$

[$\because$ each Rated 10 MVA for 3ϕ $\Rightarrow$ 3 × 10 MVA = 30 MVA]

$$= 0.118\ pu.$$

At Generator-3:

The Base kV at Gen. 3 side = Previous Base kV × T/F Ratio

$$= 110 \times \frac{11.95}{119.51}$$

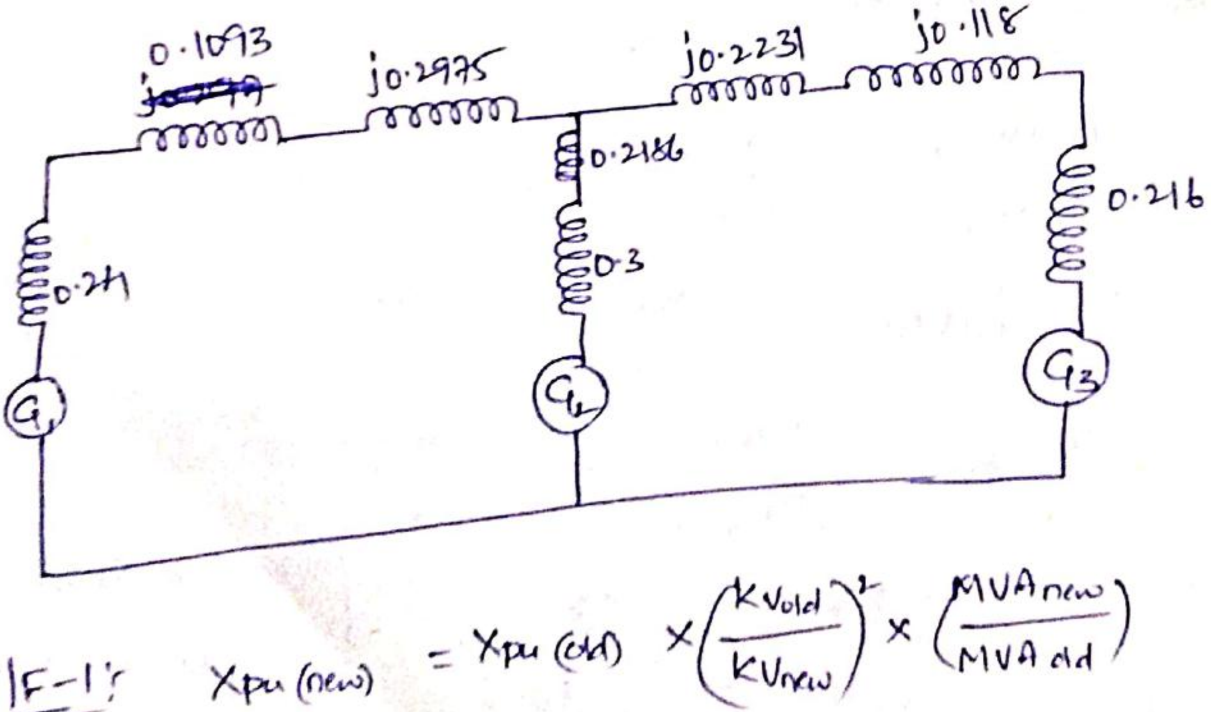
$$= 11 \text{ kV}$$

$$\therefore X_{pu\,(new)} = X_{pu\,(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.15 \times \left(\frac{13.2}{11}\right)^2 \times \left(\frac{30}{30}\right)$$

$$= 0.216 \text{ pu.}$$

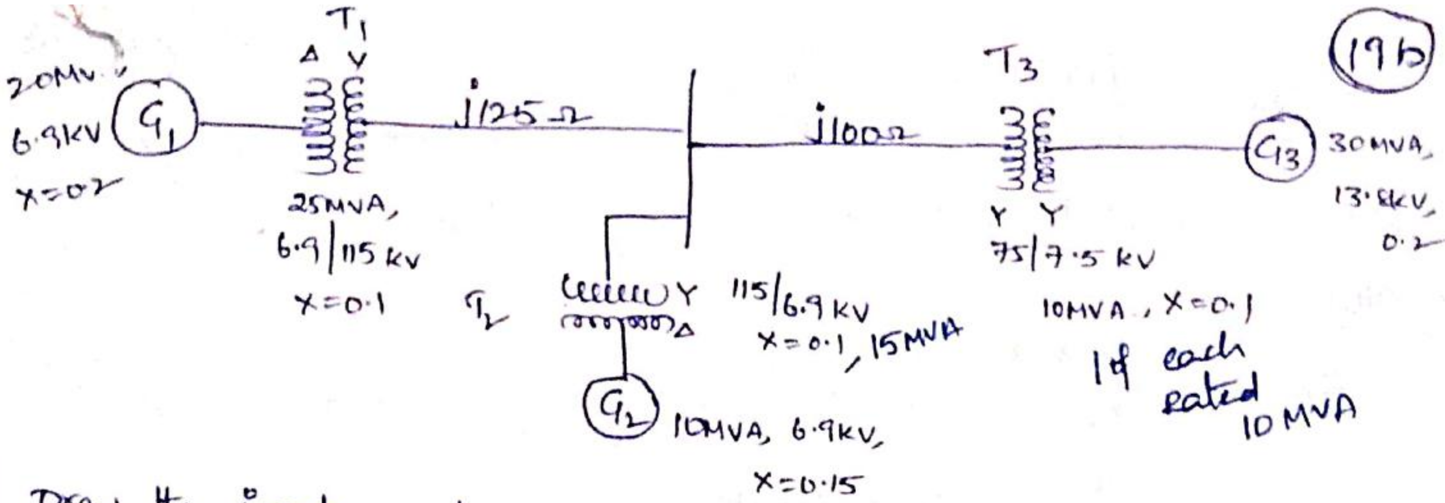
∴ The Reactance diagram will be,



At T/F-1: $X_{pu\,(new)} = X_{pu\,(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$

$$= 0.1 \times \left(\frac{6.9}{6.6}\right)^2 \times \left(\frac{30}{30}\right)$$

$$= 0.1092 \text{ pu}$$



Draw the impedance diagram, choosing the base of 20MVA, 6.9KV for Generator-1.

Sol. Base MVA = 20 MVA

KV = 6.9 kV

At Generator-1:

$$X_{pu} = X_{pu(old)} \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.2 \times \left(\frac{6.9}{6.9}\right)^2 \times \left(\frac{20}{20}\right)$$

$$= 0.2 pu.$$

At T/F-1:

$$X_{pu} = X_{pu(old)} \times \left(\frac{KV_{old}}{KV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.1 \times \left(\frac{6.9}{6.9}\right)^2 \times \left(\frac{20}{25}\right)$$

$$= 0.08 pu.$$

At T/L-1:

Base KV = Previous kV × T/F Ratio

$$= 6.9 \times \frac{115}{6.9} = 115 KV$$

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}} \Rightarrow Z_{base} = \frac{(Base\ KV)^2}{Base\ MVA} = \frac{(115)^2}{20}$$

$$= 661.25\ \Omega$$

$$\therefore Z_{pu} = \frac{j125}{661.25}$$

$$= j0.189 \text{ pu.}$$

At T/L-2:

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$$

$$= \frac{j100}{661.25} = 0.151 \text{ pu.}$$

At T/F-2:

$$X_{pu} = X_{pu\,(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.15 \times \left(\frac{115}{115}\right)^2 \times \frac{20}{15}$$

$$= 0.2 \text{ pu.}$$

At Generator-2:

$$\text{Base kV} = \text{Previous kV} \times \text{T/F Ratio}$$

$$= \cancel{115} \times \frac{6.9}{\cancel{115}}$$

$$= 6.9 \text{ kV}$$

$$X_{pu} = X_{pu\,(old)} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{old}}\right)$$

$$= 0.15 \times \left(\frac{6.9}{6.9}\right)^2 \times \left(\frac{20}{10}\right)$$

$$= 0.3 \text{ pu.}$$

At Generator-2:

Base KV = Previous KV × T/F Ratio

$$= 115 \times \frac{6.9}{115} = 6.9\,KV$$

$$X_{pu} = X_{pu}(old) \times \frac{MVA_{new}}{MVA_{old}} \times \left(\frac{KV_{old}}{KV_{new}}\right)^2$$

$$= 0.3.$$

At T/F-3:

for 3ϕ $\Rightarrow$ $\frac{7.5}{75} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{13\,KV}{130\,KV}$

$$X_{pu} = X_{pu}(old) \times \frac{MVA_{new}}{MVA_{old}} \times \frac{KV_{old}}{KV_{new}}$$

$$= 0.1 \times \left(\frac{20}{30}\right) \times \left(\frac{130}{115}\right)^2$$

$$= 0.0851\ pu.$$

At Generator-3:

Base KV = Previous Base KV × T/F Ratio

$$= 115 \times \frac{13}{130}$$

$$= 11.5\ KV$$

$$X_{pu} = X_{pu}(old) \times \frac{MVA_{new}}{MVA_{old}} \times \left(\frac{KV_{old}}{KV_{new}}\right)^2$$

$$= 0.2 \times \left(\frac{20}{30}\right) \times \left(\frac{13.8}{11.5}\right)^2$$

$$= 0.192\ pu$$

5/12/12

Unit - I

Representation of Power System Components :-

→ The balanced 3φ Networks are always solved as a single phase (or) per phase equivalent ckt comprising one of the 3-lines and a Neutral return, power System Components such as transformers generators and loads etc can be indicated by standard symbols, while a transmission line is represented by a single-line between its two ends. The diagram is simplified further by omitting the Neutral wire. Such a simplified diagram of Electric system is called single-line Diagram.

The single line Diagram gives the significant information about the system parameters, which was indicated in the single line diagram.

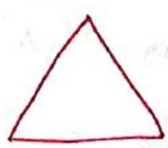
The standard symbols for power system components are given as follows.

1. Two winding transformer

2. 3-winding transformer.

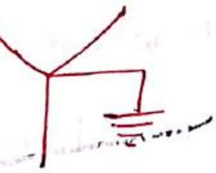
3. Rotating Machine (or) Armature

4. Three phase three wire Delta-Connection.



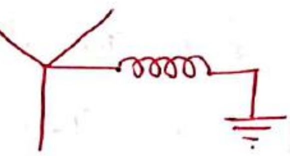
5. Three phase Star Connection with Neutral Ungrounded.

6. Three Phase Star Connection with solidly grounded



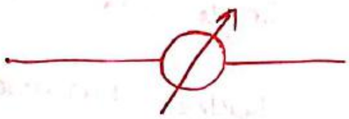
7. Three Phase Star Connection grounded with Resistance

8. Three phase Star Connection grounded with Reactor



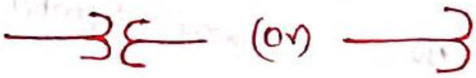
9. Air Circuit Breaker

10. phase shifting transformer.



11. Current transformer

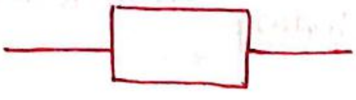
12. potential transformer



13. Voltmeter and Ammeter

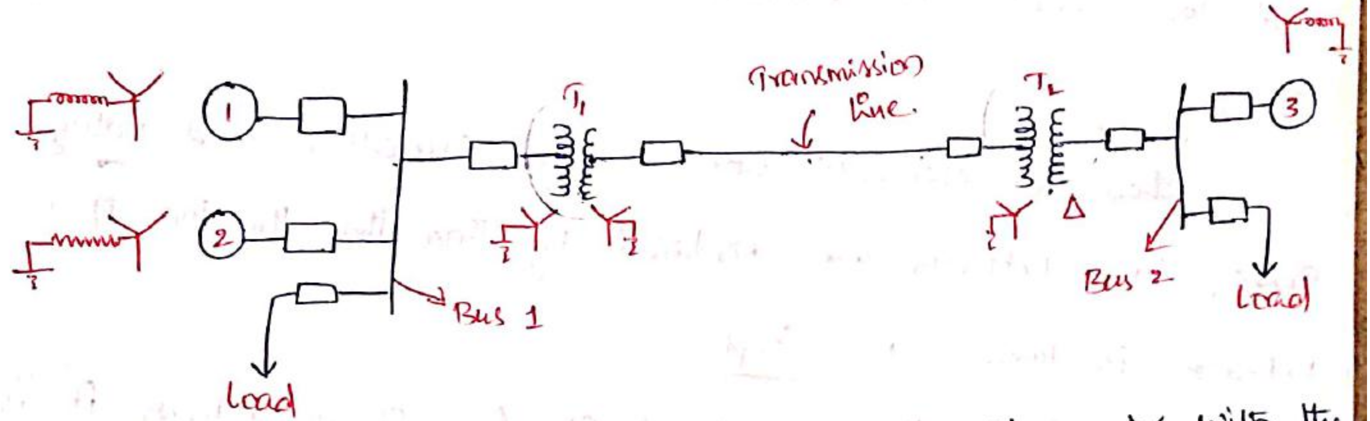
14. Fuse

15. Power circuit Breaker



The purpose of one-line diagram (or) single-line diagram is to supply in concise form the significant information about the system. The importance and the ~~of~~ different features of a system varies with the problem under consideration, and the amount of information included in the diagram depends up on the purpose for which the diagram is intended. Also, the information found on a one-line diagram must be expected to vary according to the problem.

For example the one-line diagram of sample power system is shown below



The standard symbol to designate a three phase Y with the Neutral solidly grounded is shown in above components. But from the above sample diagram it is shown that the Y connection to the generators are grounded through Resistance and Reactance. This means that when ever a fault occurs in the power system the fault current is going to be limited to the flow of ground.

Most of the transformer Neutrals in the transmission systems are solidly grounded. And the Generator Neutrals are usually grounded through fairly high Resistances and some times through the inductances to limit the fault current flow to the ground during the fault conditions.

* Graph Theory :-

Graphs are very usefull in Several fields like Engineering, physical and Social etc.. Many applications of Several electrical Components Such as Machines and Power System Components are Representing in a Simple way in graph form and for analysis of electrical circuits it plays very important role..

When the elements like Resistors, capacitors and voltage sources in a network are replaced by lines, then the type of a network is known as Graph.

To describe the geometrical structure of a network, it is Sufficient to Replace the network Components by one-line Segments irrespective of the characteristics of the components. And the line Segments are joined by means of nodes.

To Assemble a network matrix, the knowledge of physical layout of Electrical power System i.e, how the various System components such as generators, transformers and transmission lines are Connected is required.

Graph Theory is a very handy tool in describing the physical structure of a power system. Some terms Commonly used in the graph theory is defined as follows, i, Element :- An individual Component such as generator, transformer transmission lines or load which is irrespective of their characteristics is represented in a single line diagram of a power system by a single line segment is known as Element.

ii, Node :- The Terminals of an Element is known as Node

(iii) Graph :- A graph shows the physical interconnections of the elements of a Network.

(iv) Sub-Graph :- A Sub-graph is any subset of elements of a graph.

(v) path :- A path is a sub-graph of connected elements with no more than two elements connected to any one Node.

(vi) Connected graph :- If there is a path between every pair of nodes then the graph is said to be a connected graph.

(vii) Oriented graph :- when all the elements of a Connected graph are assigned directions, then the graph is called Oriented graph.

(viii) Tree :- A Tree is defined as a Connected sub-graph Constituting all the nodes but containing no closed loops.

(ix) Branch :- A Branch is an element of a Tree $(b=n-1)$

(x) Links :- Those elements of the Connected graph that are not Included in the tree are called links. $(l=e-n+1)$

(xi) Co-tree :- A Co-tree is a Sub-graph that is Constituted of links. Co-tree is a Complement of a tree.

b = branches.

$$\left[\begin{array}{l} \because b=n-1 \\ l=e-n+1 \end{array}\right]$$

n = no. of Nodes
e = " Elements
l = links

* Types of Incidence Matrices :-

1. Element node incidence Matrix (A')
2. Bus incidence Matrix. (A)
3. Branch Path incidence Matrix. (K)
4. Basic Cut-set incidence Matrix. (B)
5. Augmented Cut-set Matrix. (B')
6. Basic loop incidence Matrix. (C)
7. Augmented loop incidence Matrix. (C')

1. Element Node Incidence Matrix :-

The incidence of elements to Nodes in a Connected graph is given by Element node incidence Matrix and is denoted by (A').

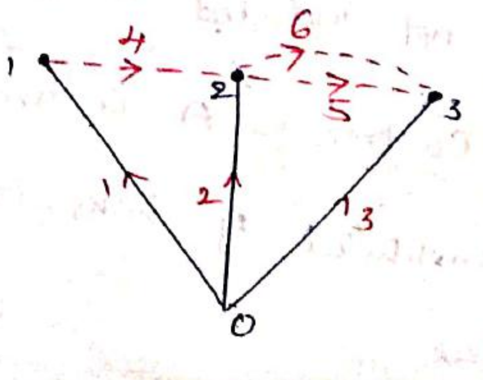
To determine the elements of A' Matrix consider a graph as shown below.

$a_{ij} = 1$, if the i^{th} element is incidence & oriented away from j^{th} node.

$a_{ij} = -1$, if the i^{th} element is incidence and oriented towards the j^{th} Node.

$a_{ij}' = 0$, if the i^{th} element is not incidence to j^{th} node.

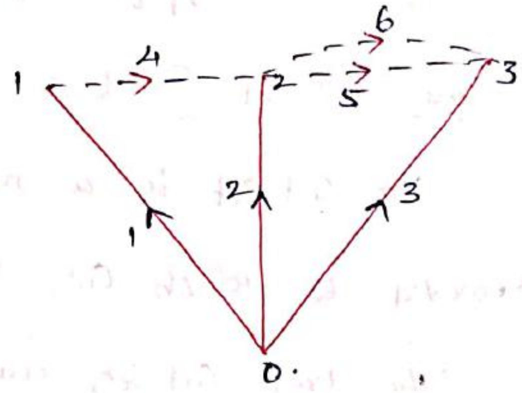
$$A' = \begin{array}{c|c} e \backslash n & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \end{array}$$



2. Bus Incidence Matrix :-

Any Node of a Connected graph can be selected as a Reference node. Then the other nodes are Reffered to as Busses. The Matrix obtained from the Element Node Matrix (A') by eliminating the Reference Node column is the Bus Incidence Matrix A.

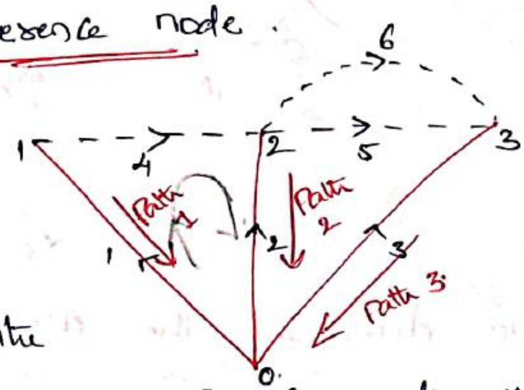
$$\therefore A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \end{array}$$



3. Branch Path Incidence Matrix :-

The branch path incidence matrix shows the incidence of branches to the paths in a tree, where a path is oriented from a bus (or) Node to the Reference node.

Now, we can determine the elements of a Matrix, K.



$K_{ij} = 1$, if i^{th} ~~node~~ branch is in the Path from j^{th} bus to Reference oriented in Same direction.

$K_{ij} = -1$, if i^{th} branch is in Path from j^{th} bus to Reference Oriented in opposite direction.

$K_{ij} = 0$, if i^{th} branch is not in path from j^{th} bus to the Reference node

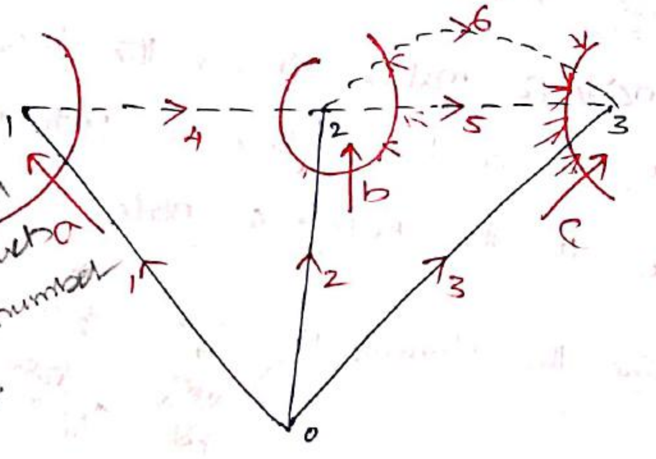
Now, the branch path incidence Matrix associated with the tree shown in above fig.

$$K = \begin{array}{c|ccc} \text{branch} \backslash \text{Path} & 1 & 2 & 3 \\ \hline 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{array} \qquad \therefore K = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

4. Basic Cut-set Incidence Matrix :-

The Cut set is a minimal set of branches of the graph, Removal of which Cuts the graph into two parts.

The Basic Cut Sets Consists of only one branch. so number of basic Cut Sets is equal to the number of branches.



'Cut set' should contain only one branch and any number of links.

[NOTE :- The basic Cut set Matrix is taken between the number of elements & the number of basic cut sets]

To determine the elements of a matrix. as follows,

B_{ij} = 1, when i^{th} branch is in Cut set j. and orientation Coincide

= -1, when i^{th} branch is in the Cut set j. and the orientation do not Coincide

= 0, when the i^{th} branch is not in the Cut Set j.

Cut set orientation will be direction of branch.

Now the Cut set Matrix B is

basic cut sets / elements	a	b	c
1	1	0	0
2	0	1	0
3	0	0	1
4	-1	1	0
5	0	-1	1
6	0	-1	1

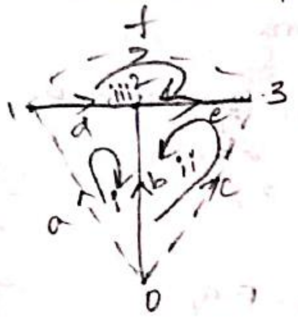
$$\therefore \underline{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

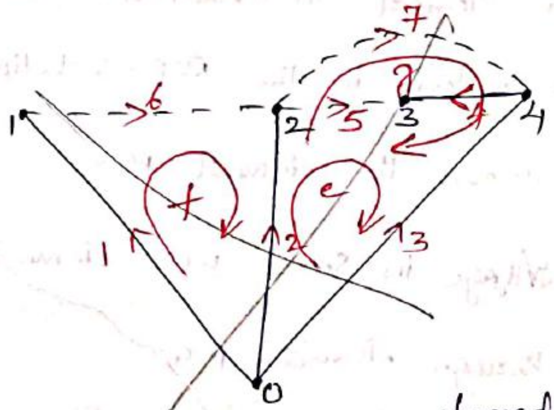
5. Basic loop Incidence Matrix :- (or) Tie-set Matrix

The basic loop Incidence matrix gives the incidence of elements to basic loops of a connected graph. It is denoted with C

Should contain only one link and any number of branches.

loop direction will be the link direction





The elements of a matrix can be formed as,

$C_{ij} = 1$, when the element i is in basic loop j and directions are same

$= -1$, when the element i is in the basic loop j and the directions are not same

$= 0$, when the element i is not in the jth loop.

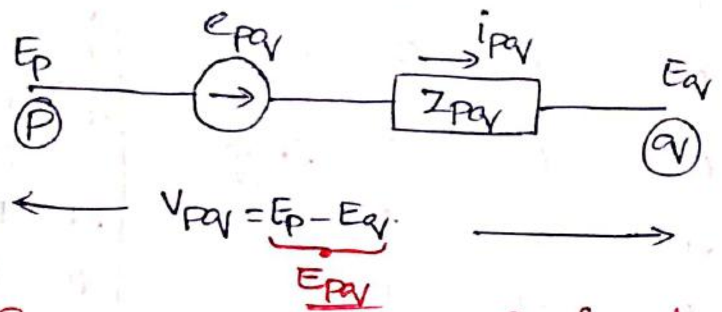
	i	ii	iii
a	1	0	0
b	-1	-1	0
c	0	1	0
d	1	0	-1
e	0	-1	-1
f	0	0	1

$C =$

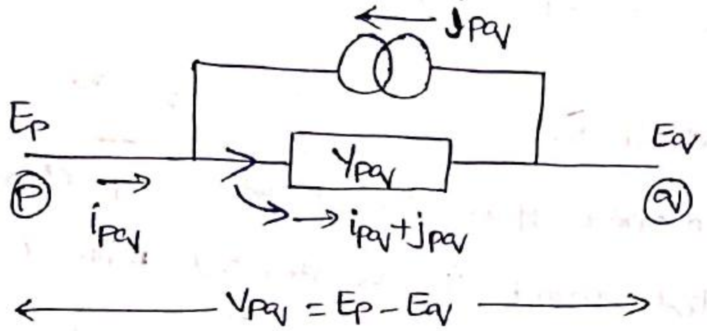
loops / elements	e	f	g
1	0	1	0
2	1	-1	1
3	-1	0	-1
4	-1	0	0
5	1	0	0
6	0	1	0
7	0	0	1

$$\therefore C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Representation of an element p-q in impedance form and Admittance form :-



(a) Representation of element in impedance form,



(b) Representation of an Element in Admittance form.

The variables and parameters in the Representation of networks are v_{pq} = Voltage across the element p-q

e_{pq} = Source voltage in series with element p-q

i_{pq} = Current through element p-q.

j_{pq} = Source current in parallel with element p-q

z_{pq} = self-impedance of element p-q

y_{pq} = self-admittance of element p-q.

Now, the performance equation of element in impedance form

$$\Rightarrow \boxed{v_{pq} + e_{pq} = z_{pq}\, i_{pq}} \quad \text{--- (1)}$$

Similarly, the performance equation of Element in Admittance form

$$\Rightarrow \boxed{i_{pq} + j_{pq} = y_{pq}\, v_{pq}} \quad \text{--- (2)}$$

* Primitive Network :-

The data obtained from any power system Network will be in the form of Primitive Network. Primitive Network is a set of the elements which provides the information regarding to the characteristics of individual elements of a particular power system Network.

Now, the performance equations of Primitive Network can be derived from the eq. ① and ② by expressing the variables as vectors and the parameters as Matrices.

A set of information Regarding to uncoupled elements is defined as primitive Network

∴ Primitive Network in impedance form is,

$$① \Rightarrow \boxed{\bar{v} + \bar{e} = [Z]\,\bar{i}} \quad \text{——} ③$$

Similarly, Primitive Network in Admittance form is,

$$② \Rightarrow \boxed{\bar{i} + \bar{j} = [Y]\,\bar{v}} \quad \text{——} ④$$

Here, from the equations ③ and ④ [Z] and [Y] are the Primitive impedance Matrix and Primitive Admittance Matrix Respectively.

The Diagonal elements of a Primitive Matrices are self impedance (or) self Admittance elements. And Rest of the diagonals are Mutual impedance (or) Mutual Admittance Elements.

Self Elements

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \qquad [Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

Mutual Elements

* Formation of Y-BUS :

The Admittance Matrix Y can be determined by using the following methods.

1. Based on the Incidence Matrices
   (a) Singular transformation method.
   (b) Non-singular transformation method.
2. Based on the Network Analysis equations.
   (a) By Direct Inspection method.

→ Singular Transformation Method :-

The branch voltages of any graph is expressed as.

$$V = A\,V_{bus}. \quad \text{——} \; (5)$$

→ This equation from the stevenson text book Page-260

where, A = Bus Incidence matrix.

from the equation (4) we get,

$$I + J = Y \cdot V \quad \text{——} \; (6)$$

$$\Rightarrow I+J = Y \cdot A \cdot V_{bus}$$

[∵ Substituted eq (5) in (6)]

Now, Pre-Multiplying A^T on both sides.

$$\Rightarrow A^T \cdot I + A^T \cdot J = A^T Y \cdot A \cdot V_{bus}. \quad \text{——} \; (A)$$

where, $A^T \cdot I$ is algebraic sum of currents at each bus.

$A^T \cdot J$ is algebraic sum of source currents at a bus.

According to Kirchoff's current law, Algebraic sum of currents at a node (or) bus is 0.

$$\therefore A^T \cdot I = 0 \cdot \quad \text{——} \; (7)$$

Similarly, algebraic sum of source currents at a bus is given by

$$A^T \cdot J = J_{Bus} \quad \text{——} \; (8)$$

Substituting ⑦ and ⑧ in Ⓐ

$\therefore$ Ⓐ $\Rightarrow J_{bus} = A^T \cdot y \cdot A\, V_{bus}$. $\quad [\because J_{bus} = I_{bus}]$

$$\therefore \frac{J_{bus}}{V_{bus}} = A^T \cdot y \cdot A.$$

$$\therefore \boxed{Y_{Bus} = A^T \cdot y \cdot A}$$ ⇒ Formation of Y_{BUS} using Singular Transformation Method.

where, y = Premitive Admittance Matrix

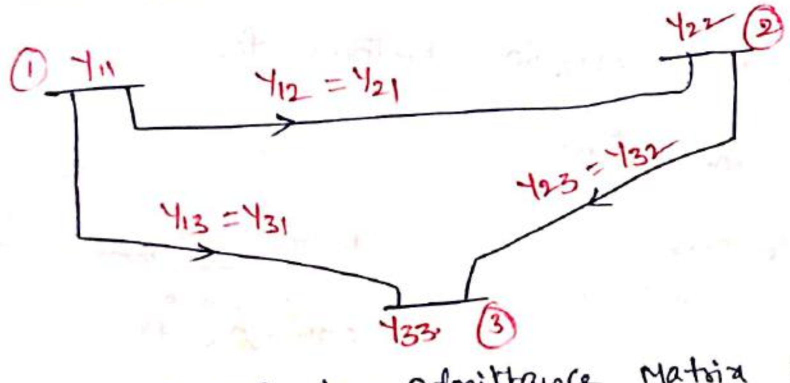
and A = Bus incidence Matrix.

A^T = Transpose of Bus incidence Matrix,

→ Direct Inspection Method :-

When there is no Mutual Coupling between the transmission lines of a networks, the bus admittance Matrix Y_{BUS} can be formulated by inspecting the Network itself.

For Example, let us Consider a 3 bus system.



But the general bus admittance Matrix of 3-bus system is,

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

Here, Y_{11} in the sense, the Sum of the admittances linked to the Bus ①.

$$\therefore Y_{11} = Y_{12} + Y_{13}.$$

Similarly, $Y_{22} = Y_{21} + Y_{23}$

$Y_{33} = Y_{31} + Y_{32}$

*** Now, the formation of Ybus is going to be framed as given below,

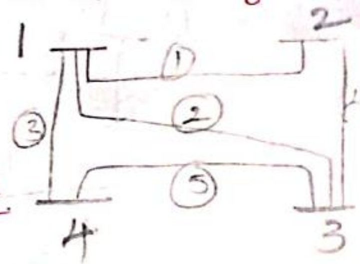
i, The diagonal elements of a Matrix to a particular bus is the sum of the admittances connected to that bus.

ii, The off-diagonal elements is the Negative of the admittance connected between the any two buses.

$$\therefore Y_{bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} \\ -Y_{21} & Y_{22} & -Y_{23} \\ -Y_{31} & -Y_{32} & Y_{33} \end{bmatrix}$$

Problems :-

① Form the Ybus by Inspection Method for a 4-bus system if the impedances are given,



Line (bus to bus)	Impedance
1. 1-2	0.15 + j0.6 pu.
2. 1-3.	0.1 + j0.4 pu
3. 1-4	0.15 + j0.6 pu.
4. 2-3	0.05 + j0.2 pu
5. 3-4	0.05 + j0.2 pu.

Sol: we are now trying to calculate the Bus Admittance matrix YBUS. Firstly, the Admittance values are to be calculated from the given Impedances values.

Line	Impedance	Admittance
1-2	0.15+j0.6 pu.	0.392+j(−1.569)
1-3	0.1+j0.4 pu	0.588−j 2.353
1-4	0.15+j0.6 pu	0.392 −j1.569
2-3	0.05+j0.2 pu	1.176 − j4.706
3-4	0.05+j0.2 pu.	1.176 − j4.706

$$\therefore Y_{11} = Y_{12} + Y_{13} + Y_{14}$$
$$= (0.392 - j1.569) + (0.588 - j2.353) + (0.392 - j1.569)$$
$$= 1.372 - j5.49$$

$$Y_{22} = Y_{12} + Y_{23}$$
$$= [0.392 + (-j1.569)] + (1.176 - j4.706)$$
$$= 1.568 - j6.275.$$

$$Y_{33} = Y_{31} + Y_{34} + Y_{32}$$
$$= (0.588 - j2.353) + (1.176 - j4.706) + (1.176 - j4.706)$$
$$= 2.94 - j11.765$$

$$Y_{44} = Y_{14} + Y_{34}$$
$$= (0.392 - j1.569) + (1.176 - j4.706)$$
$$= 1.568 - j6.275$$

$Y_{12} = -Y_{21} = -0.392 + j(1.569)$; $Y_{24} = -Y_{42} = 0.$

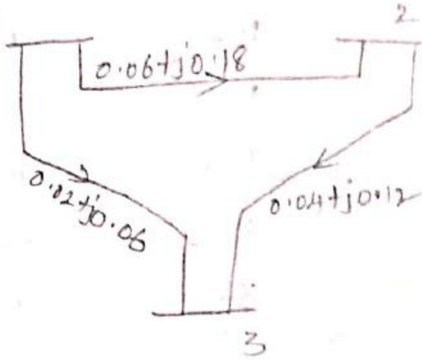
$Y_{13} = -Y_{31} = -0.588 + j2.353$; $Y_{23} = -Y_{32} = -1.176 + j4.706$

$Y_{14} = -Y_{41} = -0.392 + j1.569$; $Y_{34} = -Y_{43} = -1.176 + j4.706$

$$\therefore Y_{BUS} = \begin{bmatrix} 1.372 - j5.491 & -0.392 + j1.569 & -0.588 + j2.353 & -0.392 + j1.569 \\ -0.392 + j1.569 & 1.568 - j6.275 & -1.176 + j4.706 & 0 \\ -0.588 + j2.353 & -1.196 + j4.706 & 2.94 - j11.765 & -1.176 + j4.706 \\ -0.392 + j1.569 & 0 & -1.176 + j4.706 & 1.568 - j6.275 \end{bmatrix}$$

(2) Calculate the Y_{BUS} from the given data.

Bus to Bus	Impedance
1 - 2	0.06+j0.18
1-3	0.02+j0.06
2-3	0.04+j0.12



Sol: given data,

Bus to Bus	Impedance	Admittance
1-2	0.06+j0.18	1.667-j5
1-3	0.02+j0.06	5-j15
2-3	0.04+j0.12	2.5-j7.5

$$Y_{11} = Y_{12}+Y_{13} \qquad ; Y_{12} = -Y_{21} = -1.667+j5$$

$$=(1.667-j5) + (5-j15) \qquad ; Y_{13} = -Y_{31} = -5+j15$$

$$= 6.667-j20.$$

$$Y_{22} = Y_{12}+Y_{23} \qquad ; Y_{23} = -Y_{32} = -2.5+j7.5$$

$$=(1.667-j5) + 2.5+(-j7.5)$$

$$= 4.1667-j12.5$$

$$Y_{33} = Y_{13}+Y_{23}$$

$$=(5-j15)+(2.5-j7.5)$$

$$= 7.5-j22.5$$

$$\therefore Y_{BUS} = \begin{bmatrix} 6.667-j20 & -1.667+j5 & -5+j15 \\ -1.667+j5 & 4.1667-j12.5 & -2.5+j7.5 \\ -5+j15 & -2.5+j7.5 & 7.5+j(-22.5) \end{bmatrix}$$

(3) Calculate the Y_{BUS} for the above problem's data when the line charging Admittance also given.

Bus to Bus	Impedance	Bus Code	charging Y.
1-2	0.06+j0.18	1	j0.05
1-3	0.02+j0.06	2	j0.06
2-3	0.04+j0.12	3	j0.05

Sol. when ever a line charging Admittance is given in the problem of Formation of Y_{BUS}. Add the Admittance values to the only the Diagonal Elements of the Y_{BUS}. Diagonal elements in the sense Y_{11}, Y_{22}, Y_{33} .....

In the above problem it is directly given that for the Bus-1 the charging Admittance value is j0.05. similarly, for Bus-2 ⇒ j0.06 and for Bus-3 it is j0.05

Now, the Diagonal elements are,

$$Y_{11} = 6.667 - j20 + j0.05$$
$$= 6.667 - j19.95$$

$$Y_{22} = 4.1667 - j12.5 + j0.06$$
$$= 4.1667 - 12.44j$$

$$Y_{33} = 7.5 - j22.5 + j0.05$$
$$= 7.5 - j22.45$$

[The Remaining Elements are same]

Now, the Y_{BUS} ⇒

$$Y_{BUS} = \begin{bmatrix} 6.667 - j19.95 & -1.667 + j5 & -5 + j15 \\ -1.667 + j5 & 4.1667 - j12.44 & -2.5 + j7.5 \\ -5 + j15 & -2.5 + j7.5 & 7.5 - j22.45 \end{bmatrix}$$

4. Form the Y-Bus by Inspection Method for a 4-bus System

Bus to Bus	Line Impedance	Charging Admittance
1-2	0.2+j0.8	j0.02
2-3	0.3+j0.9	j0.03
2-4	0.25+j1	j0.04
3-4	0.2+j0.8	j0.02
1-3	0.1+j0.4	j0.01

Sol. The Admittances for the given impedance values.

$$y_{12} = \frac{1}{0.2+j0.8} = 0.294 - j1.176 = -y_{21}$$

$$y_{23} = \frac{1}{0.3+j0.9} = 0.333 - j1 = -y_{32}$$

$$y_{24} = \frac{1}{0.25+j1} = 0.235 - j0.941 = -y_{42}$$

$$y_{34} = \frac{1}{0.2+j0.8} = 0.294 - j1.176 = -y_{43}$$

$$y_{13} = \frac{1}{0.1+j0.4} = 0.588 - j2.353 = -y_{31}$$

for the Bus ① the total Charging Admittance is

$$y_1 = (y_{12} + y_{13}) = j0.02 + j0.01 = j0.03.$$

Similarly, $y_2 = (y_{23} + y_{24} + y_{12}) = j0.03 + j0.04 + j0.02 = j0.09$

$$y_3 = (y_{23} + y_{34} + y_{13}) = j0.02 + j0.01 + j0.03 = j0.06$$

$$y_4 = (y_{24} + y_{34}) = j0.04 + j0.02 = j0.06.$$

NOTE:- Charging Admittance (total) values are added to only $Y_{11}, Y_{22}, Y_{33}, Y_{44}$ because we are calculating total charging Admittances at the Buses itself

$\therefore Y_{11} = (y_{12} + y_{13} + y_1)$

$= (0.294 - j1.176) + (0.588 - j2.353) + j0.03$

$= 0.882 - j3.499$

$Y_{22} = (y_{12} + y_{23} + y_{24}) + y_2$

$= (0.294 - j1.176) + (0.333 - j1) + (0.235 - j0.941) + j0.09$

$= 0.862 - j3.027$

$Y_{33} = (y_{23} + y_{34} + y_{13}) + y_3$

$= (0.333 - j1) + (0.294 - j1.176) + (0.588 - j2.353) + j0.06$

$= 1.215 - j4.469$

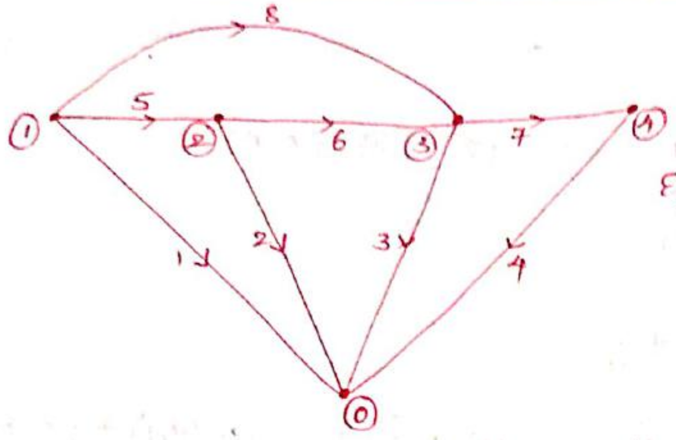
$Y_{44} = (y_{24} + y_{34}) + y_4$

$= (\cancel{0.294 - j1.176}\ 0.235 - j0.941) + (\cancel{0.588 - j2.353}\ 0.294 - j1.176) + 0.06$

$= \cancel{0.882 - j3.469}\ 0.529 - j2.057$

$$\therefore Y_{BUS} = \begin{bmatrix} 0.882 - j3.499 & -0.294 + j1.176 & \cancel{-0.333+j1}\ -0.588 + j2.353 & 0 \\ -0.294 + j1.176 & 0.862 - j3.027 & -0.333 + j1 & \cancel{-0.294+j1.176}\ -0.235 + j0.941 \\ -0.588 + j2.353 & -0.333 + j1 & 1.215 - j4.469 & -0.294 + j1.176 \\ 0 & -0.235 + j0.941 & -0.294 + j1.176 & 0.529 - j2.057\ \cancel{0.882 - j3.469} \end{bmatrix}$$

(5) For the figure and the impedance values are provided below. All the impedances values are in p.u. values. Determine Y_{BUS} using Direct Inspection method.



Element	Bus code	Self impedances
1	0-1	0.1
2	0-2	0.2
3	0-3	0.25
4	0-4	0.5
5	1-2	0.1
6	2-3	0.4
7	3-4	0.3
8	1-3	0.6.

Sol. From the above figure and the data available from the table; we need to calculate the Y_{Bus} by Direct Inspection Method.

The self Admittances are, $Y_{11} = Y_{01} + Y_{12} + Y_{13}$

$$= \frac{1}{0.1} + \frac{1}{0.1} + \frac{1}{0.6}$$

$$= 10 + 10 + 1.667 = 21.667$$

$$Y_{22} = Y_{02} + Y_{12} + Y_{23}$$

$$= \frac{1}{0.2} + \frac{1}{0.1} + \frac{1}{0.4} = 5 + 10 + 2.5 = 17.5.$$

$$Y_{33} = Y_{03} + Y_{23} + Y_{13} + Y_{34}$$

$$= \frac{1}{0.25} + \frac{1}{0.4} + \frac{1}{0.3} + \frac{1}{0.3} = 4 + 2.5 + 3.333 + 3.333 = 11.5 \checkmark$$

$$Y_{44} = Y_{04} + Y_{34}$$

$$= \frac{1}{0.5} + \frac{1}{0.3} = 2 + 3.333 = 5.333.$$

$Y_{12} = Y_{21} = \frac{-1}{0.1} = -10.$; $Y_{13} = -Y_{31} = \frac{-1}{0.6} = -1.67$

$Y_{14} = Y_{41} = 0$; $Y_{23} = -Y_{32} = \frac{-1}{0.4} = -2.5$; $Y_{24} = 0. = -Y_{42}$

$Y_{34} = -Y_{43} = \frac{-1}{0.3} = -3.333.$

$$\therefore Y_{BUS} = \begin{bmatrix} 21.67 & -10 & -1.67 & 0 \\ -10 & 17.5 & -2.5 & 0 \\ -1.67 & -2.5 & 11.5 & -3.33 \\ 0 & 0 & -3.33 & 5.33 \end{bmatrix}$$

6. Determine the Y_{BUS} formation for the above problem using Singular Transformation method.

Sol. With the help of singular Transformation method

$$Y_{BUS} = A^T Y A$$

where, Y = primitive incidence Matrix

A = Bus incidence Matrix.

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

[$\because$ from the directed graph of above problem]

$$A^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$Y = Z^{-1} = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 \end{bmatrix}^{-1}$$

$$\therefore Y = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.67 \end{bmatrix}$$

∴ Y_{BUS} formation using singular Transformation Method is $A^T Y A$

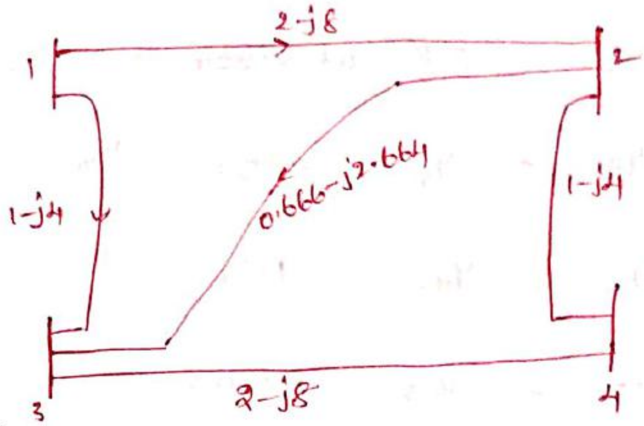
$$\Rightarrow Y_{BUS} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.67 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

∴ In Y_{BUS} all the Diagonal elements are of positive and the rest of elements are with Negative sign.

$$\Rightarrow \therefore Y_{BUS} = \begin{bmatrix} 21.67 & -10 & -1.67 & 0 \\ -10 & 17.5 & -2.5 & 0 \\ -1.67 & -2.5 & 11.5 & -3.33 \\ 0 & 0 & -3.33 & 5.33 \end{bmatrix}$$

∴ <u>NOTE</u>: From this the <u>Y_{BUS}</u> should be same for both the inspection & Transformation <u>Methods</u>. But not in <u>All Cases</u>.

7. Form the Y_{BUS} for the 4-bus System shown below. The Admittances are in p.u. Values.



Sol given, the admittances are in p.u. values. Now calculate the self Admittances and Mutual Admittances of a 4-bus Network to form the Y_{BUS}.

Bus Code	Admittances
1-2	2-j8
1-3	1-j4
2-3	0.666-j2.664
2-4	1-j4
3-4.	2-j8

$\therefore Y_{11} = Y_{12} + Y_{13}$

$= (2-j8) + (1-j4) = 3-j12$

$Y_{22} = Y_{23} + Y_{24} + Y_{21}$

$= (0.666-j2.664) + (1-j4) + (2-j8)$

$= 1.666 - j6.664 + (2-j8)$

$= 3.666 - j14.664.$

$Y_{33} = Y_{13} + Y_{23} + Y_{34}$

$= (1-j4) + (0.666-j2.664) + (2-j8)$

$= 3.666 - j14.664.$

$Y_{44} = Y_{24} + Y_{34}$

$= (1-j4) + (2-j8) = 3-j12$

Y_{BUS} is known as 'Sparse Matrix' while, Z_{BUS} is known as 'Full Matrix'

⇒ Sparsity :- Containing more no. of Zero Elements in an obtained System Matrix

* If Sparsity & no. of busses is given then Min. no of tr. lines Required to be Calculated as,

$t = \frac{n^2(1-x) - n}{2}$

where, n = no. of busses

x = Sparsity in %.

t = tr. lines

$Y_{12} = -Y_{21} = -(2-j8) = -2+j8$

$Y_{23} = -0.666+j2.664 = -Y_{32}$

$Y_{13} = -Y_{31} = -1+j4$; $Y_{14} = -Y_{41} = 0.$

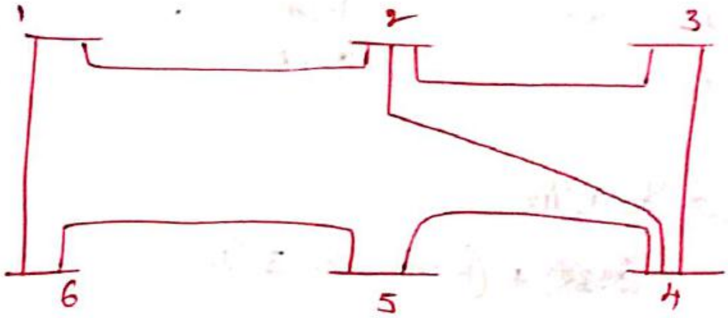
$Y_{24} = -Y_{42} = -1+j4.$

$Y_{34} = -Y_{43} = -2+j8$

$$\therefore Y_{BUS} = \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

(8) Construct the Y_{BUS} by Singular Transformation Method. For the given Network.

Sol:



Element	Reactances
1-2	0.04
1-6	0.06
2-4	0.03
2-3	0.02
3-4	0.08
4-5	0.06
5-6	0.05

Sol: given Elements are in Reactances. There by the Primitive impedance Matrix 'Z' is given by,

$$Z = \begin{bmatrix} j0.04 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j0.06 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j0.03 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j0.08 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j0.06 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & j0.05 \end{bmatrix}$$

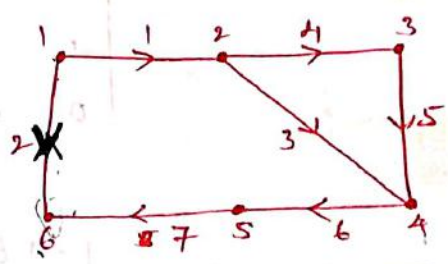
Now, the Admittance Primitive Matrix y is given by,

$$y = [Z]^{-1} = \begin{bmatrix} j0.04 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j0.06 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j0.03 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j0.08 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j0.06 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & j0.05 \end{bmatrix}^{-1}$$

By Singular Transformation Method $Y_{BUS} = A^T y \cdot A$

where, A is Bus incidence Matrix.

$$\therefore A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



[Considered "6" as Reference]

$$\therefore Y_{BUS} = A^T y A \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} [y].$$

$$\Rightarrow A^T Y = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} -j \begin{bmatrix} 25 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16.67 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 33.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix}$$

$$\Rightarrow A^T . Y = -j \begin{bmatrix} 25 & 16.67 & 0 & 0 & 0 & 0 & 0 \\ -25 & 0 & 33.33 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 12.5 & 0 & 0 \\ 0 & 0 & -33.33 & 0 & -12.5 & 16.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & -16.67 & 20 \end{bmatrix}$$

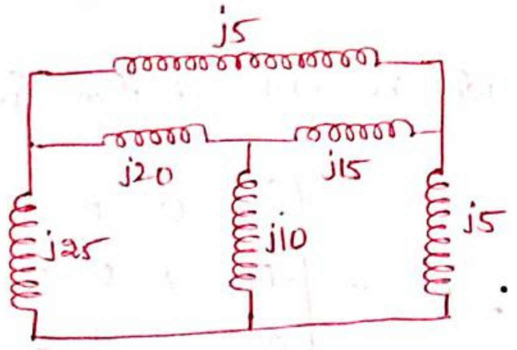
$$\therefore Y_{BUS} = A^T . Y . A$$

$$= -j \times \begin{pmatrix} 25 & 16.67 & 0 & 0 & 0 & 0 & 0 \\ -25 & 0 & 33.33 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 12.5 & 0 & 0 \\ 0 & 0 & -33.33 & 0 & -12.5 & 16.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & -16.67 & 20 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\therefore Y_{BUS} = -j \times \begin{bmatrix} 41.67 & -25 & 0 & 0 & 0 \\ -25 & 108.33 & -50 & -33.33 & 0 \\ 0 & -50 & 62.5 & -12.5 & 0 \\ 0 & -33.33 & -12.5 & 62.5 & -16.67 \\ 0 & 0 & 0 & -16.67 & 36.67 \end{bmatrix}$$

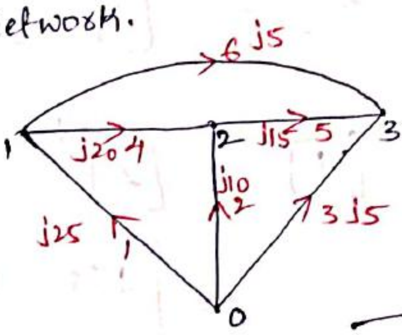
Note:- According to the bus that we are considering it as Reference, the Y_{BUS} is going to be Determined

(10) Determine Y_{Bus} by Direct Inspection Method and by using Singular Transformation Method for the following Network. impedances are in p.u. values.



Sol. given impedances are in p.u. values.

If we draw a graph for the above Network.



Now by Inspection method.

$$Y_{11} = Y_{10} + Y_{12} + Y_{13}$$

$$= \frac{1}{j25} + \frac{1}{j20} + \frac{1}{j5}$$

$$= -j0.04 - j0.05 - j0.2 = -j0.29$$

What ever may be the directions in a graph, it doesn't affect the overall Y_{Bus}

$$Y_{22} = Y_{21} + Y_{20} + Y_{23}$$

$$= \frac{1}{j20} + \frac{1}{j10} + \frac{1}{j15}$$

$$= -j0.05 - j0.1 - j0.0666 = -j0.2166.$$

Either from 1,2,3 to 0 (or) 0 to 1,2,3 The Y_{Bus} will be same. I have Verified

$$Y_{33} = Y_{30} + Y_{31} + Y_{32}$$

$$= \frac{1}{j5} + \frac{1}{j5} + \frac{1}{j15}$$

$$= -j0.2 - j0.2 - j0.0666 = -j0.4666$$

$Y_{12} = Y_{21} = -j0.05$, $Y_{13} = Y_{31} = -j0.2$

$j_{23} = Y_{32} = -j0.0666$,

$$\therefore Y_{BUS} = \begin{bmatrix} -j0.29 & j0.05 & j0.2 \\ j0.05 & -j0.2166 & j0.0666 \\ j0.2 & j0.0666 & -j0.4666 \end{bmatrix}$$

Now by Singular Transformation Method.

$$Y_{BUS} = A^T [Y] A$$

A = Bus Incidence Matrix, will be obtained from Graph.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} ; \quad A^T = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix}$$

$$[Y] = [Z]^{-1}$$

$$= \begin{bmatrix} j25 & 0 & 0 & 0 & 0 & 0 \\ 0 & j10 & 0 & 0 & 0 & 0 \\ 0 & 0 & j5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j20 & 0 & 0 \\ 0 & 0 & 0 & 0 & j15 & 0 \\ 0 & 0 & 0 & 0 & 0 & j5 \end{bmatrix}^{-1}$$

$$= -j \times \begin{bmatrix} 0.04 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.066 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}$$

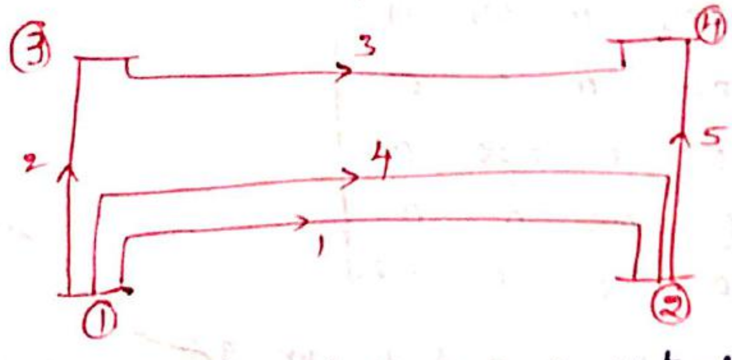
$$\therefore Y_{BUS} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} -j \times \begin{bmatrix} 0.04 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.066 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= -j\begin{bmatrix} -0.04 & 0 & 0 & 0.05 & 0 & 0.2 \\ 0 & -0.1 & 0 & -0.05 & 0.066 & 0 \\ 0 & 0 & -0.2 & 0 & -0.066 & -0.2 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\therefore\ Y_{BUS} = -j\begin{bmatrix} 0.29 & -0.05 & -0.2 \\ -0.05 & 0.2166 & -0.066 \\ -0.2 & -0.0666 & 0.466 \end{bmatrix}$$

(9) Determine the Y_{BUS} Matrix by singular Transformation Method

Element	Self Impedance		Mutual Impedance	
	Bus-Code	Impedance	Bus Code	Impedance
1	1-2 (1)	0.2	—	—
2	1-3	0.4	1-2 (1)	0.05
3	3-4	0.5	—	—
4	1-2 (2)	0.25	1-2 (1)	0.1
5	2-4	0.2	—	—



Sol: given self Impedances and Mutual Impedances.

Now, from the above oriented Network directly calculate the Bus Incidence Matrix, by taking (1) as Reference.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} ; \quad A^T = \begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}$$

Now Primitive Impedance Matrix.

$$Z = \begin{array}{c|ccccc} & 1\text{-}2(1) & 1\text{-}3 & 3\text{-}4 & 1\text{-}2(2) & 2\text{-}4 \\ \hline 1\text{-}2(1) & 0.2 & 0.05 & 0 & 0.1 & 0 \\ 1\text{-}3 & 0.05 & 0.4 & 0 & 0 & 0 \\ 3\text{-}4 & 0 & 0 & 0.5 & 0 & 0 \\ 1\text{-}2(2) & 0.1 & 0 & 0 & 0.25 & 0 \\ 2\text{-}4 & 0 & 0 & 0 & 0 & 0.2 \end{array}$$

$$[Y] = [Z]^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

To Calculate the inverse of 5x5 Matrix

Interchange the rows 3 and 4 we get,

$$\Rightarrow [Y] = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

Similarly Interchange columns 3 and 4 we get

$$\Rightarrow [Y]' = \left[\begin{array}{ccc|cc} 0.2 & 0.05 & 0.1 & 0 & 0 \\ 0.05 & 0.4 \; A_1 & 0 & 0 & 0 \; A_2 \\ 0.1 & 0 & 0.25 & 0 & 0 \\ \hline 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 \; A_3 & 0 & 0 & 0.2 \; A_4 \end{array}\right]^{-1}$$

Now Calculate the Inverses for the Sub-Matrices of above Matrix we get,

$$A_1^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0.1 \\ 0.05 & 0.4 & 0 \\ 0.1 & 0 & 0.25 \end{bmatrix}^{-1} = \begin{bmatrix} 6.5 & -0.81 & -2.6 \\ -0.81 & 2.60 & 0.32 \\ -2.6 & 0.32 & 5.04 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}.$$

$$[Y]' = \left[\begin{array}{ccc|cc} 6.5 & -0.81 & -2.6 & 0 & 0 \\ -0.81 & 2.60 & 0.32 & 0 & 0 \\ -2.6 & 0.32 & 5.04 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{array}\right]$$

To obtain the original Primitive Admittance Matrix, Again interchange the Rows and Coloumns of 3 & 4. to their original positions of $[Y]'$.

$$\therefore [Y] = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ -0.81 & 2.6 & 0 & 0.32 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -2.6 & 0.33 & 0 & 5.04 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Now, $Y_{BUS} = A^T [Y] A$

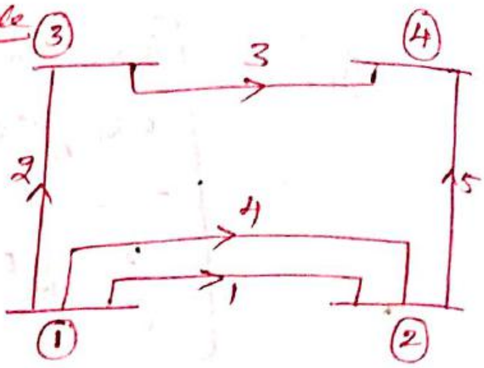
$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ -0.81 & 2.6 & 0 & 0.32 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -2.6 & 0.33 & 0 & 5.04 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

Therefore, on symplifying we get,

$$Y_{BUS} = \begin{bmatrix} 11.3 & -0.48 & -5 \\ \begin{matrix} -0.48 \\ -\cancel{3.077} \end{matrix} & 4.6 & -2 \\ -5 & -2 & 7 \end{bmatrix}$$

(11) Determine the Y_{BUS} using singular Transformation Method

Element	Self impedance		Mutual impedance	
1	1-2 (1)	→ 0·6	—	—
2	1-3	→ 0·5	1-2 (1)	0·1
3	3-4	→ 0·5	—	—
4	1-2 (2)	→ 0·4	1-2 (1)	0·2
5	2-4	→ 0·2	—	—



Sol. Same as above problem,

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \; ; \; A^T = \begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}$$

Now, the Primitive Admittance Matrix.

$$[Z] = \begin{bmatrix} 0·6 & 0·1 & 0 & 0·2 & 0 \\ 0·1 & 0·5 & 0 & 0 & 0 \\ 0 & 0 & 0·5 & 0 & 0 \\ 0·2 & 0 & 0 & 0·4 & 0 \\ 0 & 0 & 0 & 0 & 0·2 \end{bmatrix}^{-1}$$

Now, Interchange the Rows and columns of 3 and 4 we get,

$$\Rightarrow Y = \begin{bmatrix} 0·6 & 0·1 & 0 & 0·2 & 0 \\ 0·1 & 0·5 & 0 & 0 & 0 \\ 0·2 & 0 & 0 & 0·4 & 0 \\ 0 & 0 & 0·5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0·2 \end{bmatrix}^{-1}$$

⇒ Interchanged the 3,4 Rows.

Interchanging the 3,4 columns. ⇒

$$\Rightarrow Y = \left[\begin{array}{ccc|cc} 0·6 & 0·1 & 0·2 & 0 & 0 \\ 0·1 & 0·5 & 0 & 0 & 0 \\ 0·2 & 0 & 0·4 & 0 & 0 \\ \hline 0 & 0 & 0 & 0·5 & 0 \\ 0 & 0 & 0 & 0 & 0·2 \end{array}\right]^{-1}$$

A_1, A_2, A_3, A_4

Now, $A_1^{-1} = \begin{bmatrix} 0.6 & 0.1 & 0.2 \\ 0.1 & 0.5 & 0 \\ 0.2 & 0 & 0.4 \end{bmatrix}^{-1} = \begin{bmatrix} 2.08 & -0.416 & -1.041 \\ -0.416 & 2.08 & 0.208 \\ -1.041 & 0.208 & 3.024 \end{bmatrix}$

$A_4^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$

$$\therefore \; Y' = \begin{bmatrix} 2.08 & -0.416 & -1.04 & 0 & 0 \\ -0.416 & 2.08 & 0.208 & 0 & 0 \\ -1.041 & 0.208 & 3.024 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Again Interchanging the Rows & Columns of 3 & 4 to get its original position

Now, Interchanging both the Rows and Columns of 3 & 4 we get,

$$Y = \begin{bmatrix} 2.08 & -0.416 & 0 & -1.04 & 0 \\ -0.416 & 2.08 & 0 & 0.208 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -1.041 & 0.208 & 0 & 3.024 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Now, $Y_{BUS} = A^T [Y] A$

$$\Rightarrow \underbrace{\begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}}_{3\times5} \underbrace{\begin{bmatrix} 2.08 & -0.416 & 0 & -1.04 & 0 \\ -0.416 & 2.08 & 0 & 0.208 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -1.04 & 0.208 & 0 & 3.024 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}}_{5\times5} \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}}_{5\times3}$$

on Simplification

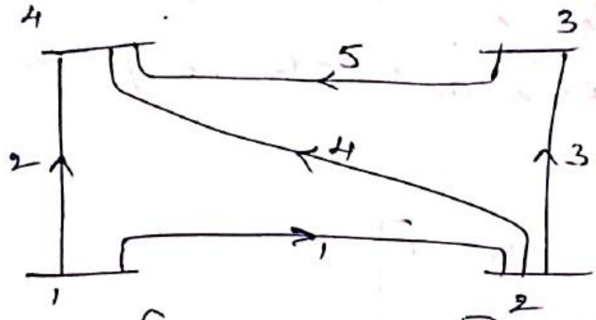
$$\Rightarrow Y_{BUS} = \underbrace{\begin{bmatrix} 8.024 & -0.208 & -5 \\ -0.208 & 4.08 & -2 \\ -5 & -2 & 7 \end{bmatrix}}_{3\times3}$$

(12) Determine the Y_{Bus} using Singular Transformation Method for a given data. Take (1) as Reference.

Element	Self		Mutual	
1	1-2 → j0.5		–	–
2	1-4 → j0.4		–	–
3	2-3 → j0.3		1-2 (1)	j0.2
4	2-4 → j0.5		–	–
5	3-4 → j0.2		–	–

Sol. from the given data,

the Network will be as below.



$$\Rightarrow A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & -1 & -1 \end{bmatrix}$$

$$[Y] = [Z]^{-1} = \begin{array}{c} \\ 1\text{-}2 \\ 1\text{-}4 \\ 2\text{-}3 \\ 2\text{-}4 \\ 3\text{-}4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1\text{-}2 & 1\text{-}4 & 2\text{-}3 & 2\text{-}4 & 3\text{-}4 \end{array} \\ \begin{bmatrix} j0.5 & 0 & j0.2 & 0 & 0 \\ 0 & j0.4 & 0 & 0 & 0 \\ j0.2 & 0 & j0.3 & 0 & 0 \\ 0 & 0 & 0 & j0.5 & 0 \\ 0 & 0 & 0 & 0 & j0.2 \end{bmatrix}^{-1} \end{array}$$

$$\Rightarrow [Y] = [Z]^{-1} = -j \left[\begin{array}{ccc|cc} 0.5 & 0 & 0.2 & 0 & 0 \\ 0 & 0.4 \; (A_1) & 0 & 0 & A_2 \; 0 \\ 0.2 & 0 & 0.3 & 0 & 0 \\ \hline 0 & 0 \; (A_3) & 0 & 0.5 & 0 \; (A_4) \\ 0 & 0 & 0 & 0 & 0.2 \end{array}\right]^{-1}$$

$$\therefore A_1^{-1} = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0 & 0.4 & 0 \\ 0.2 & 0 & 0.3 \end{bmatrix}^{-1} = \begin{bmatrix} 2.7 & 0 & -1.81 \\ 0 & 2.5 & 0 \\ -1.81 & 0 & 4.54 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\therefore\ y = -j \begin{bmatrix} 2.7 & 0 & -1.81 & 0 & 0 \\ 0 & 2.5 & 0 & 0 & 0 \\ -1.81 & 0 & 4.54 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\therefore\ Y_{BUS} = A^T [y] A$$

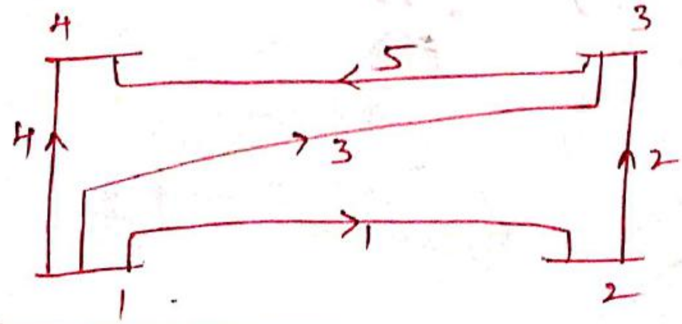
$$= \begin{bmatrix} -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & -1 & -1 \end{bmatrix} -j \begin{bmatrix} 2.7 & 0 & -1.81 & 0 & 0 \\ 0 & 2.5 & 0 & 0 & 0 \\ -1.81 & 0 & 4.54 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$= -j \begin{bmatrix} 12.88 & -6.35 & -6.53 \\ -6.35 & 9.54 & 6.81 \\ -6.52 & 6.81 & 2.22 \end{bmatrix}$$

(13) Find the Y_{BUS} by using Singular transformation method. Take 2 as Reference

Element	Self	
1	1-2 →	0.3
2	2-3 →	0.4
3	1-3 →	0.3
4	1-4 →	0.2
5	3-4 →	0.5

Sol. from the above given data, the Networks will be,



Now, find Bus Incidence matrix, primitive admittance matrices to find Y_{BUS}.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad ; \quad A^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$$[Y] = [Z]^{-1} = \begin{bmatrix} 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}^{-1}$$

$$\therefore [Y] = \begin{bmatrix} 3.33 & 0 & 0 & 0 & 0 \\ 0 & 2.5 & 0 & 0 & 0 \\ 0 & 0 & 3.33 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

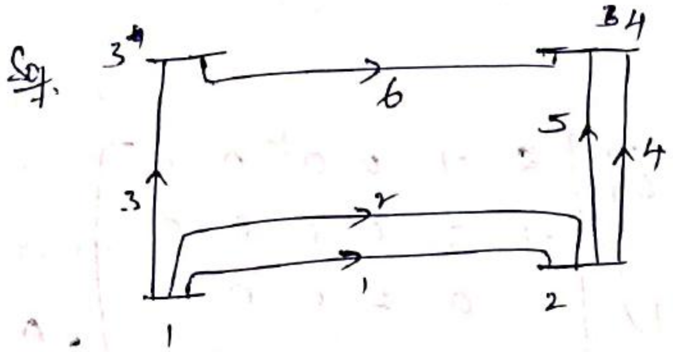
Now, $Y_{BUS} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 3.33 & 0 & 0 & 0 & 0 \\ 0 & 2.5 & 0 & 0 & 0 \\ 0 & 0 & 3.33 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$

$$\therefore Y_{BUS} = \begin{bmatrix} 11.66 & -3.33 & -5 \\ -3.33 & 7.833 & -2 \\ -5 & -2 & 7 \end{bmatrix}$$

(14) Find the Y_{BUS} using singular Transformation Method from the given data

Element	Self	Mutual		
1	1-2 → 0.6	—	—	Take (1) As Reference
2	1-2 → 0.4	1-2 (1)	0.2	
3	1-3 → 0.5	—	—	
4	2-4 → 0.2	—	—	
5	2-4 → 0.4	2-4 (4)	0.1	
6	3-4 → 0.5	—	—	

Sol.



$$\therefore A = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

$$[Y] = [Z]^{-1} = \begin{array}{c} \\ 1-2 \\ 1-2 \\ 1-3 \\ 2-4 \\ 2-4 \\ 3-4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1-2 & 1-2 & 1-3 & 2-4 & 2-4 & 3-4 \end{array} \\ \left[\begin{array}{ccc|ccc} 0.6 & 0.2 & 0 & 0 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0.2 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{array}\right] \end{array}$$

A_1, A_2, A_3, A_4

$$A_1^{-1} = \begin{bmatrix} 0.6 & 0.2 & 0 \\ 0.2 & 0.4 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.4 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}^{-1} = \begin{bmatrix} 5.7 & -1.4 & 0 \\ -1.4 & 2.85 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$Y_{BUS} = A^T [Y] A$

$$\therefore [Y] = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.7 & -1.42 & 0 \\ 0 & 0 & 0 & -1.42 & 2.85 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}_{6\times 6}$$

(∵ Substituting the Sub-matrices in the Main Matrix. (Their inverses))

$$\therefore Y_{BUS} = A^T (Y) A$$

$$= \begin{bmatrix} -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}_{3\times 6} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.7 & -1.42 & 0 \\ 0 & 0 & 0 & -1.42 & 2.85 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}_{6\times 6} \cdot A$$

$$= \begin{bmatrix} -1 & -2 & 0 & 4.3 & 1.45 & 0 \\ 0 & 0 & -2 & 0 & 0 & 2 \\ 0 & 0 & 0 & -4.3 & -1.45 & -2 \end{bmatrix}_{3\times 6} \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}_{6\times 3}$$

$$\therefore Y_{BUS} = \begin{bmatrix} 8.75 & 0 & -5.75 \\ 0 & 4 & -2 \\ -5.75 & -2 & 7.75 \end{bmatrix}_{3\times 3}$$

24/12/17

Unit-II

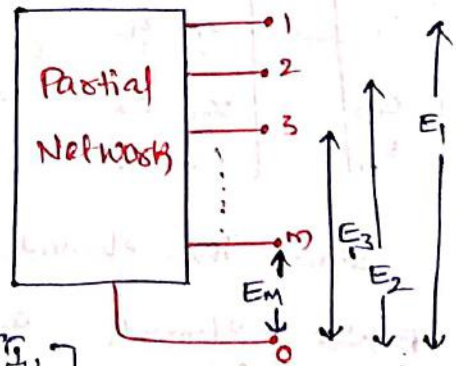
In previous chapter we have discussed about Graph Theory and Formation of Network Matrices by Direct Inspection and the Singular Transformation Methods. With this techniques there are some disadvantages. The main drawback is as the number of busses increases the determination of Network Matrices will be some what difficult.

To overcome the above drawbacks another method is proposed in the year 1960. This is known as Building Algorithm Method. The step by step procedure to determine the Z bus Matrix is known as Building Algorithm Method.

If YBUS is calculated from the Network, Zbus can be obtained by finding the inverse of the YBUS. The Bus Impedance Matrix can be found directly by using Building Algorithm. is shown below

***Performance equation of Partial Network:**

Assume that Zbus is known for a Partial Network of m busses and a Reference Node 0.



Now, the performance equation of this matrix is,

$$E_{bus} = Z_{bus} \cdot I_{bus}$$

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_m \end{bmatrix}_{m\times 1} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} \\ \vdots & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mm} \end{bmatrix}_{m\times m} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_m \end{bmatrix}_{m\times 1}$$

When an element p-q is added to the Network it may be Branch or a link.

If p-q is a branch, a new bus q is added to the Network and the order of Zbus becomes $(m+1) \times (m+1)$.

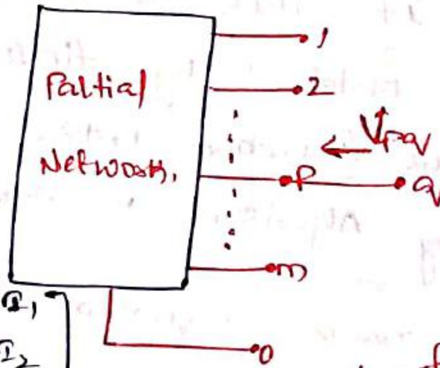
Similarly the orders of Ebus and Ibus become $(m+1) \times 1$.

Now, to Determine the New bus impedance matrix, it requires only the calculation of elements in the new row and column.

If p-q is a link, no new bus is added to the network. Here, the order of a Matrix will not changes but the elements of Zbus will be calculated to include the effect of added. link.

* Addition of Branch :-

If a element p-q is added as a Branch to the partial Network as shown below.



Then the performance equation to the newly added branch will be,

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix}$$

Derivations from Hemalatha and Jayachrista

Since the elements in the network consists of Bilateral passive Elements. So, $Z_{qi} = Z_{iq}$

where, $i = 1,2,3 \ldots m$
$\neq q$

The element Z_{qi} can be determined by injecting a current at the i^{th} bus and calculating the voltage at the q^{th} bus with respect to the Reference bus.

$$\therefore E_1 = Z_{1i} I_i$$

$$E_2 = Z_{2i} I_i$$

$$\vdots$$

$$E_p = Z_{pi} I_i$$

$$E_m = Z_{mi} I_i$$

$$E_q = Z_{qi} I_i.$$

Also, it is assumed that the added branch p-q is mutually coupled to one or more elements ρ-σ of the Partial Network.

Now, the currents in the Added element p-q and mutually coupled element ρ-σ is given by the Primitive Matrix as,

$$\begin{bmatrix} i_{pq} \\ i_{\rho\sigma} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & y_{pq,\rho\sigma} \\ y_{\rho\sigma,pq} & y_{\rho\sigma,\rho\sigma} \end{bmatrix} \begin{bmatrix} v_{pq} \\ v_{\rho\sigma} \end{bmatrix} \quad \text{---} \ (1)$$

since, pq subscript Refers to added element and the subscript ρσ Refers to Mutual element.

where,

i_{pq}, v_{pq} are Current through and voltage across the Added element

$i_{\rho\sigma}, v_{\rho\sigma}$ are Current through and voltage across the Mutual element

$y_{pq,pq}$ is the Self-Admittance of Added element

$y_{pq,\rho\sigma}$ are Mutual Admittance of p-q and ρ-σ.

As we know that, the voltage across the element p-q is given by, $v_{pq} = E_p - E_q$

Similarly, the voltage across the Mutual element is given by $v_{\rho\sigma} = E_\rho - E_\sigma$.

and the Current through the element p-q is 0.

$$\therefore i_{pq} = 0.$$

[why because to determine ~~Zp~~ Z_{qi} the Current injection is done at the i^{th} bus and there is no Relation between i^{th} bus and q^{th} bus

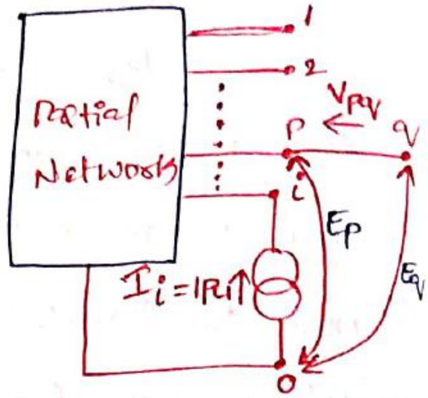
So, thereby ~~Zp~~ $i_{pq} = 0$. $\left[\because i = 1,2,\ldots n \neq q\right]$]

Now from the equation (1)

$$\Rightarrow i_{pq} = y_{pq,pq} v_{pq} + y_{pq,\rho\sigma} v_{\rho\sigma} = 0$$

$$\Rightarrow v_{pq} = \frac{-y_{pq,\rho\sigma} \cdot v_{\rho\sigma}}{y_{pq,pq}}$$

$$= \frac{-y_{pq,\rho\sigma}(E_\rho - E_\sigma)}{y_{pq,pq}}$$



$$\Rightarrow E_p - E_q = \frac{-y_{pq,\rho\sigma}(E_\rho - E_\sigma)}{y_{pq,pq}}$$

$$\Rightarrow E_q = E_p + \frac{y_{pq,\rho\sigma}(E_\rho - E_\sigma)}{y_{pq,pq}}$$

Now, $E_q = Z_{qi} I_i$, $E_p = Z_{pi} I_i$ etc

If $I_i = 1$ pu then $E_q = Z_{qi}$, $E_p = Z_{pi}$ .... Similarly $E_\rho = Z_{\rho i}$, $E_\sigma = Z_{\sigma i}$

Now. $$\boxed{Z_{qi} = Z_{pi} + \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq,pq}}}$$ → when 'p' is not the Reference bus and with Mutual Coupling.

where, $i = 1, 2, 3, \ldots, m$ — (2)
$\neq q$

Similarly, the element Z_{qq} can be calculated by injecting a current at the q^{th} bus and to calculate the voltage at that bus.

$$\therefore E_1 = Z_{1q} I_q$$

$$E_2 = Z_{2q} I_q$$

$$\vdots$$

$$E_p = Z_{pq} I_q$$

$$\vdots$$

$$E_m = Z_{mq} I_q$$

$$E_q = Z_{qq} I_q.$$

Now Similarly, $V_{pq} = E_p - E_q$

and $V_{p\sigma} = E_p - E_\sigma$

Now Coming to i_{pq}, the direction of i_{pq} is in Reverse direction. So, thereby assuming $i_{pq} = -1$, and assuming $I_q = 1$ p.u.

Now, from eq. ①

$$\Rightarrow i_{pq} = Y_{pq,pq} V_{pq} + Y_{pq\,p\sigma} V_{p\sigma}$$

$$\Rightarrow -1 = Y_{pq,pq} V_{pq} + Y_{pq\,p\sigma} V_{p\sigma}$$

$$\Rightarrow -1 = Y_{pq\cdot pq}(E_p - E_q) + Y_{pq\,p\sigma}(E_p - E_\sigma)$$

$$\Rightarrow \therefore E_q = E_p + \frac{1 + Y_{pq,p\sigma}(E_p - E_\sigma)}{Y_{pq,pq}}$$

$$\Rightarrow \boxed{Z_{qq} = Z_{pq} + \frac{1 + Y_{pq,p\sigma}(E_p - E_\sigma)}{Y_{pq,pq}}} \quad \text{— ③}$$

$\left[\begin{array}{l} E_p = Z_{pq} \\ E_\sigma = Z_{\sigma q} \end{array}\right]$

If there is no mutual Coupling between the added branch then ② and ③ will be,

$$\left.\begin{array}{ll} Z_{qi} = Z_{pi}, & i = 1, 2, \ldots m \\ & \neq q \\ Z_{qq} = Z_{pq} + Z_{pq,pq} & \end{array}\right\} \text{— ④}$$

even if 'p' is the Reference Node and there is no mutual Coupling then,

$Z_{pi} = 0$ $\quad i = 1, 2, \ldots m$
$\neq q$

and hence, $Z_{qi} = 0$

$\therefore Z_{pq} = 0$ $\quad$ [∴ from ④]

and $Z_{qq} = Z_{pq,pq}$.

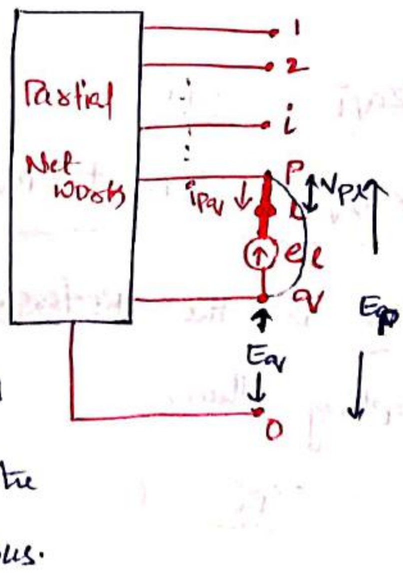
* Addition of a link :-

If the added element p-q is a link, then the procedure for recalculating the elements of the Bus impedance Matrix is to connect in series with added element a voltage source e_l

This creates a fictitious node l which will be eliminated later. The voltage source e_l is selected such that the current through the added link is zero.

The performance equation of the partial network with the added element p-l and source series voltage e_l is,

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ e_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \cdots & Z_{1p} \cdots & Z_{1m} & Z_{1l} \\ Z_{21} & Z_{22} \cdots & Z_{2p} \cdots & Z_{2m} & Z_{2l} \\ \vdots & & & & \\ Z_{p1} & Z_{p2} \cdots & Z_{pp} \cdots & Z_{pm} & Z_{pl} \\ \vdots & & & & \\ Z_{m1} & Z_{m2} \cdots & Z_{mp} \cdots & Z_{mm} & Z_{ml} \\ Z_{l1} & Z_{l2} \cdots & Z_{lp} \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \quad \text{---} (1)$$



Since, the voltage source e_l is connected between the nodes l and q.

$$\therefore \boxed{e_l = E_l - E_q}$$

Now, the element Z_{li} is determined by injecting a current at the i^{th} bus and calculating the voltage at l^{th} bus w.r.t. q^{th} bus.

from the equation (1) we get,

$$e_l = Z_{li} I_i \quad \text{---} (2)$$

Similarly, $E_1 = Z_{1i} I_i$

$E_2 = Z_{2i} I_i$

$\vdots$

$E_m = Z_{mi} I_i$

If $I_i = 1$ pu, Z_{li} can be calculated by calculating e_l.

But from the Partial Network.

$$e_l = E_p - E_q - V_{pl} \quad \text{——— (3)}$$

Since, the current through the added link $i_{pl} = 0$. and the element p-l is treating as a branch. The current in this element in terms of Primitive Admittances and the voltage across the element is,

$$i_{pl} = Y_{pl,pl} V_{pl} + Y_{pl,\rho\sigma} V_{\rho\sigma} \qquad (\because i_{pl} = 0)$$

$$\Rightarrow V_{pl} = \frac{-Y_{pl,\rho\sigma} V_{\rho\sigma}}{Y_{pl,pl}}$$

since, $Y_{pl,\rho\sigma} = Y_{pq,\rho\sigma}$ and $Y_{pl,pl} = Y_{pq,pq}$.

$$\therefore V_{pl} = \frac{-Y_{pq\,\rho\sigma} V_{\rho\sigma}}{Y_{pq\,pq}} \quad \text{——— (4)}$$

Now substituting (2) and (4) in eq. (3).

$$\Rightarrow Z_{li} = E_p - E_q - \left[\frac{-Y_{pq\,\rho\sigma}(E_\rho - E_\sigma)}{Y_{pq\,pq}}\right]$$

$$\therefore \boxed{Z_{li} = Z_{pi} - Z_{qi} + \frac{Y_{pq\,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{Y_{pq,pq}}}$$

where, $i = 1, 2, 3, \ldots, n$

$\neq l$

Now the element Z_{ll} can be calculated by injecting a current at the l^{th} bus and calculating the voltage at that bus with respect to q^{th} bus as Reference from the equation ① we get,

$$e_l = Z_{ll} I_l \quad \text{——} ⑤$$

If $I_l = 1 \cdot pu$ and $i_{\rho l} = -1$ then the current interms of Primitive admittances and the voltage across the element is,

$$\Rightarrow i_{\rho l} = Y_{\rho l \rho l} V_{\rho l} + Y_{\rho l \rho \sigma} V_{\rho \sigma}$$

$$\Rightarrow -1 = Y_{\rho l \rho l} V_{\rho l} + Y_{\rho l \rho \sigma} V_{\rho \sigma}$$

$$\Rightarrow V_{\rho l} = \frac{-1 - Y_{\rho l \rho \sigma} V_{\rho \sigma}}{Y_{\rho l \rho l}}$$

As we know that, $Y_{\rho l \rho l} = Y_{pq\,pq}$ and $Y_{\rho l \rho \sigma} = Y_{pq\,\rho \sigma}$.

$$\therefore V_{\rho l} = -\frac{(1 + Y_{pq\,\rho\sigma} V_{\rho\sigma})}{Y_{pq,pq}} \quad \text{——} ⑥$$

But from the partial Networks $e_l = E_p - E_q - V_{\rho l}$ —— ⑦

$$\therefore e_l = E_p - E_q + \frac{(1 + Y_{pq\,\rho\sigma} V_{\rho\sigma})}{Y_{pq\,pq}}$$

from ⑤

$$\Rightarrow Z_{ll} = E_p - E_q + \frac{(1 + Y_{pq\,\rho\sigma} V_{\rho\sigma})}{Y_{pq\,pq}} \quad (\because I_l = 1 \cdot pu)$$

$$\Rightarrow Z_{ll} = Z_{\rho l} - Z_{ql} + \frac{(1 + Y_{pq\,\rho\sigma} V_{\rho\sigma})}{Y_{pq\,pq}}$$

$$\therefore Z_{ll} = Z_{\rho l} - Z_{ql} + \left(\frac{1 + Y_{pq\,\rho\sigma}(E_\rho - E_\sigma)}{Y_{pq\,pq}}\right)$$

$$\therefore \boxed{Z_{ll} = Z_{\rho l} - Z_{ql} + \frac{\left[1 + Y_{pq\,\rho\sigma}(Z_{\rho l} - Z_{\sigma l})\right]}{Y_{pq\,pq}}}$$

If there is no Mutual Coupling

$$\therefore \boxed{Z_{li} = Z_{pi} - Z_{qi} \quad \text{for } i = 1,2,\dots,m \neq l}$$

$$\text{and} \quad \boxed{Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq,pq}}$$

If 'p' is the Reference bus and there is no mutual coupling

$Z_{pi} = 0$. and $Z_{pl} = 0$.

$$\therefore \boxed{Z_{li} = -Z_{qi}.}$$

$$\text{and} \quad \boxed{Z_{ll} = -Z_{ql} + Z_{pq,pq}.}$$

After Determining Z_{bus} (including the link), the Modified Z_{bus} (elimination of l^{th} node) is given as,

$$\boxed{Z_{ij}\,(\text{Modified}) = Z_{ij}\,(\text{original}) - \frac{Z_{il} \cdot Z_{lj}}{Z_{ll}}}.$$

where, $i, j = 1, 2, \dots, m$.

* Formula for Z-Bus Building Algorithm :-

→ with mutual Coupling when 'p' is not the Reference Bus.

when the added element is a Branch.

1. $Z_{qi} = Z_{pi} + \dfrac{Y_{pq,\rho\sigma}\,(Z_{\rho i} - Z_{\sigma i})}{Y_{pq,pq}}$ for $i = 1, 2, 3, \dots, m \neq q$.

2. $Z_{qq} = Z_{pq} + \dfrac{1 + Y_{pq,\rho\sigma}\,(Z_{\rho q} - Z_{\sigma q})}{Y_{pq\text{-}pq}}$

when 'p' is the Reference Bus.

3. $Z_{qi} = \dfrac{Y_{pq,p\sigma}(Z_{pi} - Z_{\sigma i})}{Y_{pq,pq}}$ for $i = 1, 2, \ldots, m$ $\neq q$

4. $Z_{qq} = \dfrac{1 + Y_{pq,p\sigma}(Z_{pq} - Z_{\sigma q})}{Y_{pq,pq}}$

* For the added element is a link when p is not Reference

5. $Z_{li} = Z_{pi} - Z_{qi} + \dfrac{Y_{pq,p\sigma}(Z_{pi} - Z_{\sigma i})}{Y_{pq,pq}}$ for $i = 1, 2, 3, \ldots, m$ $\neq l$

6. $Z_{ll} = Z_{pl} - Z_{ql} + \dfrac{1 + Y_{pq,p\sigma}(Z_{pl} - Z_{\sigma l})}{Y_{pq,pq}}$

when 'p' is Reference Bus.

7. $Z_{li} = -Z_{qi} + \dfrac{Y_{pq,\sigma p}(Z_{pi} - Z_{\sigma i})}{Y_{pq,pq}}$ for $i = 1, 2, 3, \ldots, m$ $\neq l$

8. $Z_{ll} = -Z_{ql} + \dfrac{1 + Y_{pq,p\sigma}(Z_{pl} - Z_{\sigma l})}{Y_{pq,pq}}$

* With No mutual Coupling when 'p' is NOT Reference bus.

$$\left.\begin{aligned} &9.\quad Z_{qi} = Z_{pi} \quad \text{for } i = 1, 2, 3, \ldots, m \neq q \\ &10.\quad Z_{qq} = Z_{pq} + Z_{pq,pq} \end{aligned}\right\} \text{for Branch}$$

$$\left.\begin{aligned} &11.\quad Z_{li} = Z_{pi} - Z_{qi} \quad \text{for } i = 1, 2, 3, \ldots, m \neq l \\ &12.\quad Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq,pq} \end{aligned}\right\} \text{for a link.}$$

When 'P' is Reference Bus:

13. $Z_{qi} = 0$ for $i = 1,2,3,\ldots,m$ $\neq q$ } for a Branch.

14. $Z_{qq} = Z_{pq\,pq}$

15. $Z_{li} = -Z_{qi}$ for $i = 1,2,\ldots,m$ $\neq l$ } for a link.

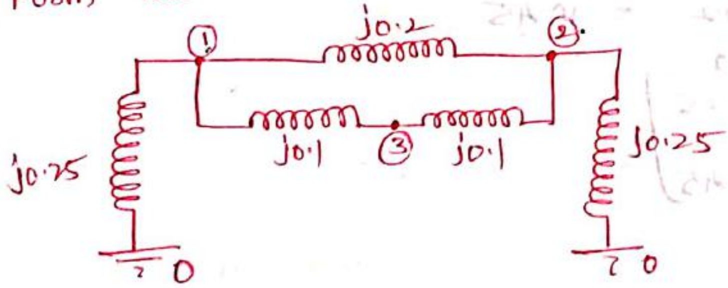
16. $Z_{ll} = -Z_{ql} + Z_{pq\,pq}$

* Modification of Elements for elimination of i^{th} node

17. $$Z_{ij}\,(\text{Modified}) = Z_{ij}\,(\text{before Modification}) - \frac{Z_{il}\,Z_{lj}}{Z_{ll}} \quad \text{for } i,j = 1,2,\ldots,m$$

* Problems :-

(1) Form the Bus Impedance Matrix for the Network shown



Now, firstly Adding an element $0-1 \Rightarrow p-q$

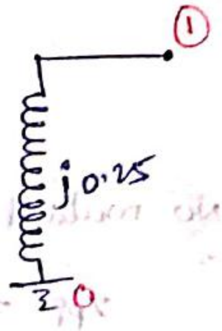
Now, firstly Adding an element and No mutual Coupling.

step-1 $\therefore p = 0,\ q = 1.$ Since it is a branch and No mutual Coupling

[$\therefore$ The Z_{BUS} for added element to be in order, 2x2]

$$Z_{BUS} = \begin{bmatrix} 00 & 01 \\ 10 & 11 \end{bmatrix}$$

$\because$ 'p' is a Reference and No mutual Coupling



$Z_{qi} = 0$ and $Z_{qq} = Z_{pq\,pq}$

$\Rightarrow Z_{11} = j0.25$

$$\therefore Z_{BUS} = \begin{pmatrix} 0 & 0 \\ 0 & j0.25 \end{pmatrix} \Rightarrow 1\,(j0.25)$$

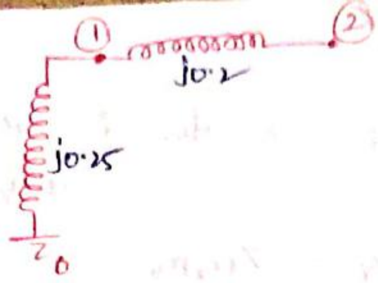
[$\therefore$ Neglecting the Ref. bus]

step-2

Now add another element $1-2 \Rightarrow p-q$

$\therefore p = 1,\ q = 2$

∵ P is not Reference bus and No mutual Coupling.



Now, Z_{bus} will be $Z_{bus} = \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix}$

$\Rightarrow Z_{BUS} = \begin{bmatrix} j0.25 & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

∵ P is not Reference bus $Z_{qi} = Z_{pi}$ for $i=1$.

$\Rightarrow \therefore Z_{21} = Z_{11}$

$\Rightarrow Z_{21} = j0.25$

$\therefore Z_{12} = Z_{21} \Rightarrow j0.25$

Now, $Z_{qq} = Z_{pq} + Z_{pq\,pq} \Rightarrow Z_{12} + Z_{12,12}$

$= j0.25 + j0.2 = j0.45$

$$\therefore Z_{BUS} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} j0.25 & j0.25 \\ j0.25 & j0.45 \end{bmatrix} \end{array}.$$

Step-3.

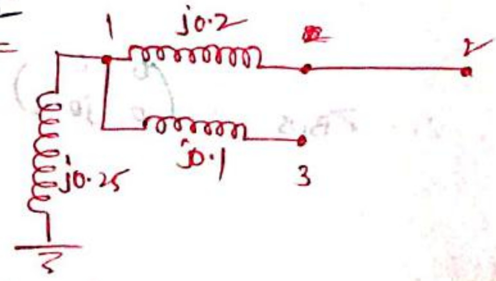
Adding element 1-3. It is also a branch $1\text{-}3 \Rightarrow p\text{-}q$

∴ $p=1$ $q=3$ and No mutual Coupling.

$$\therefore Z_{BUS} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} j0.25 & j0.25 & Z_{13} \\ j0.25 & j0.45 & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \end{array}$$

∵ No mutual Coupling and p is not Reference bus,

$Z_{pi} = Z_{qi}$ for $i = 1,2$



$\Rightarrow Z_{31} = Z_{11}$

$\Rightarrow Z_{31} = j0.25 = Z_{13}$.

Similarly, $Z_{32} = Z_{12}$

$\therefore Z_{32} = j0.25 = Z_{23}$.

$$Z_{qq} = Z_{pq} + Z_{pq,pq}$$

$$Z_{33} = Z_{13} + Z_{13,13}$$

$$= j0.25 + j0.1$$

$$\therefore \underline{Z_{33} = j0.35}$$

$$\therefore \quad Z_{BUS} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} j0.25 & j0.25 & j0.25 \\ j0.25 & j0.45 & j0.25 \\ j0.25 & j0.25 & j0.35 \end{pmatrix} \end{array}$$

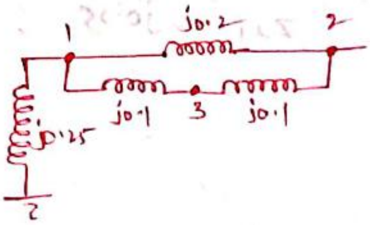
Step-4 : Now Adding element 3-2, it is a link, p-q.

$\therefore$ $p = 3$, $q = 2$ and no mutual coupling

$$Z_{BUS} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & \begin{pmatrix} j0.25 & j0.25 & j0.25 & Z_{1l} \\ j0.25 & j0.45 & j0.25 & Z_{2l} \\ j0.25 & j0.25 & j0.35 & Z_{3l} \\ Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{pmatrix} \end{array}$$

[$\because$ It is a link, i.e, This adds a fictitious node 'l'.]

Since, it is a link and no mutual coupling and p is not the Reference bus.



$$\underline{Z_{li} = Z_{pi} - Z_{qi}} \quad \text{for } \underline{i = 1,2,3}$$

$$Z_{l1} = Z_{31} - Z_{21}$$

$$= j0.25 - j0.25 = 0.$$

$$Z_{l2} = Z_{32} - Z_{22}$$

$$= j0.25 - j0.45 = -j0.2$$

$$Z_{l3} = Z_{33} - Z_{23}$$

$$= j0.35 - j0.25 = +j0.1$$

$$\underline{Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq,pq}}$$

$$= Z_{3l} - Z_{2l} + Z_{32,32}$$

$$= j0.1 - (-j0.2) + j0.1$$

$$= j0.1 + j0.2 + j0.1 = j0.4$$

$$\therefore\ Z_{BUS} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & j0.25 & j0.25 & j0.25 & j0 \\ 2 & j0.25 & j0.45 & j0.25 & -j0.2 \\ 3 & j0.25 & j0.25 & j0.35 & j0.1 \\ l & j0 & -j0.2 & j0.1 & j0.4 \end{array}$$

→ we have to eliminate the fictitious Node l

$$\boxed{Z_{ij(m)} = Z_{ij(or)} - \frac{Z_{il} Z_{lj}}{Z_{ll}}} \quad \text{for } i,j = 1,2,3.$$

$$Z_{11(m)} = Z_{11(ori)} - \frac{Z_{1l} Z_{l1}}{Z_{ll}}$$

$$= j0.25 - \frac{0}{j0.4} = j0.25$$

$$Z_{12} = Z_{12} - \frac{Z_{1l} Z_{l2}}{Z_{ll}}$$

$$= j0.25 - \frac{0}{j0.45} = j0.25$$

$$Z_{13} = Z_{13(or)} - \frac{Z_{1l} Z_{l3}}{Z_{ll}}$$

$$= j0.25 - \frac{0}{j0.4} = j0.25$$

$$Z_{21} = Z_{12}, \quad Z_{22} = Z_{22} - \frac{Z_{2l} Z_{l2}}{Z_{ll}}$$

$$Z_{21} = j0.25, \quad = j0.45 - \frac{(-j0.2 \times -j0.2)}{j0.4}$$

$$= j0.45 - \frac{j0.04}{j0.4} = j0.45 - j0.1$$

$$= j0.35$$

$$Z_{23} = Z_{23} - \frac{Z_{2l} Z_{l3}}{Z_{ll}}$$

$$= j0.25 - \frac{(-j0.2 \times j0.1)}{j0.4} = j0.25 + j0.05$$

$$= j0.3.$$

$$Z_{13} = Z_{31}, \quad Z_{32} = Z_{23},$$

$$Z_{33} = Z_{33} - \frac{Z_{3l} Z_{l3}}{Z_{ll}}$$

$$= j0.35 - \left(\frac{j0.1 \times j0.1}{j0.4}\right)$$

$$= j0.35 - j0.025$$

$$= j0.325$$

$$\therefore\ Z_{BUS\,(\text{Modified})} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.25 & j0.25 & j0.25 \\ 2 & j0.25 & j0.35 & j0.3 \\ 3 & j0.25 & j0.3 & j0.325 \end{array}$$

Step - 5 :- Now Adding last element 0-2. And it is also a link and P-Q. 'P' is Reference bus.

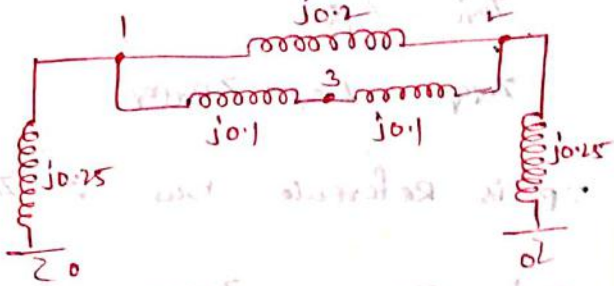
$$Z_{BUS} = \begin{bmatrix} j0.25 & j0.25 & j0.25 & Z_{1l} \\ j0.25 & j0.35 & j0.3 & Z_{2l} \\ j0.25 & j0.3 & j0.325 & Z_{3l} \\ Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{bmatrix}$$

→ fictitious Node l

$\because$ p is Reference bus and No mutual Coupling

$[\because p = 0,\ q = 2]$

$$\boxed{Z_{li} = -Z_{qi}} \quad \text{for } i = 1,2,3.$$



$Z_{l1} = -Z_{21}$
$= -j0.25$

$Z_{l2} = -Z_{22}$
$= -j0.35$

$Z_{l3} = -Z_{23} = -j0.3$

$Z_{ll} = -Z_{ql} + Z_{pq,pq}$
$= -Z_{2l} + Z_{02,02}$
$= -(-j0.35) + j0.25$
$= j0.6$

$$\therefore Z_{BUS} = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & l \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ l \end{matrix} & \begin{bmatrix} 0.25 & 0.25 & 0.25 & -0.25 \\ 0.25 & j0.35 & j0.3 & -0.35 \\ 0.25 & 0.3 & 0.325 & -0.3 \\ -0.25 & -0.35 & -0.3 & 0.6 \end{bmatrix} \end{array}$$

Have to Eliminate this node 'l'.

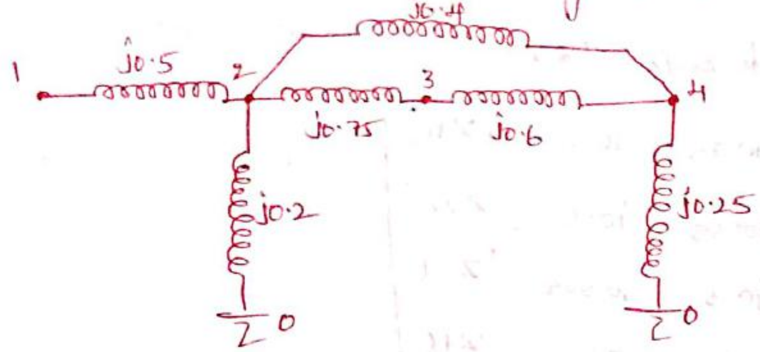
Eliminating fictitious Node 'l' is given by

$$Z_{ij\,(\text{Modified})} = Z_{ij\,(\text{original})} - \frac{Z_{il} Z_{lj}}{Z_{ll}}$$

After calculating all the Modified impedance elements

$$Z_{BUS} = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} j0.1458 & j0.1041 & j0.125 \\ j0.1041 & j0.1458 & j0.125 \\ j0.125 & j0.125 & j0.175 \end{bmatrix} \end{array}$$

2 Obtain the Z_{bus} for the given Network.



Sol step-1 :- Adding the Element between buses Reference to bus-2

$\therefore p=0,\ q=2$ and is not a link and no mutual Coupling

$Z_{qi} = Z_{pi}$

$$Z_{BUS} = \begin{bmatrix} Z_{00} & Z_{02} \\ Z_{20} & Z_{22} \end{bmatrix}$$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$\because p$ is Reference bus $\therefore Z_{qi} = 0.$ and $Z_{pq} = 0.$

and $Z_{qq} = Z_{pq\,pq}$

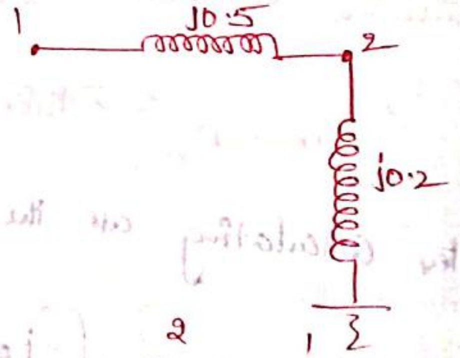
$= j0.2$

$$\therefore Z_{BUS} = \begin{bmatrix} 0 & 0 \\ 0 & j0.2 \end{bmatrix} = \begin{array}{c} 2 \\ 2\,[j0.2] \end{array}$$

Step-2 :- Now adding element buses between 2 and 1 $\therefore p=2, q=1$

No mutual Coupling . and P is not Reference bus.

$$Z_{BUS} = \begin{bmatrix} Z_{22} & Z_{21} \\ Z_{12} & Z_{11} \end{bmatrix}$$



$Z_{qi} = Z_{pi}$ for $i=2$

$Z_{12} = Z_{22} = j0.2 = Z_{21}$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$Z_{11} = Z_{12} + Z_{21\,21}$

$= j0.2 + j0.5$

$= j0.7$

$$\therefore Z_{BUS} = \begin{array}{cc} & \begin{array}{cc} 2 & 1 \end{array} \\ \begin{array}{c} 2 \\ 1 \end{array} & j\begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.7 \end{bmatrix} \end{array}$$

Step-3 :- Adding element from 2 and 3 $\therefore p=2,\ q=3.$

and no mutual coupling even p is not Reference bus.

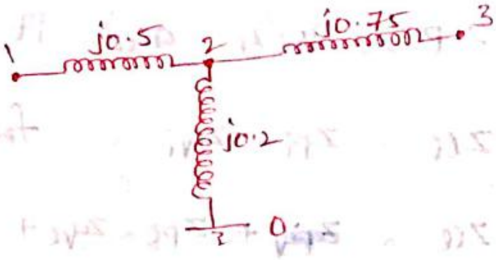
$\therefore Z_{qi} = Z_{pi} =$ for $i = 2,1$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$$\therefore Z_{BUS} = \begin{bmatrix} j0.2 & j0.2 & Z_{23} \\ j0.2 & j0.7 & Z_{13} \\ Z_{32} & Z_{31} & Z_{33} \end{bmatrix}$$

$Z_{31} = Z_{21}$

$= j0.2 = Z_{13}.$



$Z_{32} = Z_{22} = j0.2 = Z_{23}$

$Z_{33} = Z_{23} + Z_{23\,23}$

$= j0.2 + 0.75 = j0.95$

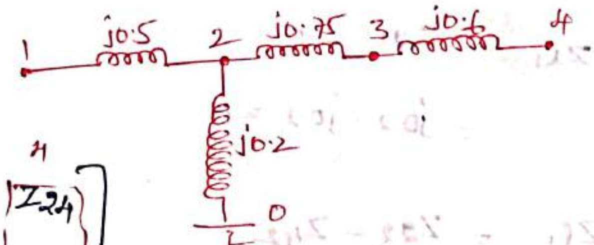
$$\text{Now,}\quad Z_{BUS} = \begin{array}{c c} & \begin{array}{ccc} 2 & 1 & 3 \end{array} \\ \begin{array}{c} 2 \\ 1 \\ 3 \end{array} & \begin{bmatrix} j0.2 & j0.2 & j0.2 \\ j0.2 & j0.7 & j0.2 \\ j0.2 & j0.2 & j0.95 \end{bmatrix} \end{array}$$

Step-4 :- Add the Element from 3 and 4 $\therefore p=3,\ q=4.$

and no mutual coupling

p is not Reference bus

$Z_{qi} = Z_{pi}$ and for $i = 1,2,3$



$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$$Z_{BUS} = \begin{array}{c c} & \begin{array}{cccc} 2 & 1 & 3 & 4 \end{array} \\ \begin{array}{c} 2 \\ 1 \\ 3 \\ 4 \end{array} & \begin{bmatrix} j0.2 & j0.2 & j0.2 & Z_{24} \\ j0.2 & j0.7 & j0.2 & Z_{14} \\ j0.2 & j0.2 & j0.95 & Z_{34} \\ Z_{42} & Z_{41} & Z_{34} & Z_{44} \end{bmatrix} \end{array}$$

$Z_{qi} = Z_{pi}$ for $i = 1,2,3.$

$Z_{41} = Z_{31}$

$= j0.2 = Z_{14}$

$Z_{42} = Z_{32}$

$= j0.2 = Z_{24}$

$Z_{43} = Z_{33} = j0.95 = Z_{34}$

$$Z_{44} = Z_{pq} + Z_{pqpq}$$
$$= Z_{34} + Z_{3434}$$
$$= j0.95 + j0.6$$
$$= j1.55$$

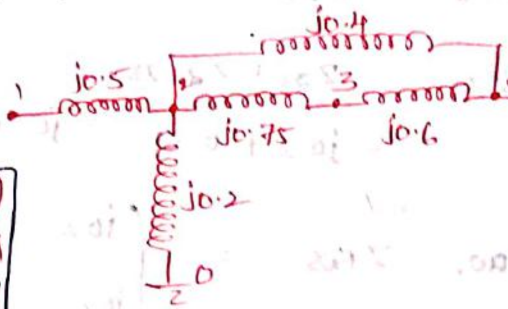
$$\therefore Z_{BUS} = \begin{array}{c|cccc} & 2 & 1 & 3 & 4 \\ \hline 2 & j0.2 & j0.2 & j0.2 & j0.2 \\ 1 & j0.2 & j0.7 & j0.2 & j0.2 \\ 3 & j0.2 & j0.2 & j0.95 & j0.95 \\ 4 & j0.2 & j0.2 & j0.95 & j1.55 \end{array}$$

step-5 :- Addition of Element from buses 2 and 4

$\therefore p=2,\ q=4$ and it is a link & 'p' is not Reference.

$Z_{li} = Z_{pi} - Z_{qi}$ for $i = 1,2,3,4$.

$Z_{ll} = \cancel{Z_{pq}} + Z_{pl} - Z_{ql} + Z_{pqpq}$



$$Z_{BUS} = \begin{array}{c|ccccc} & 2 & 1 & 3 & 4 & l \\ \hline 2 & j0.2 & j0.2 & j0.2 & j0.2 & Z_{2l} \\ 1 & j0.2 & j0.7 & j0.2 & j0.2 & Z_{1l} \\ 3 & j0.2 & j0.2 & j0.95 & j0.95 & Z_{3l} \\ 4 & j0.2 & j0.2 & j0.95 & j1.55 & Z_{4l} \\ l & Z_{l2} & Z_{l1} & Z_{l3} & Z_{l4} & Z_{ll} \end{array}$$

$Z_{li} = Z_{pi} - Z_{qi}$

$Z_{l1} = Z_{21} - Z_{41}$
$= j0.2 - j0.2 = 0$

$Z_{l2} = Z_{22} - Z_{42}$
$= j0.2 - j0.2$
$= 0$

$Z_{l3} = Z_{23} - Z_{43}$
$= j0.2 - j0.95$
$= -j0.75$

$Z_{l4} = Z_{24} - Z_{44}$
$= j0.2 - j1.55$
$= -j1.35$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq}$
$= Z_{2l} - Z_{4l} + Z_{2424}$
$= 0 + j1.35 + j0.4$
$= 1.75$

$$\therefore Z_{BUS} = \begin{array}{c|ccccc} & 2 & 1 & 3 & 4 & l \\ \hline 2 & j0.2 & j0.2 & j0.2 & j0.2 & 0 \\ 1 & j0.2 & j0.7 & j0.2 & j0.2 & 0 \\ 3 & j0.2 & j0.2 & j0.95 & j0.95 & -j0.75 \\ 4 & j0.2 & j0.2 & j0.95 & j1.55 & -j1.35 \\ l & 0 & 0 & -j0.75 & -j1.35 & j1.75 \end{array}$$

Eliminating the fictitious node 'l' we have,

$$Z_{ij}\text{(modified)} = Z_{ij}\text{(original)} - \frac{Z_{il}Z_{lj}}{Z_{ll}} \quad \text{for } i,j = 1,2,3\ldots$$

$Z_{11} = Z_{11} - \dfrac{Z_{1l}Z_{l1}}{Z_{ll}}$

$= j0.7 - 0 = j0.7$

$Z_{12} = Z_{12} - \dfrac{Z_{1l}Z_{l2}}{Z_{ll}}$

$= j0.2$ $\quad [\because Z_{1l} = 0]$.

$Z_{13} = Z_{13} - \dfrac{Z_{1l}Z_{l3}}{Z_{ll}}$

$= j0.2$

$Z_{14} = Z_{14} - \dfrac{Z_{1l}Z_{l4}}{Z_{ll}}$

$= j0.2$

$Z_{21} = Z_{21} - \dfrac{Z_{2l}Z_{l1}}{Z_{ll}}$

$= j0.2 - 0 = j0.2$

$Z_{22} = Z_{22} - \dfrac{Z_{2l}Z_{l2}}{Z_{ll}}$

$= j0.2 - 0 = j0.2$

$Z_{23} = Z_{23} - \dfrac{Z_{2l}Z_{l3}}{Z_{ll}}$

$= j0.2 - 0 = j0.2$

$Z_{24} = Z_{24} - \dfrac{Z_{2l}Z_{l4}}{Z_{ll}}$

$= j0.2 - 0 = j0.2$

$Z_{31} = Z_{31} - \dfrac{Z_{3l}Z_{l1}}{Z_{ll}}$

$= j0.2 - 0 = j0.2$

$Z_{32} = Z_{32} - \dfrac{Z_{3l}Z_{l2}}{Z_{ll}}$

$= j0.2 - 0 = j0.2$

$Z_{33} = Z_{33} - \dfrac{Z_{3l}Z_{l3}}{Z_{ll}}$

$= j0.95 - \dfrac{(-j0.75 \times -j0.75)}{j1.75}$

$= +j0.628$

$Z_{34} = Z_{34} - \dfrac{Z_{3l}Z_{l4}}{Z_{ll}}$

$= j0.95 - \dfrac{(-j0.75 \times -j1.35)}{j1.75}$

$= +j0.3714$.

$Z_{41} = Z_{41} - \dfrac{Z_{4l}Z_{l1}}{Z_{ll}}$

$= j0.2 - 0 = j0.2$

$Z_{42} = Z_{42} - \dfrac{Z_{4l}Z_{l2}}{Z_{ll}}$ $\quad [\because Z_{l2} = 0]$

$= j0.2 - 0 = j0.2$

$Z_{43} = Z_{43} - \dfrac{Z_{4l}Z_{l3}}{Z_{ll}}$

$= j0.95 - \dfrac{(-j1.35 \times -j0.75)}{j1.75}$

$= +j0.3714$

$Z_{44} = Z_{44} - \dfrac{Z_{4l}Z_{l4}}{Z_{ll}}$

$= j1.55 - \dfrac{(-j1.35 \times -j1.35)}{j1.75}$

$= j0.5085$

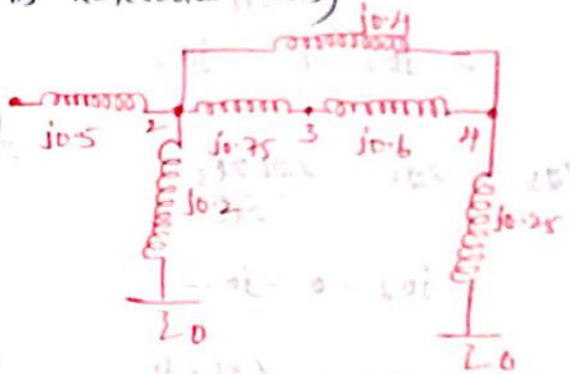
$$\therefore Z_{BUS} = \begin{array}{c|cccc} & 2 & 1 & 3 & 4 \\ \hline 2 & j0.2 & j0.2 & j0.2 & j0.2 \\ 1 & j0.2 & j0.7 & j0.2 & j0.2 \\ 3 & j0.2 & j0.2 & j0.629 & j0.3714 \\ 4 & j0.2 & j0.2 & j0.3714 & j0.5085 \end{array}$$

Step-6 :- Addition of element from 0 to 4 $\therefore p=0, q=4$ and it is a link, No mutual coupling and p is Reference bus

$\therefore Z_{li} = Z_{pi} - Z_{qi} \qquad i = 1,2,3,4$

$Z_{l1} = -Z_{41}$ (∵ P is Reference bus)

$= -j0.2$



$Z_{l2} = -Z_{42}$

$= -j0.2$

$Z_{l3} = -Z_{43} = -j0.3714$

$Z_{l4} = -Z_{44} = -j0.5085$

$\therefore Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$ (∵ P is Reference bus)

$= -Z_{ql} + Z_{pq\,pq}$

$Z_{ll} = -Z_{4l} + Z_{0404}$

$= +j0.5085 + j0.25$

$= j0.75$

$$Z_{BUS} = j \begin{array}{c|ccccc} & 2 & 1 & 3 & 4 & l \\ \hline 2 & 0.2 & 0.2 & 0.2 & 0.2 & Z_{2l} \\ 1 & 0.2 & 0.7 & 0.2 & 0.2 & Z_{1l} \\ 3 & 0.2 & 0.2 & 0.629 & 0.3174 & Z_{3l} \\ 4 & 0.2 & 0.2 & 0.3714 & 0.5085 & Z_{4l} \\ l & Z_{2l} & Z_{1l} & Z_{l3} & Z_{4l} & Z_{ll} \end{array}$$

$$\therefore Z_{BUS} = \begin{bmatrix} j0.2 & j0.2 & j0.2 & j0.2 & -j0.2 \\ j0.2 & j0.7 & j0.2 & j0.2 & -j0.2 \\ j0.2 & j0.2 & j0.629 & j0.3714 & -j0.3714 \\ j0.2 & j0.2 & j0.3714 & j0.5085 & -j0.5085 \\ -j0.2 & -j0.2 & -j0.3714 & -j0.5085 & j0.75 \end{bmatrix}$$

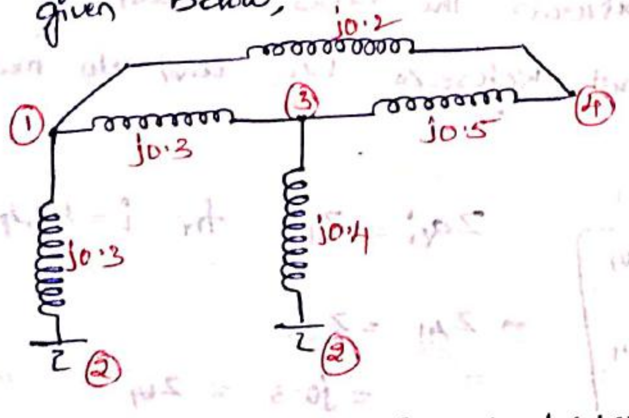
Now again do the same procedure to eliminate the fictitious node 'l' using the formula $Z_{ij} = Z_{ij} - \frac{Z_{il} Z_{lj}}{Z_{ll}}$ for $i,j = 1,2,3,4$

$$\therefore \text{final } Z_{BUS} = \begin{array}{c|cccc} & 2 & 1 & 3 & 4 \\ \hline 2 & j0.1472 & j0.1472 & j0.102 & j0.0659 \\ 1 & j0.1472 & j0.6472 & j0.102 & j0.0659 \\ 3 & j0.102 & j0.102 & j0.4417 & j0.1224 \\ 4 & j0.0659 & j0.00659 & j0.1224 & j0.1676 \end{array}$$

3. Using Building Algorithm Construct Z_{BUS} for the network whose line data is given below. Take bus-2 as Reference.

Element	Bus Code	Reactance
1	1-2	0.3
2	2-3	0.4
3	1-3	0.3
4	1-4	0.2
5	3-4	0.5

Sol: The schematic network for the given line data will be shown as given below,



Step-1: Adding the element buses between 2 and 1

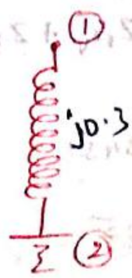
p is Reference bus & No mutual Coupling

$\therefore p=2, q=1$ and p is Reference

$Z_{pi} = 0, \quad Z_{pq} = 0$

$\therefore Z_{qi} = Z_{pi}$ for $i=0,$

$Z_{qq} = Z_{pq,pq}$



$$Z_{BUS} = \begin{bmatrix} Z_{22} & Z_{21} \\ Z_{12} & Z_{11} \end{bmatrix}$$

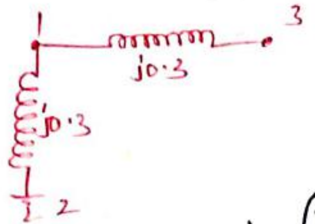
$Z_{10} = 0 = Z_{01}$ and $Z_{11} = Z_{12,12}$

$= j0.3$

$$\therefore Z_{BUS} = \begin{array}{c} \\ 1 \end{array}\begin{array}{c} 1 \\ \begin{bmatrix} j0.3 \end{bmatrix} \end{array}$$

Step-2: Adding the element between buses (1) and (3)

$\therefore p=1, q=3$. 'P' is not Reference bus & no mutual Coupling



$\therefore Z_{qi} = Z_{pi}$ for $i = 3, 1$

$Z_{31} = Z_{11}$

$= j0.3 = Z_{13}$

$$Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 3 \end{array} \\ \begin{bmatrix} Z_{11} & Z_{13} \\ Z_{31} & Z_{33} \end{bmatrix} \end{array}$$

$\therefore Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$Z_{33} = Z_{13} + Z_{1313}$

$= j0.3 + j0.3 = j0.6$

$$\therefore Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 3 \end{array} \\ \begin{bmatrix} j0.3 & j0.3 \\ j0.3 & j0.6 \end{bmatrix} \end{array}$$

Step-3: Adding the Element between the buses 3 and 4

$\Rightarrow p=3, q=4$ and p is not Reference bus and No mutual Coupling.

$$Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} Z_{11} & Z_{13} & Z_{14} \\ Z_{31} & Z_{33} & Z_{34} \\ Z_{41} & Z_{43} & Z_{44} \end{bmatrix} \end{array}$$

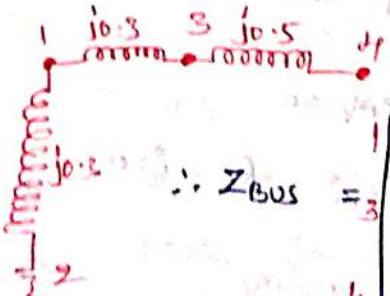
$Z_{qi} = Z_{pi}$ for $i = 1, 3, 4$

$\Rightarrow Z_{41} = Z_{31}$

$= j0.3 = Z_{14}$

$Z_{43} = Z_{33} = j0.6 = Z_{34}$

$\therefore Z_{44} \Rightarrow Z_{qq} = Z_{pq} + Z_{pq\,pq}$



$\Rightarrow Z_{44} = Z_{34} + Z_{3434}$

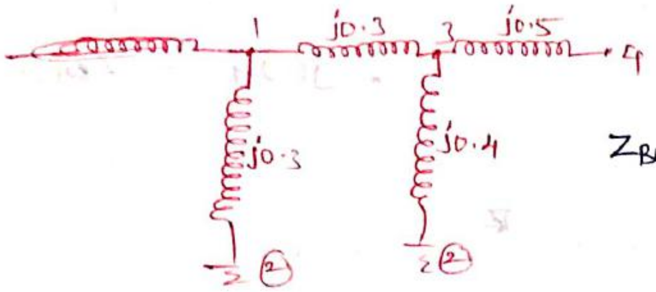
$= j0.6 + j0.5$

$= j1.1$

$$\therefore Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.3 & j0.3 & j0.3 \\ j0.3 & j0.6 & j0.6 \\ j0.3 & j0.6 & j1.1 \end{bmatrix} \end{array}$$

Step-4 :- Now Adding element from ② and ③, It is a link.

$\therefore p=2,\ q=3$, $\because p$ is Reference bus and no mutual coupling



$$Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \\ 4 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 3 & 4 & l \end{array} \\ \begin{bmatrix} j0.3 & j0.3 & j0.3 & Z_{1l} \\ j0.3 & j0.6 & j0.6 & Z_{3l} \\ j0.3 & j0.6 & j1.1 & Z_{4l} \\ Z_{l1} & Z_{l3} & Z_{l4} & Z_{ll} \end{bmatrix} \end{array}$$

$\because p$ is Reference bus,

$$Z_{li} = Z_{pi} - Z_{qi} \quad (\because p \text{ is Reference})$$
$$= -Z_{qi} \quad (i = 1,3,4)$$

$$Z_{l1} = -Z_{31} = -j0.3$$

$$Z_{l3} = -Z_{33} = -j0.6$$

$$Z_{l4} = -Z_{34} = -j0.6$$

$$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$$
$$= -Z_{ql} + Z_{pq\,pq} \quad (\because p \text{ is Reference})$$
$$Z_{ll} = -Z_{3l} + Z_{2323}$$
$$= +j0.6 + j0.4$$
$$= j1.0.$$

$$\therefore Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \\ 4 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 3 & 4 & l \end{array} \\ \begin{bmatrix} j0.3 & j0.3 & j0.3 & -j0.3 \\ j0.3 & j0.6 & j0.6 & -j0.6 \\ j0.3 & j0.6 & j1.1 & -j0.6 \\ -j0.3 & -j0.6 & -j0.6 & j1.0 \end{bmatrix} \end{array}$$

Now eliminate link Row and Column

$$\therefore Z_{ij} = Z_{ij} - \frac{Z_{il} Z_{lj}}{Z_{ll}} \quad \text{for } i,j = 1,2,3,4\ldots$$

$$Z_{11} = Z_{11} - \frac{Z_{1l} Z_{l1}}{Z_{ll}}$$
$$= j0.3 - \frac{(0.3 \times 0.3)}{1}$$
$$= j0.21$$

$$Z_{33} = Z_{33} - \frac{Z_{3l} Z_{l3}}{Z_{ll}}$$
$$= j0.6 - \frac{(0.6 \times 0.6)}{1}$$
$$= j0.24$$

$$Z_{44} = Z_{44} - \frac{Z_{4l} Z_{l4}}{Z_{ll}}$$

$$= j1.1 - \left(\frac{0.6 \times 0.6}{1}\right)$$

$$= j0.74.$$

$$Z_{34} = Z_{34} - \frac{Z_{3l} Z_{l4}}{Z_{ll}}$$

$$= j0.6 - \frac{(j0.6 \times j0.6)}{1}$$

$$= j0.24 = Z_{43}.$$

$$Z_{13} = Z_{13} - \frac{Z_{1l} Z_{3l}}{Z_{ll}}$$

$$= j0.3 - \frac{(0.3 \times 0.6)}{1}$$

$$= j0.12 = Z_{31}$$

$$Z_{14} = Z_{14} - \frac{Z_{1l} Z_{l4}}{Z_{ll}}$$

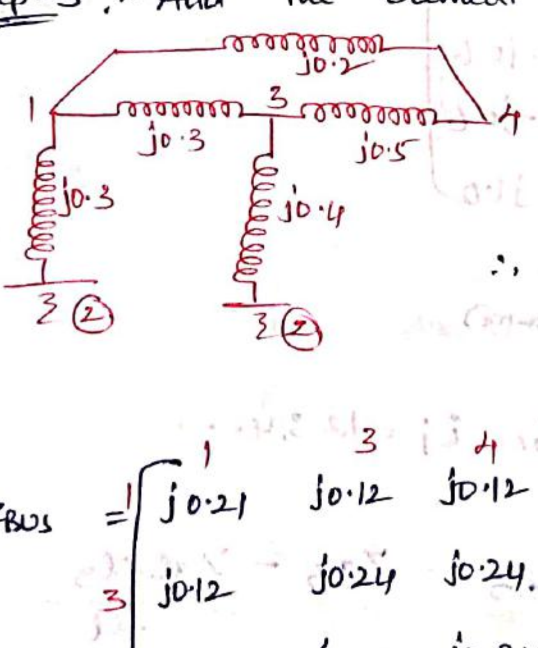
$$= j0.3 - (j0.3 \times j0.6)$$

$$= j0.12 = Z_{41}$$

Now, Modified

$$Z_{BUS} = \begin{array}{c|ccc} & 1 & 3 & 4 \\ \hline 1 & j0.21 & j0.12 & j0.12 \\ 3 & j0.12 & j0.24 & j0.24 \\ 4 & j0.12 & j0.24 & j0.74 \end{array}$$

Step-5 :- Add the element from buses ① to ④.



It is a link, $p=1$, $q=4$ and p is not Reference bus

$\therefore Z_{li} = Z_{pi} - Z_{qi}$ for $i = 1,3,4$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$

$$Z_{BUS} = \begin{array}{c|cccc} & 1 & 3 & 4 & l \\ \hline 1 & j0.21 & j0.12 & j0.12 & Z_{1l} \\ 3 & j0.12 & j0.24 & j0.24 & Z_{3l} \\ 4 & j0.12 & j0.24 & j0.74 & Z_{4l} \\ l & Z_{l1} & Z_{l3} & Z_{l4} & Z_{ll} \end{array}$$

$$Z_{l1} = Z_{11} - Z_{41}$$

$$= j0.21 - j0.12$$

$$= j0.09$$

$$Z_{l3} = Z_{13} - Z_{43}$$

$$= j0.24 - j0.24$$

$$= 0$$

$$Z_{l4} = Z_{14} - Z_{44}$$
$$= j0.12 - j0.74$$
$$= -j0.62$$

$$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$$
$$= Z_{1l} - Z_{4l} + Z_{1414}$$
$$= j0.09 + j0.62 + j0.2$$
$$= j0.9.$$

$$\therefore Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \\ 4 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 3 & 4 & l \end{array} \\ \begin{bmatrix} j0.21 & j0.12 & j0.12 & j0.09 \\ j0.12 & j0.24 & j0.24 & 0 \\ j0.12 & j0.24 & j0.74 & -j0.62 \\ j0.09 & 0 & -j0.62 & j0.9 \end{bmatrix} \end{array}$$

Now again eliminate the fictitious node Row and column by using $Z_{ij} = Z_{ij} - \dfrac{Z_{il} Z_{lj}}{Z_{ll}}$ for $i,j = 1,3,4.$

$$Z_{11} = Z_{11} - \frac{Z_{1l} Z_{l1}}{Z_{ll}}$$
$$= j0.21 - \frac{(j0.09 \times 0.09)}{j0.9}$$
$$= j0.201$$

$$Z_{13} = Z_{13} - \frac{Z_{1l} Z_{l3}}{Z_{ll}}$$
$$= 0.12 - (0.09 \times 0)$$
$$= j0.12 = Z_{31}$$

$$Z_{14} = Z_{14} - \frac{Z_{1l} Z_{l4}}{Z_{ll}}$$
$$= j0.12 - \frac{(0.09 \times -0.62)}{0.9}$$
$$= j0.182 = Z_{41}$$

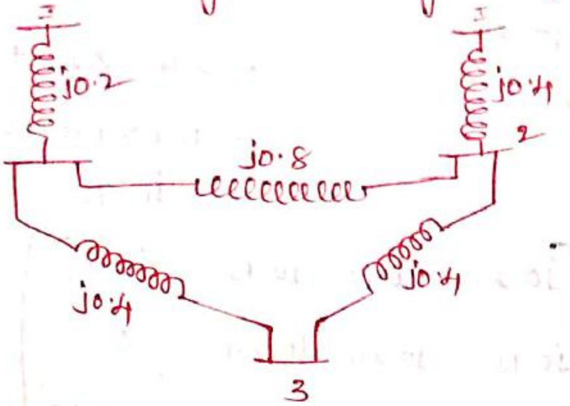
$$Z_{34} = Z_{34} - \frac{Z_{3l} Z_{l4}}{Z_{ll}}$$
$$= j0.24 - 0$$
$$= j0.24 = Z_{43}.$$

$$Z_{33} = Z_{33} - \frac{Z_{3l} Z_{l3}}{Z_{ll}}$$
$$= j0.24 - 0$$
$$= j0.24$$

$$Z_{44} = Z_{44} = \frac{Z_{4l} Z_{l4}}{Z_{ll}}$$
$$= j0.74 - \frac{(0.62 \times 0.62)}{j0.9}$$
$$= j0.312$$

$$\therefore Z_{BUS} = \begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.201 & j0.12 & j0.182 \\ j0.12 & j0.24 & j0.24 \\ j0.182 & j0.24 & j0.312 \end{bmatrix} \end{array} \Leftarrow \text{Modified } Z_{BUS}$$

4. Construct the Z_{bus} using building Algorithm for given Network.



Sol

i) Adding the Element j0.2 between 0 and 1

$\therefore p=0,\ q=1$, p is Reference bus.

$$\therefore Z_{bus} = \begin{bmatrix} Z_{00} & Z_{01} \\ Z_{10} & Z_{11} \end{bmatrix}, \quad Z_{qi} = 0$$

$$Z_{pq} = Z_{pq}\, Z_{pq}$$

$\Rightarrow Z_{pi} = 0 \qquad \therefore Z_{qq} = Z_{pqpq}$

$\Rightarrow Z_{11} = Z_{0101}$

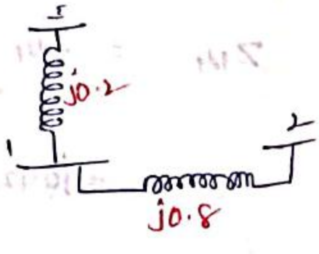
$= j0.2$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0 & 0 \\ 0 & j0.2 \end{bmatrix} \end{array} = \begin{array}{cc} & 1 \\ 1 & [j0.2] \end{array}$$

ii) Now Add the element j0.8 between 1 and 2

$\therefore p=1,\ q=2$

$$Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \end{array}$$



$Z_{qi} = Z_{pi}, \quad Z_{qq} = Z_{pq} + Z_{pqpq}$ for $i=1$

$\Rightarrow Z_{21} = Z_{11} = j0.2$

$= Z_{12}$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} j0.2 & j0.2 \\ j0.2 & j1.0 \end{bmatrix} \end{array}$$

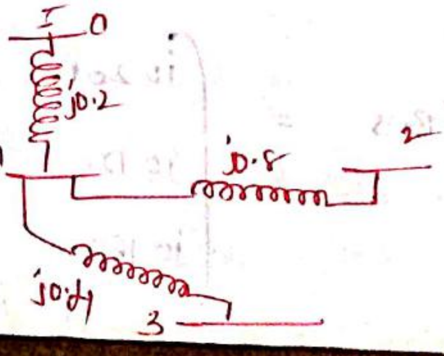
$Z_{22} = Z_{12} + Z_{1212}$

$= j0.2 + j0.8 = j1.0.$

iii) Adding the element j0.4 between 1 and 3.

$\therefore p=1 \quad q=3$

$$Z_{bus} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \end{array}$$



p is not the Reference bus.

$Z_{qq} = Z_{pq} + Z_{pqpq}$ and $Z_{qi} = Z_{pi}$ for $i = 1, 2, 3$.

$Z_{qi} = Z_{pi}$

$\Rightarrow Z_{31} = Z_{11}$

$= j0.2 = Z_{13}$.

$Z_{32} = Z_{12}$

$= Z_{21} = j0.2 = Z_{23}$.

$$\therefore Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.2 & j0.2 & j0.2 \\ 2 & j0.2 & j1.0 & j0.2 \\ 3 & j0.2 & j0.2 & j0.6 \end{array}$$

$Z_{qq} = Z_{pq} + Z_{pqpq}$

$\Rightarrow Z_{33} = Z_{13} + Z_{1313}$

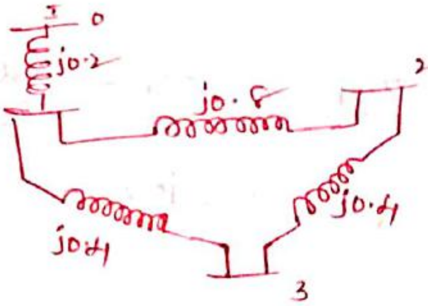
$= j0.2 + j0.4$

$= j0.6$

(iv) Now Adding element $j0.4$ from 2 and 3. where 'p' is not Reference and it is a link

$\therefore p = 2,\ q = 3$

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & Z_{11} & Z_{12} & Z_{13} & Z_{1l} \\ 2 & Z_{21} & Z_{22} & Z_{23} & Z_{2l} \\ 3 & Z_{31} & Z_{32} & Z_{33} & Z_{3l} \\ l & Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{array}$$



$\therefore Z_{li} = Z_{pi} - Z_{qi}$ for $i = 1, 2, 3$.

$\Rightarrow Z_{l1} = Z_{21} - Z_{31}$

$= 0 = Z_{1l}$

$Z_{l2} = Z_{22} - Z_{32}$

$= j1.0 - j0.2$

$= j0.8 = Z_{2l}$

$Z_{l3} = Z_{23} - Z_{33}$

$= j0.2 - j0.6$

$= -j0.4$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq}$

$= Z_{2l} - Z_{3l} + Z_{2323}$

$= j0.8 + j0.4 + j0.4$

$= j1.6$

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & j0.2 & j0.2 & j0.2 & 0 \\ 2 & j0.2 & j1.0 & j0.2 & j0.8 \\ 3 & j0.2 & j0.2 & j0.6 & -j0.4 \\ l & 0 & j0.8 & -j0.4 & j1.6 \end{array}$$

Now eliminating the links Row and column using the given formula

$$Z_{ij(\text{Modified})} = Z_{ij(\text{original})} - \frac{Z_{il}Z_{lj}}{Z_{ll}}$$

$$\Rightarrow Z_{11} = Z_{11} - \frac{Z_{l1}Z_{l1}}{Z_{ll}} = j0.2 - 0 = j0.2$$

$$Z_{12} = Z_{12} - \frac{Z_{1l}Z_{l2}}{Z_{ll}} = j0.2 - 0 = j0.2$$

$$Z_{13} = Z_{13} - \frac{Z_{1l}Z_{l3}}{Z_{ll}} = j0.2 - 0 = j0.2$$

$$Z_{21} = Z_{21} - \frac{Z_{2l}Z_{l1}}{Z_{ll}} = j0.2 - 0 = j0.2$$

$$Z_{22} = Z_{22} - \frac{Z_{2l}Z_{l2}}{Z_{ll}} = j1.0 - \frac{(j0.8 \times -j0.8)}{j1.6} = j1.0 + j0.4 = j1.4$$

$$Z_{23} = Z_{23} - \frac{Z_{2l}Z_{l3}}{Z_{ll}} = j0.2 - \frac{(j0.8 \times -j0.4)}{j1.6} = j0.4.$$

$$Z_{31} = Z_{31} - \frac{Z_{3l}Z_{l1}}{Z_{ll}} = j0.2 - 0 = j0.2$$

$$Z_{32} = Z_{32} - \frac{Z_{3l}Z_{l2}}{Z_{ll}} = j0.2 - \frac{(-j0.4 \times j0.8)}{j1.6} = j0.4$$

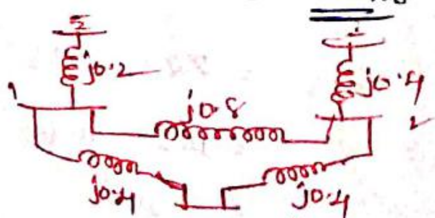
$$Z_{33} = Z_{33} - \frac{Z_{3l}Z_{l3}}{Z_{ll}} = j0.6 - \left(\frac{Z_{l3}Z_{l3}}{Z_{ll}}\right) = j0.6 - \frac{(-j0.4 \times -j0.4)}{j1.6} = j0.6 - j0.1 = j0.5$$

$$\therefore Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.2 & j0.2 & j0.2 \\ 2 & j0.2 & j1.4 & j0.4 \\ 3 & j0.2 & j0.4 & j0.5 \end{array}$$

(V) Now add the last element $j0.4$ from 0 to 2.

$\therefore P=0,\ q=2$ P is Reference bus and element is a link

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & j0.2 & j0.2 & j0.2 & Z_{1l} \\ 2 & j0.2 & j1.4 & j0.4 & Z_{2l} \\ 3 & j0.2 & j0.4 & j0.5 & Z_{3l} \\ l & Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{array}$$



$\because p$ is a Reference bus.

$\cancel{Z_{qi} = 0}$, $Z_{li} = -Z_{qi}$ for $i = 1,2,3$.

$Z_{l1} = -Z_{21} = -j0.2$ | $Z_{l2} = -Z_{22} = -j1.4$ | $Z_{l3} = -Z_{23} = -j0.4$

$Z_{ll} = -Z_{ql} + Z_{pq\,pq}$

$= -Z_{2l} + Z_{0202}$

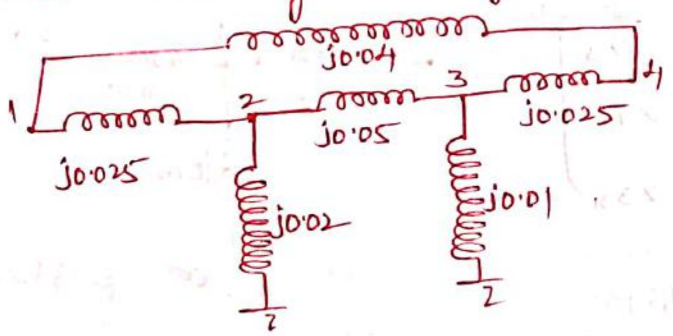
$= +j1.4 + j0.4 = j1.8$

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ 1 & j0.2 & j0.2 & j0.2 & -j0.2 \\ 2 & j0.2 & j1.4 & j0.4 & -j1.4 \\ 3 & j0.2 & j0.4 & j0.5 & -j0.4 \\ l & -j0.2 & -j1.4 & -j0.4 & j1.8 \end{array}$$

Now Again eliminating the Row and coloumn of a link we get a Modified Zbus.

$$\therefore Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0.177 & 0.044 & 0.155 \\ 2 & 0.044 & 0.311 & 0.088 \\ 3 & 0.155 & 0.088 & 0.411 \end{array}$$

5. Construct Zbus by Building Algorithm for given Network



Sol: Add the element j0.02 from 0 to 2

$\therefore p=0,\ q=2$. 'p' is Reference bus.

$$Z_{bus} = \begin{bmatrix} Z_{00} & Z_{02} \\ Z_{20} & Z_{22} \end{bmatrix}$$

$Z_{qi} = Z_{pi} = 0$

$Z_{qq} = Z_{pq\,pq}$

$Z_{22} = Z_{0202}$

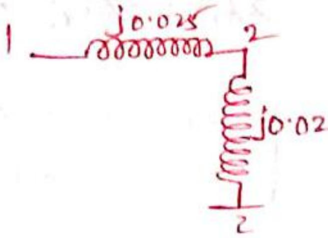
$= j0.02$

$$\therefore Z_{bus} = \begin{array}{c|cc} & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & j0.02 \end{array}$$

$$= \begin{array}{c|c} & 2 \\ 2 & j0.02 \end{array}$$

Add the Element $j0.025$ between 2 and 1 $\therefore p=2,\ q=1$

and 'p' is not Reference bus.



$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 2 & 1 \end{array} \\ \begin{array}{c} 2 \\ 1 \end{array} & \begin{bmatrix} Z_{22} & Z_{21} \\ Z_{12} & Z_{11} \end{bmatrix} \end{array}$$

$Z_{qi} = Z_{pi}$ for $i=2$

$Z_{qq} = Z_{pq} + Z_{pqpq}$

$\therefore Z_{12} = Z_{22}$

$= j0.02 = Z_{21}$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 2 & 1 \end{array} \\ \begin{array}{c} 2 \\ 1 \end{array} & \begin{bmatrix} j0.02 & j0.02 \\ j0.02 & j0.045 \end{bmatrix} \end{array}$$

$Z_{qq} = Z_{pq} + Z_{pqpq}$

$\therefore Z_{11} = Z_{21} + Z_{2121}$

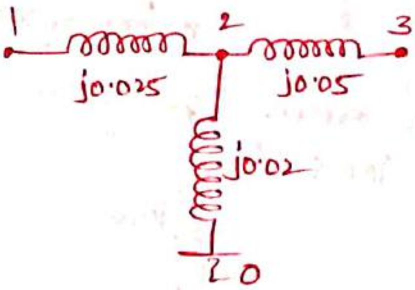
$= j0.02 + j0.025$

$= j0.045$

Now Add the element $j0.05$ between 2 and 3.

$\therefore p=2,\ q=3$ $\because p$ is not Reference bus

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{ccc} 2 & 1 & 3 \end{array} \\ \begin{array}{c} 2 \\ 1 \\ 3 \end{array} & \begin{bmatrix} Z_{22} & Z_{21} & Z_{23} \\ Z_{12} & Z_{11} & Z_{13} \\ Z_{32} & Z_{31} & Z_{33} \end{bmatrix} \end{array}$$



$Z_{qi} = Z_{pi}$ for $i = 1,2,3.$

$Z_{31} = Z_{21}$

$= j0.02$

$Z_{32} = Z_{22}$

$= j0.02$

$Z_{qq} = Z_{pq} + Z_{pqpq}$

$\therefore Z_{33} = Z_{23} + Z_{2323}$

$= j0.02 + j0.05$

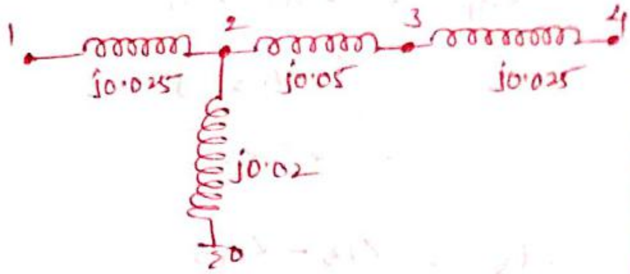
$= j0.07$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{ccc} 2 & 1 & \end{array} \\ \begin{array}{c} 2 \\ 1 \\ 3 \end{array} & \begin{bmatrix} j0.02 & j0.02 & j0.02 \\ j0.02 & j0.045 & j0.02 \\ j0.02 & j0.02 & j0.07 \end{bmatrix} \end{array}$$

Add the element j0.025 between 3 and 4.

$\therefore p=3,\ q=4.$ and p is not Reference bus.

$$\begin{array}{c|cccc} & 2 & 1 & 3 & 4 \\ \hline 2 & Z_{22} & Z_{21} & Z_{23} & Z_{24} \\ 1 & Z_{12} & Z_{11} & Z_{13} & Z_{14} \\ 3 & Z_{32} & Z_{31} & Z_{33} & Z_{34} \\ 4 & Z_{42} & Z_{41} & Z_{43} & Z_{44} \end{array}$$



$Z_{qi} = Z_{pi}$ for $i = 1, 2, 3, 4.$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$\therefore Z_{41} = Z_{31}$
$= j0.02$

$Z_{42} = Z_{32}$
$= j0.02$

$Z_{43} = Z_{33}$
$= j0.07$

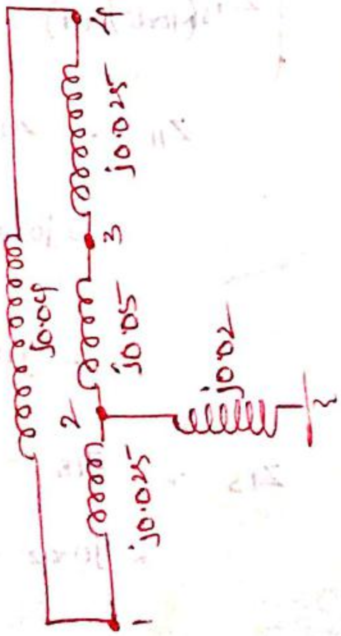
$Z_{44} = Z_{34} + Z_{3434}$
$= j0.07 + j0.025$
$= j0.095$

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 2 & 1 & 3 & 4 \\ \hline 2 & j0.02 & j0.02 & j0.02 & j0.02 \\ 1 & j0.02 & j0.045 & j0.02 & j0.02 \\ 3 & j0.02 & j0.02 & j0.07 & j0.07 \\ 4 & j0.02 & j0.02 & j0.07 & j0.095 \end{array}$$

Now Adding the element j0.04 between 1 and 4.

$p=1,\ q=4.$ and it is a link.

$$Z_{bus} = \begin{array}{c|ccccc} & 2 & 1 & 3 & 4 & l \\ \hline 2 & Z_{22} & Z_{21} & Z_{23} & Z_{24} & Z_{2l} \\ 1 & Z_{12} & Z_{11} & Z_{13} & Z_{14} & Z_{1l} \\ 3 & Z_{32} & Z_{31} & Z_{33} & Z_{34} & Z_{3l} \\ 4 & Z_{42} & Z_{41} & Z_{43} & Z_{44} & Z_{4l} \\ l & Z_{l2} & Z_{l1} & Z_{l3} & Z_{l4} & Z_{ll} \end{array}$$



where, p is not the Reference bus.

$\therefore Z_{li} = Z_{pi} - Z_{qi}$ for $i = 1,2,3,4.$

$Z_{l1} = Z_{11} - Z_{41}$
$= j0.045 - j0.02$
$= 0.025$

$Z_{l2} = Z_{12} - Z_{42}$
$= j0.02 - j0.02$
$= 0.$

$Z_{l3} = Z_{13} - Z_{43}$
$= j0.02 - j0.07$
$= -j0.05$

$Z_{l4} = Z_{14} - Z_{44}$
$= j0.02 - 0.075$
$= -j0.075$

$\therefore Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$
$= Z_{1l} - Z_{4l} + Z_{1414}$
$= 0.025 + j0.075 + j0.04$
$= j0.14.$

$$\therefore Z_{bus} = \begin{array}{c|ccccc} & 2 & 1 & 3 & 4 & l \\ \hline 2 & j0.02 & j0.02 & j0.02 & j0.02 & 0 \\ 1 & j0.02 & j0.045 & j0.02 & j0.02 & j0.025 \\ 3 & j0.02 & j0.02 & j0.07 & j0.07 & -j0.05 \\ 4 & j0.02 & j0.02 & j0.07 & j0.075 & -j0.075 \\ l & 0 & j0.025 & -j0.05 & -j0.075 & j0.14 \end{array}$$

Again eliminating the Row and Column of a links we get,

$$\boxed{Z_{ij}(\text{modified}) = Z_{ij}(\text{original}) - \frac{Z_{il} Z_{lj}}{Z_{ll}}}$$

$Z_{11} = Z_{11} - \dfrac{Z_{1l} Z_{l1}}{Z_{ll}}$
$= j0.045 - \dfrac{(j0.025 \times j0.025)}{j0.14}$
$= j0.04$

$Z_{12} = Z_{12} - \dfrac{Z_{1l} Z_{l2}}{Z_{ll}}$
$= j0.02 - 0$
$= j0.02$

$Z_{13} = Z_{13} - \dfrac{Z_{1l} Z_{l3}}{Z_{ll}}$
$= j0.02 - \dfrac{(j^2 0.025 \times -0.05)}{j0.14}$
$= j0.0289$

$Z_{14} = Z_{14} - \dfrac{Z_{l1} Z_{4l}}{Z_{ll}}$
$= j0.02 - \dfrac{(j0.025 \times -j0.075)}{j0.14}$
$= j0.033.$

$$Z_{21} = Z_{21} - \frac{Z_{l_2} Z_{1l}}{Z_{ll}} \qquad Z_{22} = Z_{22} - \frac{Z_{l_2} Z_{2l}}{Z_{ll}}$$

$$= j0.02 - 0 \qquad = j0.02 - 0$$

$$= j0.02 \qquad = j0.02$$

$$Z_{23} = Z_{23} - \frac{Z_{l_2} l_{3l}}{Z_{ll}} \qquad Z_{24} = Z_{24} - \frac{Z_{2l} Z_{4l}}{Z_{ll}}$$

$$= j0.02 - 0 = j0.02 \qquad = j0.02$$

$$Z_{31} = j0.02 - \frac{Z_{3l} Z_{l_1}}{Z_{ll}} \qquad Z_{32} = Z_{32} - \frac{Z_{l_3} Z_{2l}}{Z_{ll}}$$

$$= j0.02 - \left(\frac{-j0.05 \times j0.025}{j0.14}\right) \qquad = j0.02 - \left(\frac{-j0.05 \times 0}{j0.14}\right)$$

$$= j0.0289 \qquad = j0.02$$

$$Z_{33} = Z_{33} - \frac{Z_{l_3} Z_{3l}}{Z_{ll}} \qquad Z_{34} = Z_{34} - \frac{Z_{3l} Z_{l_4}}{Z_{ll}}$$

$$= j0.07 - \left(\frac{-j0.05 \times -j0.05}{j0.14}\right) \qquad = j0.07 - \left(\frac{-j0.05 \times -j0.075}{j0.14}\right)$$

$$= j0.052 \qquad = j0.043.$$

$$Z_{41} = Z_{41} - \frac{Z_{4l} Z_{l_1}}{Z_{ll}} \qquad Z_{42} = Z_{42} - \frac{Z_{4l} Z_{2l}}{Z_{ll}}$$

$$= j0.02 - \left(\frac{-j0.075 \times j0.025}{j0.14}\right) \qquad = j0.02 - 0$$

$$= j0.033. \qquad = j0.02$$

$$Z_{43} = Z_{43} - \frac{Z_{4l} Z_{l_3}}{Z_{ll}} \qquad Z_{44} = Z_{44} - \frac{Z_{4l} Z_{l_4}}{Z_{ll}}$$

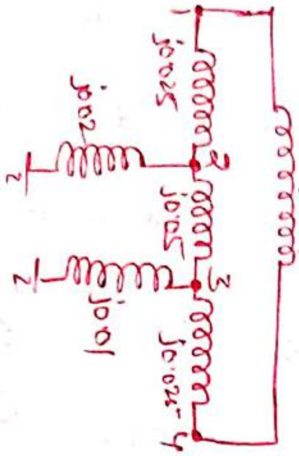
$$= j0.07 - \left(\frac{-j0.075 \times -j0.05}{j0.14}\right) \qquad = j0.095 - \left(\frac{-j0.075 \times -j0.075}{j0.14}\right)$$

$$= j0.043. \qquad = j0.054.$$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{cccc} 2 & 1 & 3 & 4 \end{array} \\ \begin{array}{c} 2 \\ 1 \\ 3 \\ 4 \end{array} & \begin{bmatrix} j0.02 & j0.02 & j0.02 & j0.02 \\ j0.02 & j0.04 & j0.0289 & j0.033 \\ j0.02 & j0.0289 & j0.052 & j0.043 \\ j0.02 & j0.033 & j0.043 & j0.054 \end{bmatrix} \end{array}$$

Add the element j0.01 between busses 0 and 3.

$\therefore p=0,\ q=3$. 'p' is Reference bus and is a link.



$\therefore Z_{li} = Z_{pi} - Z_{qi}$ for $i = 1,2,3,4$.

$= -Z_{qi}$

$Z_{l1} = -Z_{31}$
$= -0.0289$

$Z_{l3} = -Z_{33}$
$= -j0.052$

$Z_{l2} = -Z_{32}$
$= -j0.02$

$Z_{l4} = -Z_{34}$
$= -j0.043$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$

$= -Z_{ql} + Z_{pq\,pq}$

$= -Z_{3l} + Z_{0303}$

$= +j0.052 + j0.01$

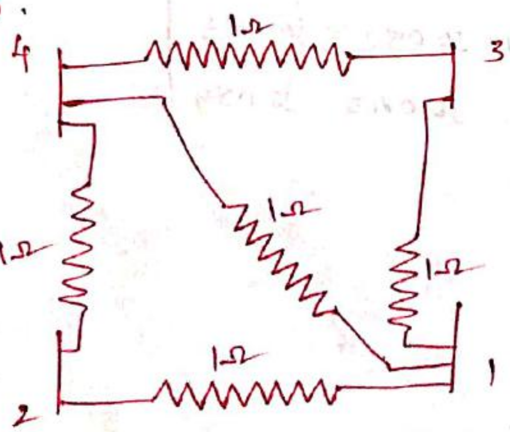
$= j0.062$

$$\therefore Z_{bus} = j\begin{array}{c} \\ 2 \\ 1 \\ 3 \\ 4 \\ l \end{array}\begin{array}{ccccc} 2 & 1 & 3 & 4 & l \\ \hline 0.02 & 0.02 & 0.02 & 0.02 & -0.02 \\ 0.02 & 0.0289 & 0.0289 & 0.033 & -0.0289 \\ 0.02 & 0.0289 & 0.052 & 0.043 & -0.052 \\ 0.02 & 0.033 & 0.043 & 0.054 & -0.043 \\ -0.02 & -0.0289 & -0.052 & -0.043 & 0.062 \end{array}$$

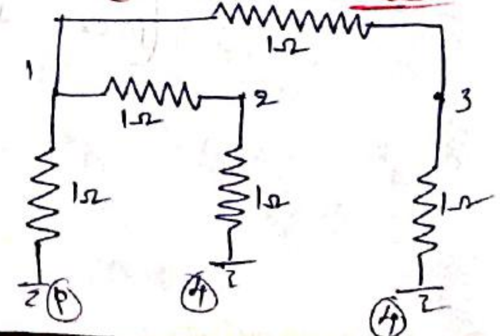
Now Again eliminating the Row and Column of a link we get a Modified Zbus.

$$\therefore Z_{bus} = \begin{array}{c} \\ 2 \\ 1 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 2 & 1 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.013 & j0.010 & j0.003 & j0.007 \\ j0.010 & j0.026 & j0.004 & j0.012 \\ j0.0043 & j0.0084 & j0.008 & j0.006 \\ \cancel{j0.012}\ j0.007 & j0.012 & j0.006 & j0.024 \end{bmatrix} \end{array}$$

6. Construct a Zbus using Building Algorithm for given Network.



Take (4) as Reference



Sol Adding the element 1Ω from 0 to 1.

$\therefore p=0,\ q=1$

$\therefore Z_{bus} = \begin{bmatrix} Z_{00} & Z_{01} \\ Z_{10} & Z_{11} \end{bmatrix}$

$Z_{qi} = Z_{pi} = 0$

$Z_{qq} = Z_{pq} + Z_{pq\,pq} = Z_{pq\,pq}$

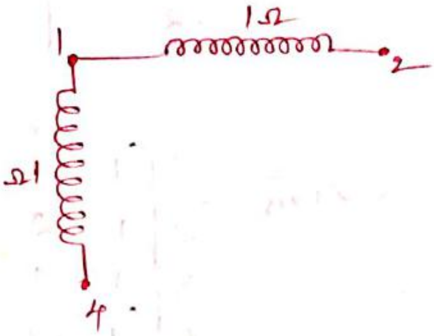
$\therefore Z_{bus} = \begin{array}{cc} & 1 \\ 1 & \begin{bmatrix} 1 \end{bmatrix} \end{array}$

$Z_{11} = Z_{0101}$

$= 1\Omega$

Adding element 1Ω between 1 and 2.

$\Rightarrow P=1,\ q=2$



$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$Z_{qi} = Z_{pi} \qquad i=1$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$Z_{21} = Z_{11}$

$= 1\Omega = Z_{12}$

$\therefore Z_{bus} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$Z_{22} = Z_{12} + Z_{1212}$

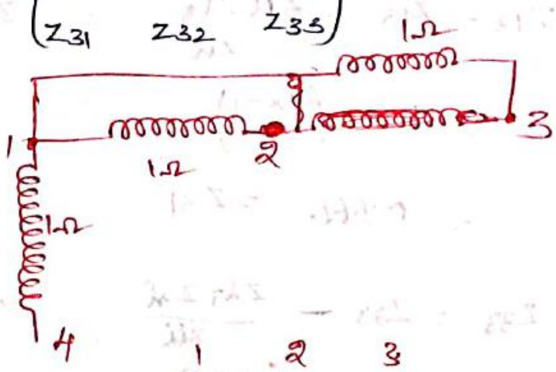
$= 1+1 = 2$

Now Adding element between 1 and 3.

$\Rightarrow P=1,\ q=3.$

$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$

$Z_{qi} = Z_{pi} \qquad \therefore$ for $i=1,2,.$



$Z_{31} = Z_{11}$

$= 1\Omega$

$Z_{32} = Z_{12} = 1\Omega$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$\therefore Z_{33} = 1 + 1$

$= 2\Omega$

$\therefore Z_{bus} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \end{array}$

Now Adding element from between 2 and 4.

$\therefore p=4,\ q=2.$

$Z_{qi} = Z_{pi}$ ($\because$ it is a link)

$\therefore Z_{li} = -Z_{qi}$ for $i=1,2,3.$

$$\therefore Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{1l} \\ Z_{21} & Z_{22} & Z_{23} & Z_{2l} \\ Z_{31} & Z_{32} & Z_{33} & Z_{3l} \\ Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{bmatrix}$$

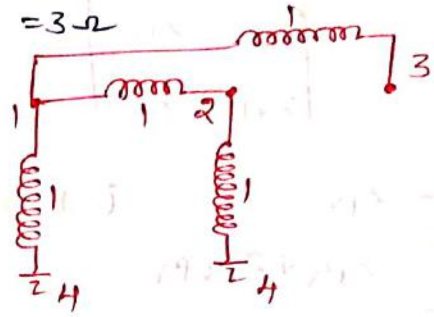
$Z_{l1} = -Z_{21}$
$= -1.$

$Z_{l2} = -Z_{22}$
$= -2$

$Z_{l3} = -Z_{23}$
$= -1$

$Z_{ll} = -Z_{ql} + Z_{pq,pq}$
$= -Z_{2l} + Z_{42\,42}$
$= +2+1 = 3\,\Omega$

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & 1 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 & -2 \\ 3 & 1 & 1 & 2 & -1 \\ l & -1 & -2 & -1 & 3 \end{array}$$



$$Z_{ij\,(\text{Modified})} = Z_{ij\,(\text{original})} - \frac{Z_{lj}\,Z_{il}}{Z_{ll}}$$

$\therefore Z_{11} = Z_{11} - \frac{Z_{1l}\,Z_{l1}}{Z_{ll}}$
$= 1 - \left(\frac{-1\times -1}{3}\right)$
$= 1 - \frac{1}{3}$
$= 0.666$

$Z_{12} = Z_{12} - \frac{Z_{1l}\,Z_{l2}}{Z_{ll}}$
$= 1 - \frac{(-1\times -2)}{3}$
$= 0.333 = Z_{21}$

$Z_{13} = Z_{13} - \frac{Z_{1l}\,Z_{l3}}{Z_{ll}}$
$= 1 - \left(\frac{-1\times -1}{3}\right)$
$= 0.666 = Z_{31}$

$Z_{23} = Z_{23} - \frac{Z_{2l}\,Z_{l3}}{Z_{ll}}$
$= 1 - \left(\frac{-2\times -1}{3}\right)$
$= 0.333 = Z_{32}$

$Z_{33} = Z_{33} - \frac{Z_{l3}\,Z_{3l}}{Z_{ll}}$
$= 2 - \left(\frac{-1\times -1}{3}\right)$
$= 1.667$

$Z_{22} = Z_{22} - \frac{Z_{l2}\,Z_{l2}}{Z_{ll}}$
$= 2 - \left(\frac{-2\times -2}{3}\right)$
$= 0.6667$

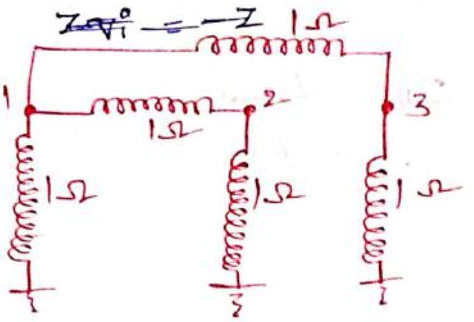
$$\therefore Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.667 & 0.33 & 0.667 \\ 2 & 0.33 & 0.667 & 0.33 \\ 3 & 0.667 & 0.33 & 1.667 \end{array}$$

Now Adding the last element 4 to 3 and is a link.

$\therefore p=4, \; q=3.$

$$Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ 1 & 0.667 & 0.33 & 0.667 & Z_{1l} \\ 2 & 0.33 & 0.667 & 0.33 & Z_{2l} \\ 3 & 0.667 & 0.33 & 1.667 & Z_{3l} \\ l & Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{array}$$

'p' is a Reference



$Z_{li} = -Z_{qi}$ for $i = 1,2,3.$

$Z_{ll} = -Z_{ql} + Z_{pq\,pq}$

$\therefore Z_{l1} = -Z_{31} = -0.667$ $\quad Z_{l2} = -Z_{32} = -0.33$ $\quad Z_{l3} = -Z_{33} = -1.667$

$$Z_{ll} = -Z_{ql} + Z_{pq\,pq}$$
$$= -Z_{3l} + Z_{4343}$$
$$= +1.667 + 1 = 2.667$$

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ 1 & 0.667 & 0.33 & 0.667 & -0.667 \\ 2 & 0.33 & 0.667 & 0.33 & -0.33 \\ 3 & 0.667 & 0.33 & 1.667 & -1.667 \\ l & -0.667 & -0.33 & -1.667 & 2.667 \end{array}$$

Now,

$$Z_{ij} = Z_{ij} - \frac{Z_{il} Z_{lj}}{Z_{ll}}$$

$$Z_{11} = Z_{11} - \frac{Z_{1l} Z_{l1}}{Z_{ll}}$$
$$= 0.667 - \frac{(0.667 \times 0.667)}{2.667}$$
$$= 0.5$$

and the process will continues to get modified Zbus.

$$\therefore Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0.5 & 0.247 & 0.25 \\ 2 & 0.247 & 0.626 & 0.12 \\ 3 & 0.25 & 0.12 & 0.625 \end{array}$$

7. Construct Zbus for the given Network.

Sol. Add the element $j0.6$ between 4 and 1.

$\therefore P=4,\ q=1 \quad \because P$ is a Reference bus.

$Z_{qi} = 0, \quad Z_{qq} = Z_{pq\,pq}$

$$Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 4 & 1 \end{array} \\ \begin{array}{c} 4 \\ 1 \end{array} & \begin{bmatrix} Z_{40} & Z_{41} \\ Z_{14} & Z_{11} \end{bmatrix} \end{array}$$

$Z_{11} = Z_{4141}$

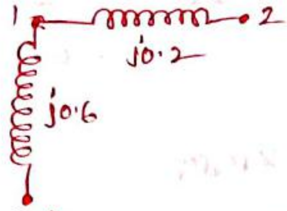
$= j0.6$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 4 & 1 \end{array} \\ \begin{array}{c} 4 \\ 1 \end{array} & \begin{bmatrix} 0 & 0 \\ 0 & j0.6 \end{bmatrix} \end{array}$$

$$\Rightarrow Z_{bus} = \begin{array}{c c} & 1 \\ 1 & \begin{bmatrix} j0.6 \end{bmatrix} \end{array}$$

Now Add the element $j0.2$ between busses 1 and 2

$\therefore P=1,\ q=2$



$$Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \end{array}$$

$Z_{qi} = Z_{pi}$ ($\because P$ is not Reference)

for $i=1$

$Z_{21} = Z_{11}$

$= j0.6 = Z_{12}$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} j0.6 & j0.6 \\ j0.6 & j0.8 \end{bmatrix} \end{array}$$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

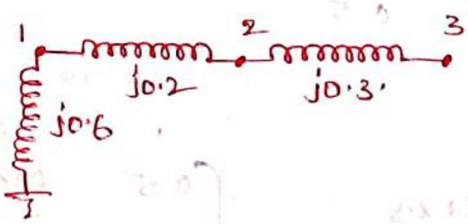
$Z_{22} = Z_{12} + Z_{1212}$

$= j0.6 + j0.2$

$= j0.8$

Now Add the Element $j0.3$ between busses 2 and 3.

$\therefore P=2,\ q=3.$



$$Z_{bus} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} j0.6 & j0.6 & Z_{13} \\ j0.6 & j0.8 & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \end{array}$$

$Z_{qi} = Z_{pi}$ for $i = 1,2,$

$Z_{31} = Z_{21}$

$= j0.6 = Z_{13}.$

$Z_{32} = Z_{22}$

$= j0.8 = Z_{23}.$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

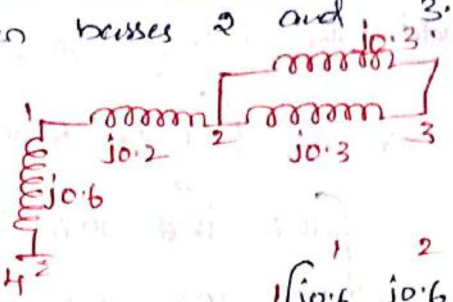
$= Z_{23} + Z_{23,23}$

$= j0.8 + j0.3 = j1.1$

$$\therefore Z_{bus} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} j0.6 & j0.6 & j0.6 \\ j0.6 & j0.8 & j0.8 \\ j0.6 & j0.8 & j1.1 \end{bmatrix} \end{array}$$

Again Add the Element $j0.3$ between busses 2 and 3.

$\therefore p=2,\ q=3$. it is a link.



$\therefore Z_{li} = Z_{pi} - Z_{qi}$ for $i = 1,2,3$.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \left[\begin{array}{ccc|c} j0.6 & j0.6 & j0.6 & Z_{1l} \\ j0.6 & j0.8 & j0.8 & Z_{2l} \\ j0.6 & j0.8 & j1.1 & Z_{3l} \\ \hline Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{array}\right] \end{array}$$

$Z_{l1} = Z_{21} - Z_{31}$

$= j0.6 - j0.6 = 0.$

$Z_{l2} = Z_{22} - Z_{32}$

$= j0.8 - j0.8 = 0.$

$Z_{l3} = Z_{23} - Z_{33}$

$= j0.8 - j1.1 = -j0.3.$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq}$

$= Z_{2l} - Z_{3l} + Z_{2323}$

$= 0 + j0.3 + j0.3$

$\therefore$ ~~Z_{bus}~~

$= j0.6.$

$$\therefore Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \left[\begin{array}{ccc|c} j0.6 & j0.6 & j0.6 & 0 \\ j0.6 & j0.8 & j0.8 & 0 \\ j0.6 & j0.8 & j1.1 & -j0.3 \\ \hline 0 & 0 & -j0.3 & j0.6 \end{array}\right] \end{array}$$

Eliminating the Row and Coloumn of a link is given by,

$$Z_{ij} = Z_{ij} - \frac{Z_{il} Z_{lj}}{Z_{ll}}$$

$Z_{11} = Z_{11} - \frac{Z_{1l} Z_{l1}}{Z_{ll}}$

$= j0.6 - 0 = j0.6$

$Z_{12} = Z_{12} - \frac{Z_{1l} Z_{l2}}{Z_{ll}}$

$= j0.6 = Z_{21}$

$Z_{22} = Z_{22} - \frac{Z_{l2} Z_{l2}}{Z_{ll}}$

$= j0.8 - 0 = j0.8$

$Z_{13} = Z_{13} - \frac{Z_{1l} Z_{l3}}{Z_{ll}}$

$= j0.6 - 0 = j0.6$

$Z_{33} = Z_{33} - \frac{Z_{l3} Z_{3l}}{Z_{ll}}$

$= j1.1 - \frac{(-j0.3 \times -j0.3)}{j0.6}$

$= j0.95$

$Z_{23} = Z_{23} - \frac{Z_{l2} Z_{l3}}{Z_{ll}}$

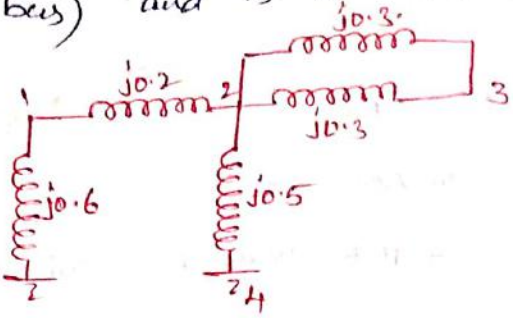
$= j0.8 - 0 = j0.8$

$$\therefore Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.6 & j0.6 & j0.6 \\ j0.6 & j0.8 & j0.8 \\ j0.6 & j0.8 & j0.95 \end{bmatrix} \end{array}$$

Now Add the element j0.5 between 2 and 4.

$\therefore p=4,\ q=2$ ($\because p$ is Reference bus) and is a link.

$$Z_{bus} = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & \begin{bmatrix} j0.6 & j0.6 & j0.6 & Z_{1l} \\ j0.6 & j0.8 & j0.8 & Z_{2l} \\ j0.6 & j0.8 & j0.95 & Z_{3l} \\ Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{bmatrix} \end{array}$$



$$Z_{li} = Z_{pi} - Z_{qi} \quad \text{for } i=1,2,3.$$
$$= -Z_{qi}$$

$$Z_{l1} = -Z_{21}$$
$$= -j0.6$$

$$Z_{l2} = -Z_{22}$$
$$= -j0.8$$

$$Z_{l3} = -Z_{23}$$
$$= -j0.8$$

$$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$$
$$= -Z_{ql} + Z_{pq\,pq}$$
$$= -Z_{2l} + Z_{4242}$$
$$= +j0.8 + j0.5$$
$$= j1.3$$

$$\therefore Z_{bus} = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & \begin{bmatrix} j0.6 & j0.6 & j0.6 & -j0.6 \\ j0.6 & j0.8 & j0.8 & -j0.8 \\ j0.6 & j0.8 & j0.95 & -j0.8 \\ -j0.6 & -j0.8 & -j0.8 & j1.3 \end{bmatrix} \end{array}$$

$$\underline{Z_{ij} = Z_{ij} - \frac{Z_{il} Z_{lj}}{Z_{ll}}}$$

$$Z_{11} = Z_{11} - \frac{Z_{l1} Z_{l1}}{Z_{ll}}$$
$$= j0.6 - \frac{j(0.6 \times 0.6)}{j1.3}$$
$$= j0.32$$

$$Z_{12} = Z_{12} - \frac{Z_{1l} Z_{l2}}{Z_{ll}}$$
$$= j0.6 - \frac{(-j0.6 \times -j0.8)}{j1.3}$$
$$= j0.23 = Z_{21}$$

$$Z_{13} = Z_{13} - \frac{Z_{1l} Z_{l3}}{Z_{ll}}$$
$$= j0.6 - \frac{(-j0.6 \times -j0.8)}{j1.3}$$
$$= j0.23 = Z_{31}$$

$$Z_{23} = Z_{23} - \frac{Z_{2l} Z_{l3}}{Z_{ll}}$$
$$= j0.8 - \frac{(-j0.8 \times -j0.8)}{j1.3}$$
$$= j030$$

$$Z_{22} = Z_{22} - \frac{Z_{l2} Z_{2l}}{Z_{ll}}$$
$$= j0.8 - \frac{(-j0.8 \times -j0.8)}{j1.3}$$
$$= +j0.3.$$

$$Z_{33} = Z_{33} - \frac{Z_{3l}\,Z_{l3}}{Z_{ll}}$$

$$= 0.95 - \frac{(-j0.8 \times -j0.8)}{j1.3}$$

$$= j0.45$$

$$\therefore Z_{bus} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} j0.32 & j0.23 & j0.23 \\ j0.23 & j0.3 & j0.3 \\ j0.23 & j0.3 & j0.45 \end{bmatrix} \end{array}$$

Add the element j0.2 between 4 and 3.

$p=4,\ q=3.$ and p is Reference bus.

$$Z_{bus} = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & \left[\begin{array}{ccc|c} j0.32 & j0.23 & j0.23 & Z_{1l} \\ j0.23 & j0.3 & j0.3 & Z_{2l} \\ j0.23 & j0.3 & j0.45 & Z_{3l} \\ \hline Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{array}\right] \end{array}$$

$$Z_{li} = Z_{pi} - Z_{qi} \quad (\because i = 1,2,3).$$

$$Z_{l1} = -Z_{31} = -j0.23.$$

$$Z_{ll} = -Z_{ql} + Z_{pq\,pq} = -Z_{3l} + Z_{4343} = j0.45 + j0.2 = j0.65$$

$$Z_{l2} = -Z_{32} = -j0.3$$

$$Z_{l3} = -Z_{33} = -j0.45$$

$$\therefore Z_{bus} = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & \left[\begin{array}{ccc|c} j0.32 & j0.23 & j0.23 & -j0.23 \\ j0.23 & j0.3 & j0.3 & -j0.3 \\ j0.23 & j0.3 & j0.45 & -j0.45 \\ \hline -j0.23 & -j0.3 & -j0.45 & +j0.65 \end{array}\right] \end{array}$$

Now, Again Eliminating the Row and Coloumn of a link to get the Actual Zbus,

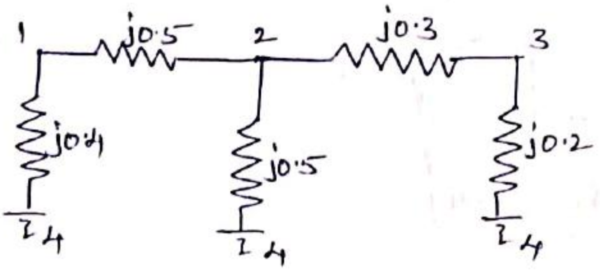
on symplification,

$$Z_{bus} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} j0.23 & j0.12 & j0.07 \\ j0.12 & j0.161 & j0.09 \\ j0.07 & j0.09 & j0.13 \end{bmatrix} \end{array}$$

8. Compute Z_{bus} using Building Algorithm for the given Network. When Mutual Coupling is given. (4) as Reference

Element	Bus Code	impedance	Mutual impedance	
1	1-2	j0.5	—	—
2	1-4	j0.4	—	—
3	2-3	j0.3	1-2	j0.2
4	2-4	j0.5	—	—
5	3-4	j0.2	—	—

Sol:



Add the element j0.4 between busses 4 and 1.

$P=4,\ q=1$

$$Z_{bus} = \begin{bmatrix} Z_{44} & Z_{41} \\ Z_{14} & Z_{11} \end{bmatrix}$$

$\because P$ is Reference bus.

$$Z_{qi} = Z_{pi} = 0$$

$$Z_{qq} = Z_{pq} + Z_{pq\,pq}$$

$$= Z_{pq\,pq}$$

$$\therefore Z_{11} = j0.4$$

$$\therefore Z_{bus} = \begin{array}{c|cc} & 4 & 1 \\ \hline 4 & 0 & 0 \\ 1 & 0 & j0.4 \end{array} = \begin{array}{c|c} & 1 \\ \hline 1 & j0.4 \end{array}$$

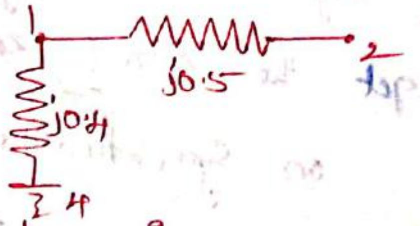
Now Add the element j0.5 between busses 1 and 2

$P=1,\ q=2$ and No mutual Coupling

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\therefore Z_{qi} = Z_{pi} \quad \text{for } i=1,2$$

$$Z_{qq} = Z_{pq} + Z_{pq\,pq}$$



$$\therefore Z_{21} = Z_{11} = j0.4$$

$$= Z_{12}$$

$$Z_{22} = Z_{12} + Z_{12\,12}$$

$$= j0.4 + j0.5$$

$$= j0.9$$

$$\therefore Z_{bus} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & j0.4 & j0.4 \\ 2 & j0.4 & j0.9 \end{array}$$

Adding element j0.3 between busses 2 and 3 and also it is mutually coupled to element 1 and 2

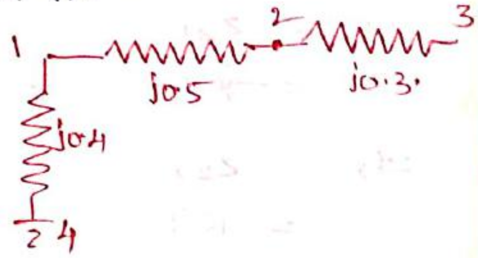
$\therefore p=2,\ q=3,$ Similarly, $\rho = 1,\ \sigma = 2.$

$$Z_{pq\rho\sigma} = \begin{array}{c|ccc} & 4\text{-}1 & 1\text{-}2 & 2\text{-}3 \\ \hline 4\text{-}1 & j0.4 & 0 & 0 \\ 1\text{-}2 & 0 & j0.5 & j0.2 \\ 2\text{-}3 & 0 & j0.2 & j0.3 \end{array}$$

$$\therefore Z_{pq\rho\sigma} = \begin{array}{c|cc} & 1\text{-}2 & 2\text{-}3 \\ \hline 1\text{-}2 & j0.5 & j0.2 \\ 2\text{-}3 & j0.2 & j0.3 \end{array}$$

from the above Matrix. determine $Y_{pq\rho\sigma}$

$$\Rightarrow Y_{pq\rho\sigma} = \begin{array}{c|cc} & 1\text{-}2 & 2\text{-}3 \\ \hline 1\text{-}2 & 2.72 & 1.81 \\ 2\text{-}3 & 1.81 & \cancel{4.54}\ 5.45 \end{array}$$



Now, $$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

$$Z_{qi} = Z_{pi} + \frac{Y_{pq\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{Y_{pq,pq}} \quad \text{for } i=1,2.$$

$$Z_{31} = Z_{21} + \frac{Y_{23,12}(Z_{11} - Z_{21})}{Y_{23,23}}$$

$$= j0.4 + \frac{1.81\,(j0.4 - j0.4)}{5.45}$$

$$= j0.4 = Z_{13}.$$

$$Z_{32} = Z_{22} + \frac{Y_{23,12}(Z_{12} - Z_{22})}{Y_{23,23}}$$

$$= j0.9 + \frac{1.81\,(j0.4 - j0.9)}{\cancel{5.45}\ 4.54}$$

$$= j0.733.$$

$$Z_{qq} = Z_{pq} + \frac{1 + Y_{pq\rho\sigma}(Z_{\rho q} - Z_{\sigma q})}{Y_{pq,pq}}$$

$$\Rightarrow Z_{33} = Z_{23} + \frac{1 + 1.81(Z_{13} - Z_{23})}{5.45}$$

$$= 0.733 + \frac{1 + 1.81(j0.4 - 0.733)}{\cancel{5.45}\ 4.54}$$

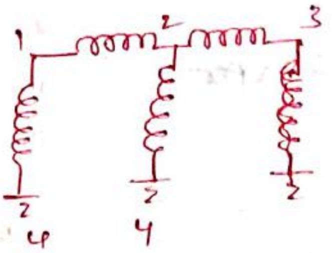
$$= 0.805$$

$$\therefore Z_{bus} = \begin{bmatrix} j0.4 & j0.4 & j0.4 \\ j0.4 & j0.9 & j0.733 \\ j0.4 & j0.733 & j0.805 \end{bmatrix}$$

Now Add the element $j0.5$ between busses 4 and 2

$\therefore p=4$, $q=2$ and is a link. & p is Reference bus.

$$\therefore Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \left[\begin{array}{ccc|c} j0.4 & j0.4 & j0.4 & Z_{1l} \\ j0.4 & j0.9 & j0.73 & Z_{2l} \\ j0.4 & j0.73 & j0.805 & Z_{3l} \\ \hline Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{array}\right] \end{array}$$



$Z_{li} = Z_{pi} - Z_{qi}$ for $i=1,2$.

$Z_{l1} = -Z_{qi}$ ($\because p$ is Reference bus)

$Z_{l1} = -Z_{21}$
$= -j0.4$

$Z_{l2} = -Z_{22}$
$= -j0.9$

$Z_{l3} = -Z_{23}$
$= -j0.73$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pq\,pq}$

$= -Z_{ql} + Z_{pq\,pq}$

$= -Z_{2l} + Z_{4242}$

$= +j0.9 + j0.5$

$= j1.4.$

$$\therefore Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \left[\begin{array}{ccc|c} j0.4 & j0.4 & j0.4 & -j0.4 \\ j0.4 & j0.9 & j0.73 & -j0.9 \\ j0.4 & j0.73 & j0.805 & -j0.73 \\ \hline -j0.4 & -j0.9 & -j0.73 & j1.4 \end{array}\right] \end{array}$$

eliminating the Row and Coloumn of a link.

$$Z_{ij} = Z_{ij} - \frac{Z_{il}Z_{lj}}{Z_{ll}}$$

$Z_{11} = Z_{11} - \frac{Z_{1l}Z_{l1}}{Z_{ll}}$

$= j0.4 - \left[\frac{-j0.4 \times -j0.4}{j1.4}\right]$

$= j0.285$

$Z_{12} = Z_{12} - \frac{Z_{1l}Z_{l2}}{Z_{ll}}$

$= j0.4 - \left(\frac{-j0.4 \times -j0.9}{j1.4}\right)$

$= j0.14$

$Z_{23} = Z_{23} - \frac{Z_{2l}Z_{l3}}{Z_{ll}}$

$= 0.73 - \frac{(j^2 0.9 \times 0.73)}{j1.4}$

$= j0.26$

$Z_{13} = Z_{13} - \frac{Z_{l1}Z_{l3}}{Z_{ll}}$

$= j0.4 - \frac{(-j0.4 \times -j0.73)}{j1.4}$

$= j0.19$

$$Z_{22} = Z_{22} - \frac{Z_{l2} Z_{2l}}{Z_{ll}} \qquad Z_{33} = Z_{33} - \frac{Z_{l3} Z_{3l}}{Z_{ll}}$$

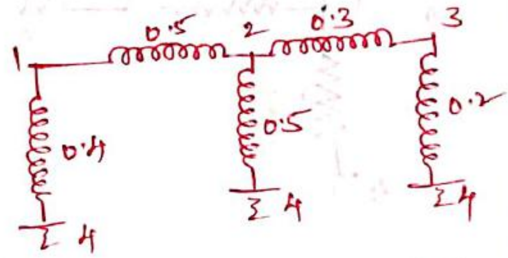
$$= j0.9 - \frac{(-j0.9 \times -j0.9)}{j1.4} \qquad = j0.805 - \frac{(j0.73 \times j0.73)}{j1.4}$$

$$= +j0.32 \qquad = +j0.424.$$

$$\therefore Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.285 & j0.14 & j0.19 \\ 2 & j0.14 & j0.32 & j0.26 \\ 3 & j0.19 & j0.26 & j0.424 \end{array}$$

Adding element j0.2 between busses 4 and 3.

$p=4$, $q=3$ and is a link where p is Reference bus.



$$Z_{li} = Z_{pi} - Z_{qi}$$

$$\Rightarrow Z_{li} = -Z_{qi} \quad \text{for } i = 1,2,3$$

$$Z_{l1} = -Z_{31} = -j0.19$$

$$Z_{l2} = -Z_{32} = -j0.26$$

$$Z_{l3} = -j0.424$$

$$Z_{ll} = -Z_{ql} + Z_{pq\,pq} = -Z_{3l} + Z_{4343} = j0.424 + j0.2 = j0.624.$$

$$\text{Now } Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & j0.28 & j0.14 & j0.19 & -j0.19 \\ 2 & j0.14 & j0.32 & j0.26 & -j0.26 \\ 3 & j0.19 & j0.26 & j0.424 & -j0.42 \\ l & -j0.19 & -j0.26 & -j0.42 & j0.624 \end{array}$$

Now again eliminating the Row and Column of a link we get,

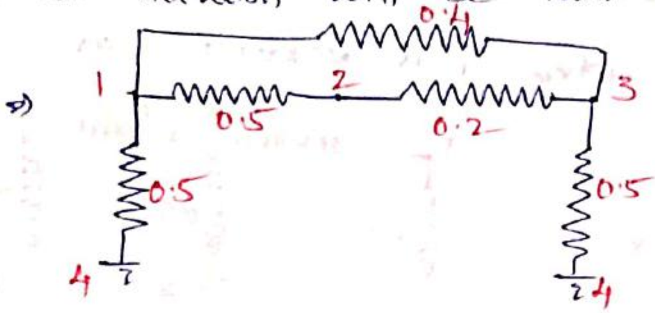
$$Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.22 & j0.06 & j0.06 \\ 2 & j0.06 & j0.211 & j0.08 \\ 3 & j0.06 & j0.08 & j0.137 \end{array}$$

7. Construct the Z_{bus} Using building Algorithm for given data

1-2	0.5	—	—
2-3	0.2	1-3	0.1
4-3	0.5	—	—
4-1	0.5	—	—
1-3	0.4	—	—

Take 4 as Reference.

Sol. The network will be from the above data



Adding element 0.5 between busses 4 & 1

$\therefore p=4,\ q=1$. 'p' is Reference bus. & No mutual coupling

$\therefore Z_{qi} = 0$

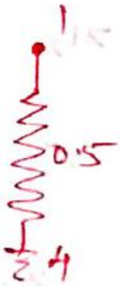
$$\therefore Z_{bus} = \begin{array}{c|cc} & 4 & 1 \\ \hline 4 & 0 & 0 \\ 1 & 0 & 0.5 \end{array}$$

$Z_{qq} = Z_{pq\,pq}$

$\Rightarrow Z_{11} = Z_{41\,41}$

$= 0.5$

$$\Rightarrow Z_{bus} = 1\,[0.5]$$



Add the element 0.5 between busses 1 and 2

$\therefore p=1,\ q=2$ & no mutually Coupled to element.

$\therefore Z_{qi} = Z_{pi}$ for $i=1$

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

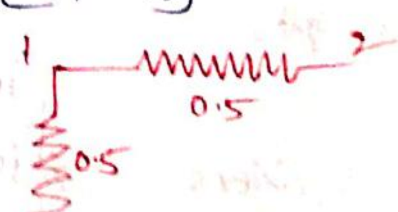
$Z_{21} = Z_{11}$

$= 0.5$

$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$= Z_{12} + Z_{12,12}$

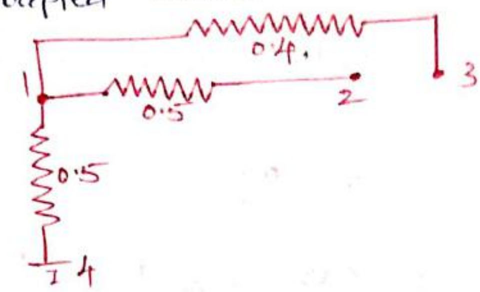
$= 0.5 + 0.5 = 1.$



$$\therefore Z_{bus} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0.5 & 0.5 \\ 2 & 0.5 & 1.0 \end{array}$$

Add the element 0.4 between busses 1 and 3.

$\therefore$ $p=1$, $q=3$ and no mutual coupled element



$$Z_{bus} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0.5 & 0.5 & Z_{13} \\ 0.5 & 1.0 & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

$Z_{qi} = Z_{pi}$ for $i=1,2$.

$Z_{31} = Z_{11}$
$= 0.5$

$Z_{32} = Z_{12}$
$= 0.5$

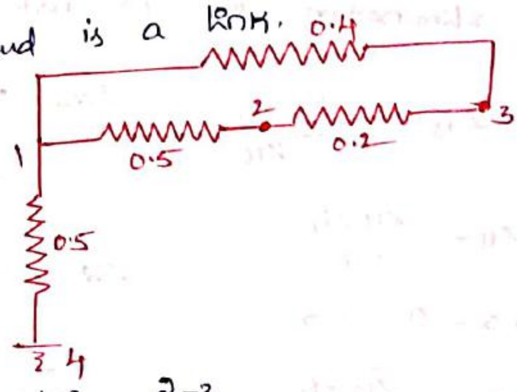
$$\therefore Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.5 & 0.5 & 0.5 \\ 2 & 0.5 & 1.0 & 0.5 \\ 3 & 0.5 & 0.5 & 0.9 \end{array}$$

$Z_{qq} = Z_{pq} + Z_{pq,pq}$

$Z_{33} = Z_{13} + Z_{1313}$
$= 0.5 + 0.4$
$= \underline{0.9}$

Now Add the element 0.2 between busses 2 and 3. and is mutually coupled to element 0.1 between busses 1 & 3. and is a link.

$\Rightarrow p=2$, $q=3$, $\rho=1$, $\sigma=3$.



Now taking the primitive matrix of $Z_{pq,\rho\sigma}$ with the help of a Networks connected upto now.

$$Z_{pq,\rho\sigma} = \begin{array}{c|cccc} & 4-1 & 1-2 & 1-3 & 2-3 \\ \hline 4-1 & 0.5 & 0 & 0 & 0 \\ 1-2 & 0 & 0.5 & 0 & 0 \\ 1-3 & 0 & 0 & 0.4 & 0.1 \\ 2-3 & 0 & 0 & 0.1 & 0.2 \end{array}$$

$$\therefore Z_{pq\,\rho\sigma} = \begin{array}{c|cc} & 1-3 & 2-3 \\ \hline 1-3 & 0.4 & 0.1 \\ 2-3 & 0.1 & 0.2 \end{array} \Rightarrow Y_{pq\,\rho\sigma} = \begin{array}{c|cc} & 1-3 & 2-3 \\ \hline 1-3 & 2.85 & -1.42 \\ 2-3 & -1.42 & 5.71 \end{array}$$

Now, $Z_{li} = Z_{pi} - Z_{qi} + \dfrac{Y_{pq\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{Y_{pqpq}}$ for $i = 1,2,3$.

$$Z_{l1} = Z_{21} - Z_{31} + \frac{Y_{23,13}\,(Z_{11} - Z_{31})}{Y_{2323}}$$

$$\Rightarrow Z_{l1} = 0.5 - 0.5 + \frac{-1.42\,(0.5-0.5)}{5.71} = 0.$$

$$Zl_2 = Z_{22} - Z_{32} + \frac{Y_{23,13}(Z_{12} - Z_{32})}{Y_{2323}}$$

$$= 1.0 - 0.5 + \left[\frac{-1.42(0.5-0.5)}{5.71}\right]$$

$$= \underline{\underline{0.5}}$$

$$Zl_3 = Z_{23} - Z_{33} + \frac{Y_{23,13}(Z_{13} - Z_{33})}{Y_{2323}}$$

$$= 0.5 - 0.9 + \left[\frac{-1.42(0.5-0.9)}{5.71}\right]$$

$$= -0.4 + 0.099$$

$$= -0.3.$$

$$Zll = Zpl - Zql + \left[\frac{1 + Y_{pq\,\rho\sigma}(Z_{\rho l} - Z_{\sigma l})}{Y_{pqpq}}\right]$$

$$\Rightarrow Zll = Z_{2l} - Z_{3l} + \left(\frac{1 + (-1.42)(Z_{13} - Z_{33})}{5.71}\right)$$

$$Zll = \underline{\underline{1.07}}$$

$$\therefore Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0 \\ 0.5 & 1.0 & 0.5 & 0 \\ 0.5 & 0.5 & 0.9 & -0.3 \\ 0 & 0 & -0.3 & 1.07 \end{bmatrix} \end{array}$$

Now eliminating the l^{th} node

$$Z_{ij} = Z_{ij} - \frac{Z_{il}\, Z_{lj}}{Z_{ll}}$$

$$Z_{11} = Z_{11} - \frac{Z_{1l}\, Zl_1}{Z_{ll}}$$

$$= 0.5 - 0 = 0.5$$

$$Z_{12} = Z_{12} - \frac{Z_{1l}\, Zl_2}{Z_{ll}}$$

$$= 0.5 - 0 = 0.5$$

$$Z_{13} = Z_{13} - \frac{Z_{1l}\, Zl_3}{Z_{ll}}$$

$$= 0.5 - 0 = 0.5$$

$$Z_{22} = Z_{22} - \frac{Zl_2\, Zl_2}{Z_{ll}}$$

$$= 1.0 - 0 = 1.0$$

$$Z_{23} = Z_{23} - \frac{Z_{2l}\, Zl_3}{Z_{ll}}$$

$$= 0.5 - 0 = 0.5$$

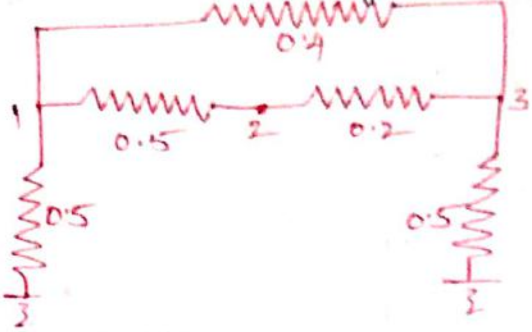
$$Z_{33} = Z_{33} - \frac{Zl_3 \cdot Z_{3l}}{Z_{ll}}$$

$$= 0.9 - \frac{(-0.3 \times -0.3)}{1.07}$$

$$= 0.81$$

$$\therefore Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 1.0 & 0.5 \\ 0.5 & 0.5 & 0.81 \end{bmatrix} \end{array}$$

Add the element 0.5 between buses 4 and 3 and is a link. and p is Reference bus, No mutual Coupling



$P=4,\ q=3.$

$$Z_{bus} = \begin{array}{c|cccc} & 1 & 2 & 3 & l \\ \hline 1 & 0.5 & 0.5 & 0.5 & Z_{1l} \\ 2 & 0.5 & 1.0 & 0.5 & Z_{2l} \\ 3 & 0.5 & 0.5 & 0.81 & Z_{3l} \\ l & Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{array}$$

$Z_{li} = -Z_{qi}$ for $i = 1,2,3$

$Z_{l1} = -Z_{31}$
$= -0.5$

$Z_{l2} = -Z_{qi}$
$= -Z_{32} = -0.5$

$Z_{l3} = -Z_{33}$
$= -0.81$

$Z_{ll} = -Z_{ql} + Z_{pq\,pq}$
$= -Z_{3l} + Z_{pq\,pq}$
$= +0.81 + 0.5 = 0.81 + 0.5$
$\therefore\ Z_{ll} = 1.31$

Again eliminate the l^{th} Node to get Actual N/w for Z_{bus}.

$$Z_{11} = Z_{11} - \frac{Z_{1l} Z_{l1}}{Z_{ll}} = 0.5 - \frac{(-0.5 \times -0.5)}{1.31} = 0.3.$$

$$Z_{22} = Z_{22} - \frac{Z_{l2} Z_{2l}}{Z_{ll}} = 1 - \frac{(-0.5 \times -0.5)}{1.31} = 0.8$$

$$Z_{12} = Z_{12} - \frac{Z_{1l} Z_{l2}}{Z_{ll}} = 0.5 - \frac{(-0.5 \times -0.5)}{1.31} = 0.3.$$

$$Z_{33} = Z_{33} - \frac{Z_{3l} Z_{l3}}{Z_{ll}} = 0.81 - \frac{(-0.81 \times -0.81)}{1.31} = 0.3.$$

$$Z_{13} = Z_{13} - \frac{Z_{l1} Z_{3l}}{Z_{ll}} = 0.5 - \frac{(-0.5 \times -0.81)}{1.31} = 0.19$$

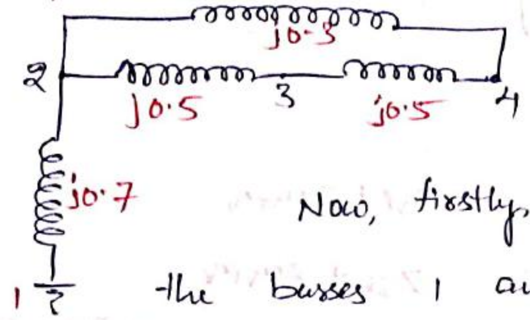
$$\therefore\ Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.3 & 0.3 & 0.19 \\ 2 & 0.3 & 0.8 & 0.19 \\ 3 & 0.19 & 0.19 & 0.3 \end{array}.$$

$$Z_{23} = Z_{23} - \frac{Z_{2l} Z_{l3}}{Z_{ll}} = 0.5 - \frac{(-0.5 \times -0.81)}{1.31} = 0.19$$

10. Construct Z_{bus} using Building Algorithm for the given data. Take 1 as Reference.

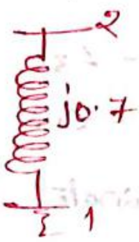
1-2	j0.7	—	—
2-3	j0.5	—	—
2-4	j0.3	1-2	j0.2
3-4	j0.5	2-3	j0.2

Sol.



Now, firstly, Add the element j0.7 between the busses 1 and 2.

$\therefore p=1, q=2$ p is Reference bus & No mutual Coupling



$Z_{qi} = Z_{pi} = 0$

$Z_{qq} = Z_{pq} + Z_{pq\,pq} = Z_{pq\,pq}$

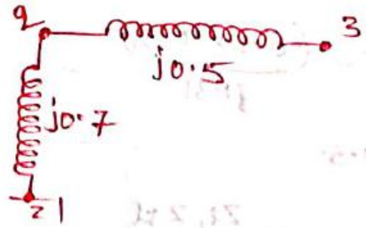
$\Rightarrow Z_{22} = Z_{1212}$
$= j0.7$

$$\therefore Z_{bus} = \begin{bmatrix} 0 & 0 \\ 0 & j0.7 \end{bmatrix} = \begin{array}{cc} & 2 \\ 2 & \begin{bmatrix} j0.7 \end{bmatrix} \end{array}$$

Now Add the element j0.5 between busses 2 and 3.

$\therefore p=2, q=3$ and no mutual Coupling

$$\therefore Z_{bus} = \begin{bmatrix} Z_{22} & Z_{23} \\ Z_{32} & Z_{33} \end{bmatrix}$$



$Z_{qi} = Z_{pi}$. for $i = 2$

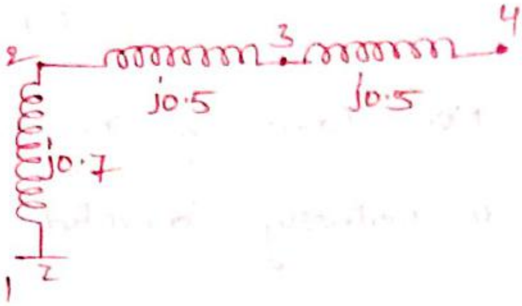
$Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$\Rightarrow Z_{32} = Z_{22}$
$= j0.7$

$Z_{33} = Z_{23} + Z_{2323}$
$= j0.7 + j0.5$
$= j1.2$

$$\therefore Z_{bus} = \begin{array}{ccc} & 2 & 3 \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} j0.7 & j0.7 \\ j0.7 & j1.2 \end{bmatrix} \end{array}$$

Adding the element j0.5 between 3 and 4. and is mutually Coupled to the element 2-3.



$$\therefore Z_{bus} = \begin{bmatrix} j0.7 & j0.7 \\ j0.7 & j1.2 \end{bmatrix}$$

$$\text{New } Z_{bus} = \begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline 2 & j0.7 & j0.7 & Z_{24} \\ 3 & j0.7 & j1.2 & Z_{34} \\ 4 & Z_{42} & Z_{43} & Z_{44} \end{array}$$

$$Z_{pq,\rho\sigma} = \begin{array}{c|ccc} & 1-2 & 2-3 & 3-4 \\ \hline 1-2 & j0.7 & 0 & 0 \\ 2-3 & 0 & j0.5 & j0.2 \\ 3-4 & 0 & j0.2 & j0.5 \end{array}$$

$$\therefore Z_{pq,\rho\sigma} = \begin{bmatrix} j0.5 & j0.2 \\ j0.2 & j0.5 \end{bmatrix}$$

$$\begin{bmatrix} p=3, \ q=4 \\ \rho=2, \sigma=3 \end{bmatrix}$$

$$\Rightarrow Y_{pq,\rho\sigma} = -j \begin{array}{c|cc} & 2-3 & 3-4 \\ \hline 2-3 & 2.38 & -0.95 \\ 3-4 & -0.95 & 2.38 \end{array}$$

Now, $$Z_{qi} = Z_{pi} + \frac{Y_{pq\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{Y_{pqpq}} \quad \text{for } i = 2,3.$$

$$Z_{42} = Z_{32} + \frac{Y_{34,23}(Z_{22} - Z_{32})}{Y_{34,34}}$$

$$= j0.7 + \frac{-0.95(j0.7 - j0.7)}{2.38}$$

$$\therefore Z_{42} = j0.7$$

$$Z_{43} = Z_{33} + \frac{Y_{34,23}(Z_{23} - Z_{33})}{Y_{34,34}}$$

$$= j1.2 + \left[\frac{(-0.95)(j0.7 - j1.2)}{2.38}\right]$$

$$Z_{43} = j1.39$$

$$Z_{qq} = Z_{pq} + \frac{1 + Y_{pq\rho\sigma}(Z_{\rho q} - Z_{\sigma q})}{Y_{pqpq}}$$

$$\Rightarrow Z_{44} = Z_{34} + \frac{1 + Y_{34,23}(Z_{24} - Z_{34})}{2.38}$$

$$= j1.39 + \frac{1 + (-0.95)(j0.7 - j1.39)}{2.38}$$

$$= j2.66.$$

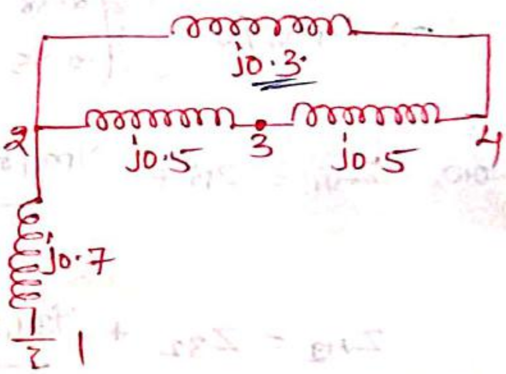
$$\therefore Z_{bus} = \begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline 2 & 0.7 & 0.7 & 0.7 \\ 3 & 0.7 & 1.2 & 1.39 \\ 4 & 0.7 & 1.39 & 2.66 \end{array} \; j$$

Now connecting the element $j0.3$ between 2 and 4 and is mutually connected to element 1-2.

Now, $Z_{pq\,pr}$ is going to be determined by connecting all the elements as primitive network up to now.

$$\therefore Z_{pq\,pr} = \begin{array}{c|cccc} & 1-2 & 2-3 & 3-4 & 2-4 \\ \hline 1-2 & j0.7 & 0 & 0 & j0.2 \\ 2-3 & 0 & j0.5 & 0 & 0 \\ 3-4 & 0 & 0 & j0.5 & 0 \\ 2-4 & j0.2 & 0 & 0 & j0.3 \end{array}$$

$$\therefore Z_{pq\,pr} = \begin{array}{c|cc} & 1-2 & 2-4 \\ \hline 1-2 & j0.7 & j0.2 \\ 2-4 & j0.2 & j0.3 \end{array}$$



and $$Y_{pq\,pr} = \begin{array}{c|cc} & 1-2 & 2-4 \\ \hline 1-2 & 1.76 & -1.176 \\ 2-4 & -1.176 & 4.11 \end{array}$$

The connected element is a link.

$\therefore p=2,\ q=4.$ p is not the Reference bus. $\rho=1, \sigma=2$

$$z_{li} = Z_{pi} - Z_{qi} + \frac{Y_{pq\,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{Y_{pq\,pq}} \quad \text{for } i = 2,3,4.$$

$$z_{l2} = Z_{22} - Z_{42} + \frac{Y_{24,12}(Z_{12} - Z_{22})}{4.11}$$

$$= j0.7 - j0.7 + \frac{(-1.176)(0 - j0.7)}{4.11}$$

$$-j0.2$$

$$Zl_3 = Z_{23} - Z_{43} + \frac{(-1.176)(Z_{13} - Z_{23})}{4.11}$$

$$= j0.7 - j1.39 + \frac{(-1.176)(0 - j0.7)}{4.11}$$

$$= -j0.89$$

$$Zl_4 = Z_{24} - Z_{44} + \frac{(-1.176)(Z_{14} - Z_{24})}{4.11}$$

$$= j0.7 - j2.66 + \frac{(-1.176)(0 - j0.7)}{4.11}$$

$$= -j2.16.$$

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + y_{pq}\rho_{\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq}\rho_{q}}$$

$$= Z_{2l} - Z_{4l} + \left[\frac{1 + (-1.176)(Z_{14} - Z_{24})}{4.11}\right]$$

$$= -j0.2 + j2.16 + \left[\frac{1 + (-1.176)(0 - j0.7)}{4.11}\right]$$

$$= j3.16$$

$$\therefore Z_{bus} = \begin{array}{c|cccc} & 2 & 3 & 4 & l \\ \hline 2 & j0.7 & j0.7 & j0.7 & -j0.2 \\ 3 & j0.7 & j1.2 & j1.39 & -j0.89 \\ 4 & j0.7 & j1.39 & j2.66 & -j2.16 \\ l & -j0.2 & -j0.89 & -j2.16 & j3.16 \end{array}$$

Now eliminating the l^{th} Node by

$$Z_{ij} = Z_{ij} - \frac{Z_{il}Z_{lj}}{Z_{ll}}$$

$$\therefore Z_{22} = Z_{22} - \frac{Z_{2l}Z_{l2}}{Z_{ll}}$$

$$= j0.7 - \frac{(-j0.2 \times -j0.2)}{j3.16}$$

$$= j0.68$$

$$Z_{23} = Z_{23} - \frac{Z_{2l}Z_{l3}}{Z_{ll}}$$

$$= j0.7 - \frac{(-j0.2 \times -j0.89)}{j3.16}$$

$$= j0.64.$$

$$Z_{24} = Z_{24} - \frac{Z_{2l} Z_{l4}}{Z_{ll}}$$
$$= j0.7 - \frac{(-j0.2 \times -j2.16)}{j3.16}$$
$$= j0.56$$

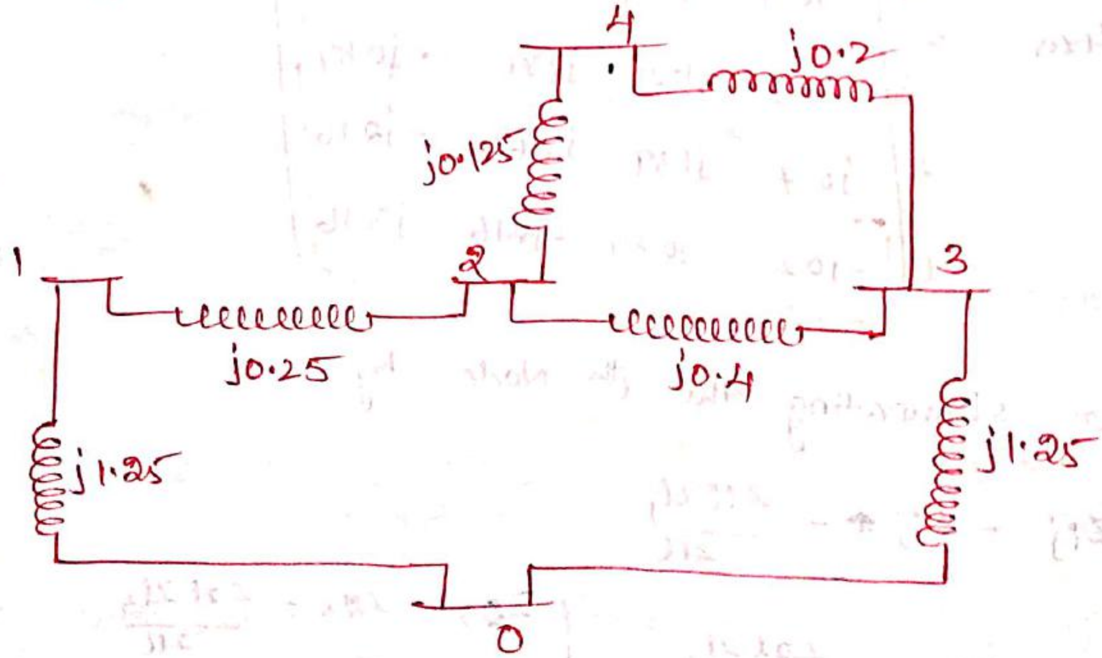
$$Z_{34} = Z_{34} - \frac{Z_{3l} Z_{l4}}{Z_{ll}}$$
$$= j1.39 - \frac{(-j0.87 \times -j2.16)}{j3.16}$$
$$= j0.78$$

$$Z_{33} = Z_{33} - \frac{Z_{3l} Z_{l3}}{Z_{ll}}$$
$$= j1.2 - \frac{(-j0.87 \times -j0.87)}{j3.16}$$
$$= j0.74.$$

$$Z_{44} = Z_{44} - \frac{Z_{4l} Z_{l4}}{Z_{ll}}$$
$$= j2.66 - \frac{(-j2.16 \times -j2.16)}{j3.16}$$
$$= j1.18$$

$$\therefore \; Z_{bus} = \begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline 2 & j0.68 & j0.64 & j0.56 \\ 3 & j0.64 & j0.74 & j0.78 \\ 4 & j0.56 & j0.78 & j1.18 \end{array}$$

(11) Construct the ZBUS by using Building Algorithm for the given Networks below.



24/3/2014

Unit-3.

Load Flow Studies

As we know that the power system is an large interconnected Networks which Consists of 'n' number of buses, 'p' number of Generators, 'q' number of loads, transformers and so on. The Main objective of any Power System Engineer is to transmit the Electrical power from one end to the another end effectively. Also maintaing the Electrical parameters (Real power, Reactive power, Voltage, Current, load-angle (or) Voltage angle 'δ') at per unit value at each and every bus.

But, this is not happening in practical point of View. The Harmonics, transients, sags, swells, power losses etc is usually happening in power system (transmission system). Due to this, the transmission of power is degrading by day to day life. Degradation of transmission of power is due to the power losses in transmission system.

One has to know about the power system, have to know firstly and must about the transmission System. How the power losses are increasing, what are the Causes, what are the techniques used in olden days to know the power losses and what are the Advanced Methods we have, and what are the Remedies to be taken to works the power system as stable ; all these Analysis is known as power System Analysis (PSA).

Now, in this chapter we are concentrating about the power losses (Calculation) at each and every bus (or) Calculating the Electrical Parameters at every bus using some methods. And this analysis (or) studies is known as "Load flow studies" (or) "power flow studies."

Up to the between 18 and 19th Centuries the classical linear and Non-linear programing techniques such as Lagrangian principle, Bellsman constant were successfull. After that unfortunately, these techniques are no longer be used to determine the load flow analysis.

Later the traditional Methods came into Existance. Those are

1. Guass - Siedel Method
2. Newton - Raphson Method
   i, Polar - Cordinates Method
   ii, Rectangular - Coordinates Method
3. Decoupled Method
4. Fast De-Coupled Method

Traditional Methods.

Even Now-a-days i.e, from the begining of 20th Century these methods are also ignored due to the invention of Advanced Methods known as "Optimization Techniques".

But in this Subject we are going to concentrate only on Traditional Methods. Any how, these (traditional) Methods are the base of Advanced Techniques.

* Load-flows ?:-

1. As we know that, Due to the growth of Electricity demands and transactions in power markets, the existing networks need to be enhanced in order to meet the load Demand.

→ To Enhance the power system Networks in future, Surely we have to know about the existing system. This Can be done by load flow Analysis.

→ To establish a new network in any locality we need to go through the load flow Analysis. After analyzing all the things by load flows with the help of MATLAB Software, then the power system Engineers go to the practical works.

→ To know the Condition of Electrical parameters, whether they are maintaining at pu values at each and every bus, must and should we have to go for Load flow Analysis.

* Types of Busses:-

The load flow problem consists of calculation of Voltage magnitude, Voltage Angle or load angle, Real power and Reactive Power at each & every busses.

Now, Depending upon the quantities specified for the busses, they are classified into 3 types.

1. Voltage Controlled Bus (or) Generation Bus (or) PV Bus
2. Load Bus (or) PQ bus
3. Reference Bus (or) Slack Bus (or) Swing Bus.

1. Load Bus:-

Specified	Unspecified
Real power,	Voltage Magnitude,
Reactive power	Voltage Angle (δ)

2. PV Bus:-

Specified	Unspecified
Real power	Reactive power
Voltage magnitude	Voltage Angle (δ).

3. Slack Bus:-

Specified	Unspecified
Voltage magnitude	Real power
Voltage Angle	Reactive power

Bus	Specified	to be Calculate
1. Load Bus (or) PQ Bus	$\rightarrow$ P, Q	$\longrightarrow$ V, δ
2. Generator Bus (or) PV Bus	$\rightarrow$ P, V	$\longrightarrow$ Q, δ.
3. Slack Bus	$\rightarrow$ V, $\delta=0$	$\longrightarrow$ P, Q.

Note :-

The Swing bus is to provide Additional Real and Reactive power to supply the transmission losses. If slack bus is not specified in a problem, then the generation bus, usually with the maximum power generation is to be taken as ~~load~~ Slack Bus (or) Reference Bus.

* Load flow Problem :-

As we know that the complex power by the Source Injected into the i^{th} bus is given by,

$$S_i = P_i + jQ_i = V_i I_i^* \quad \text{--- (1)}$$

It is quite convinient to work with I_i rather than I_i^*

So, the Complex power of above equation gives as,

$$S_i^* = P_i - jQ_i = V_i^* I_i \quad \text{--- (2)}$$

Now again in practical the power system consists of number of buses. At each and every bus the currents will be Calculated as,

$$I = VY \quad \text{--- (A)}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3i} & \cdots & Y_{3n} \\ \vdots \\ Y_{i1} & Y_{i2} & Y_{i3} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix}$$

Now,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + \cdots + Y_{i1}V_i + \cdots Y_{1n}V_n$$

Similarly,

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + \cdots + Y_{2i}V_i + \cdots + Y_{2n}V_n$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + \cdots + Y_{3i}V_i + \cdots + Y_{3n}V_n$$

$\vdots$

In general the above equations are to be written as,

$$\boxed{I_i = \sum_{K=1}^{n} Y_{iK} V_K} \quad \text{(B)}$$

Now, Substituting (B) in eq. (2)

$$(2) \Rightarrow P_i - jQ_i = V_i^* \sum_{K=1}^{n} Y_{iK} V_K$$

where, $V_i^* = |V_i| \angle -\delta_i \rightarrow$ load Angle (δ)

$\downarrow$ magnitude Voltage Angle

Similarly, $V_K = |V_K| \angle \delta_K$

$$Y_{iK} = |Y_{iK}| \angle \theta_{iK}.$$

$$\therefore P_i - jQ_i = |V_i| \angle -\delta_i \sum_{K=1}^{n} |Y_{iK}| \angle \theta_{iK} \, V_K \angle \delta_K$$

$$= |V_i| \sum_{K=1}^{n} |Y_{iK}| |V_K| \angle(-\delta_i + \theta_{iK} + \delta_K)$$

Now,

$$\left.\begin{aligned} \boxed{P_i = |V_i| \sum_{K=1}^{n} |Y_{iK}| |V_K| \cos(\delta_K + \theta_{iK} - \delta_i)} \\ \boxed{Q_i = -|V_i| \sum_{K=1}^{n} |Y_{iK}| |V_K| \sin(\delta_K + \theta_{iK} - \delta_i)} \end{aligned}\right\} \text{(C)}$$

Now, the equations from (C) are known as static load flow equations.

* Guass-Siedal Method (Iterative Method):-

Since, the static load flows are known as Non-linear equations. Such type of Non-linear algebraic equations can be solved by iterative methods.

The Guass-siedel method is an iterative algorithm for solving a set of Non-linear algebraic equations.

The iterative process is Repeated till the solution converges with in a prescribed accuracy. To solve a load-flow problem by G-S. Method, we consider two cases depending up on the type of busses present.

Case-i:- In this case we assume all the busses other than the slack bus are PQ busses.

Case-ii:- In second case we assume the presence of both PQ and PV busses, other than the slack bus.

Case-i: With the slack bus voltage assumed, the other busses voltages are found through iterative process as follows.

As we know that,

$$S = P+jQ = VI^*$$

$$\Rightarrow S^* = P-jQ = V^*I$$

for i^{th} bus $\Rightarrow S_i^* = P_i - jQ_i = V_i^* I_i$

for p^{th} bus $\Rightarrow S_p^* = P_p - jQ_p = V_p^* I_p$

$$\Rightarrow I_p = \frac{P_p - jQ_p}{V_p^*} \quad \text{---} \quad (1)$$

Now, from the performance equation,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_p \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1p} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2p} & \cdots & Y_{2n} \\ \vdots & & & & & & \\ Y_{p1} & Y_{p2} & Y_{p3} & \cdots & Y_{pp} & \cdots & Y_{pn} \\ \vdots & & & & & & \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots & & & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_p \\ \vdots \\ V_n \end{bmatrix}$$

At any bus 'p'.

$$I_p = Y_{p1}E_1 + Y_{p2}E_2 + Y_{p3}E_3 + \cdots + Y_{pp}E_p + \cdots Y_{pn}E_n$$

$$\Rightarrow I_p = \sum_{q=1}^{n} Y_{pq}E_q$$

$$I_p = Y_{pp}E_p + \sum_{\substack{q=1 \\ q\neq p}}^{n} Y_{pq}E_q$$

$$\Rightarrow Y_{pp}E_p = I_p - \sum_{\substack{q=1 \\ q\neq p}}^{n} Y_{pq}E_q$$

$$\Rightarrow E_p = \frac{1}{Y_{pp}}\left[I_p - \sum_{\substack{q=1\\q\neq p}}^{n} Y_{pq} E_q\right] \quad \text{---} (2)$$

Now, Substituting $I_p = \dfrac{P_p - jQ_p}{V_p^*}$ in (2)

$$\Rightarrow E_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1\\q\neq p}}^{n} Y_{pq} E_q\right]$$

for K^{th} number of iterations the above equation can be written as,

$$\boxed{E_p^{(K+1)} = \frac{1}{Y_{pp}}\left(\frac{P_p - jQ_p}{V_p^{K*}} - \sum_{q=1}^{p-1} Y_{pq} E_q^{(K+1)} - \sum_{q=p+1}^{n} Y_{pq} E_q^{K}\right)} \quad \text{---} (3)$$

for $K = 0,1,2,\ldots$

The equation (3) is known as load flow equation for the Gauss-Siedel Method.

For the quick convergence of solution the change in bus voltages is calculated;

$$\boxed{\Delta E_p = E_p^{K+1} - E_p^{K} \leq \varepsilon}$$

$\varepsilon = 0.0001$, ε → Tolerance value

⇒ Acceleration factor :-

Convergence in this method is speeded up by using the Acceleration factor (α). By introducing this facta the voltage correction during iteration is modified as,

$$\boxed{E_{p\,acc}^{(K+1)} = E_p^{K} + \alpha\left[E_p^{(K+1)} - E_p^{K}\right]}$$

'α' value Ranges from 1.3 to 1.7

A generally recommended value is $\boxed{\alpha = 1.6}$. A wrong choice of α may leads to Divergence of a solution.

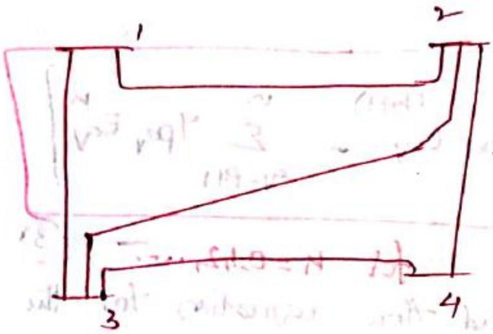
Repeating the iterative process until the voltage correction ΔE_p for all the busses is with in the prescribed limit.

Normally choosen value is $\boxed{\varepsilon = 0.0001}$

ε Value Ranges from 0.00001 to 0.00005

* Problems:-

① For the given Networks, Find out the load flows using the Gauss-Siedel Method at the end of first iteration.



Bus - Bus	Impedance
1-2	0.05 + j0.15
1-3	0.10 + j0.30
2-3	0.15 + j0.45
2-4	0.10 + j0.30
3-4	0.05 + j0.15

Bus Data:-

Bus	P_p	Q_p	V_p	Remarks.
1.	—	—	$1.04\angle 0°$	Slack bus
2	0.5	-0.2	—	PQ
3	-1.0	0.5	—	PQ
4.	0.3	-0.1	—	PQ

Sol: Impedances → Admittances

$1\text{-}2 \rightarrow 0.05 + j0.15 \longrightarrow 2 - j6$

$1\text{-}3 \rightarrow 0.10 + j0.30 \longrightarrow 1 - j3$

$2\text{-}3 \rightarrow 0.15 + j0.45 \longrightarrow 0.666 - j2$

$2\text{-}4 \rightarrow 0.10 + j0.30 \longrightarrow 1 - j3$

$3\text{-}4 \rightarrow 0.05 + j0.15 \longrightarrow 2 - j6$

$$Y_{Bus} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix}$$

$$\therefore Y_{Bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

$$E_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq} E_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq} E_q^{K}\right]$$

Assume iteration counter $K=0$ (1st iteration)

$$\Rightarrow E_2^1 = \frac{1}{Y_{pp}}\left[\frac{P_2 - jQ_2}{E_2^{0*}} - Y_{21}E_1 - Y_{23}E_3 - Y_{24}E_4\right]$$

$$E_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{E_2^{0*}} - Y_{21}E_1^1 - Y_{23}E_3^0 - Y_{24}E_4^0\right]$$

Here, E_3, E_4, E_2 is not specified.

Therefore, assuming flat voltage start is $E_3 = 1+j0$, $E_4 = 1+j0$.

$$\therefore E_2^1 = \frac{1}{3.66-j11}\left[\frac{0.5+j0.2}{1+j0} - (-2+j6)(1.04+j0) - (-0.666+j2)(1+j0) - (-1+j3)(1+j0)\right]$$

$$= \frac{1}{3.66-j11}\left[\frac{0.5+j0.2}{1+j0} - (-2.08+j6.24) - (-0.666+j2) - (-1+j3)\right]$$

$$= \frac{1}{3.66-j11}\left[\frac{0.5+j0.2}{1+j0} + 2.08 - j6.24 + 0.666 - j2 + 1 - j3\right]$$

$$= \frac{1}{3.66-j11}\left[\frac{0.5+j0.2}{1-j0} + (3.746 - j11.24)\right]$$

$$= \frac{1}{3.66-j11}\left[(0.5+j0.2) + (3.746 - j11.24)\right]$$

$$= \frac{1}{3.66-j11}(4.246 - j11.04) = 1.01923 + j0.04687$$

$$E_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.66-j11}\left[\frac{-1-j0.5}{1-j0} - 1.04(-1+j3) - (-0.666+j2)(1.0192+j0.0468) - (-2+j6)(1)\right]$$

$$= \frac{1}{3.66-j11}\left[(-1-j0.5) + 1.04 - j3.12 + 2 - j6 + 0.7723 + j2\right]$$

$$= \frac{1}{3.66-j11}\left(\cancel{1.2677 - j7.62}\right)\left(2.8123 - j11.62\right)$$

$$= 1.0276 - j0.0862.$$

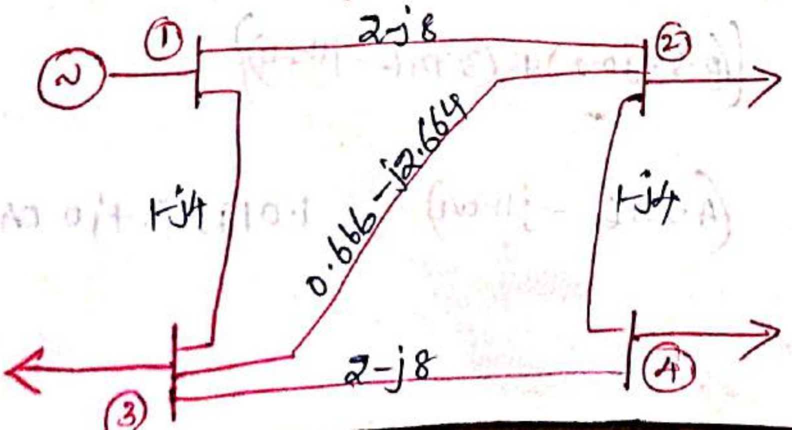
$$E_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{41}V_1^0 - Y_{42}V_2' - Y_{43}V_3'\right]$$

$$= \frac{1}{3-j9}\left[\frac{0.3+j0.1}{1-j0} - (0 - (-1+j3)(1.0192+j0.0468) - (-2+j6)(1.0276-j0.0862)\right]$$

$$= \frac{1}{3-j9}\left[(0.3+j0.1) - (-1.159+j3.01) - (-1.538+j6.338)\right]$$

$$= \frac{2.997 - j9.248}{3-j9} = 1.024 - j8.566\times10^{-3}.$$

(2) The load flow data for a 4 bus system shown in fig is given in table with bus-1 as slack. Determine the bus voltages at end of 1st iteration using Guass-Siedel Method with $\alpha = 1.6$.



Given are Admittances

Bus Data :-

Bus No.	Load P	Load Q	Assumed Bus Voltages
1.	—	—	1.06+j0
2.	0.5	0.2	1+j0
3.	0.4	0.3	1+j0
4.	0.3	0.1	1+j0.

Sol given load Real & Reactive power

$\therefore P_2 = P_{2G} - P_{2L}$
$= 0 - 0.5 = -0.5$

$Q_2 = Q_{2G} - Q_{2L}$
$= 0 - 0.2$
$= -0.2$

$P_3 = P_{3G} - P_{3L}$
$= 0 - 0.4 = -0.4$

$Q_3 = Q_{3G} - Q_{3L}$
$= 0 - 0.3 = -0.3$

$P_4 = P_{4G} - P_{4L}$
$= 0 - 0.3 = -0.3$

$Q_4 = Q_{4G} - Q_{4L}$
$= 0 - 0.1 = -0.1$

given line values are in Admittances.

$$\therefore Y_{Bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} & -Y_{14} \\ -Y_{21} & Y_{22} & -Y_{23} & -Y_{24} \\ -Y_{31} & -Y_{32} & Y_{33} & -Y_{34} \\ -Y_{41} & -Y_{42} & -Y_{43} & Y_{44} \end{bmatrix}$$

$$\therefore Y_{Bus} = \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

$\therefore$ Using Guass-Siedel Method.

$$E_p^{K+1} = \frac{1}{Y_{PP}}\left[\frac{P_p - jQ_p}{(V_p^K)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^K\right]$$

$$\therefore E_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.5+j0.2}{1-j0} \; (-2+j8)(1.06+j0) - (-0.666+j2.664)\,1. - (-1+j4)(1+j0)\right]$$

$$= \frac{1}{3.666 - j14.664}\left[-0.5+j0.2 + 2.12 - j8.48 + 0.666 - j2.664 + 1 - j4\right]$$

$$= \frac{3.286 - j14.944}{3.666 + j14.664} = 1.0118 - j0.0288$$

Now, $E^1_{2acc} = E_2^0 + \alpha\left(E_2^1 - E_2^0\right)$

$$= 1+j0 + \alpha\left[(1.0118 - j0.0288) - (1+j0)\right]$$

$$= 1.0189 - j0.0462$$

$$\therefore E_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4+j0.3}{1-j0} - (-1+j4)(1.06+j0) - (-0.666+j2.664)(1.0189 - j0.0462) - (-2+j8)\,1.\right]$$

$$= \frac{1}{3.666 - j14.664}\left[-0.4+j0.3 + 1.06 - j4.24 + \cancel{0.679 - j2.715} + 0.5555 - j2.7453 + 2 - j8\right)$$

$$= \frac{3.2155 - j14.6853}{3.666 - j14.664} = 0.9941 - j0.02955$$

$$E^1_{3acc} = E_3^0 + \alpha\left(E_3^1 - E_3^0\right)$$

$$= (1+j0) + 1.6\left[(0.9941 - j0.02955) - (1+j0)\right]$$

$$= 0.9946 - j0.0468$$

Similarly,

$$E_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3'\right]$$

$$\Rightarrow E_4^1 = \frac{1}{3-j12}\left[\frac{-0.3+j0.1}{1-j0} - 0 - (-1+j4)(1.01899-j0.0462) - (-2+j8)(0.9946-j0.0468)\right]$$

$$= \frac{1}{3-j12}\left(-0.3+j0.1 + 0.8341 - j4.1224 + 1.6148 - j8.0504\right)$$

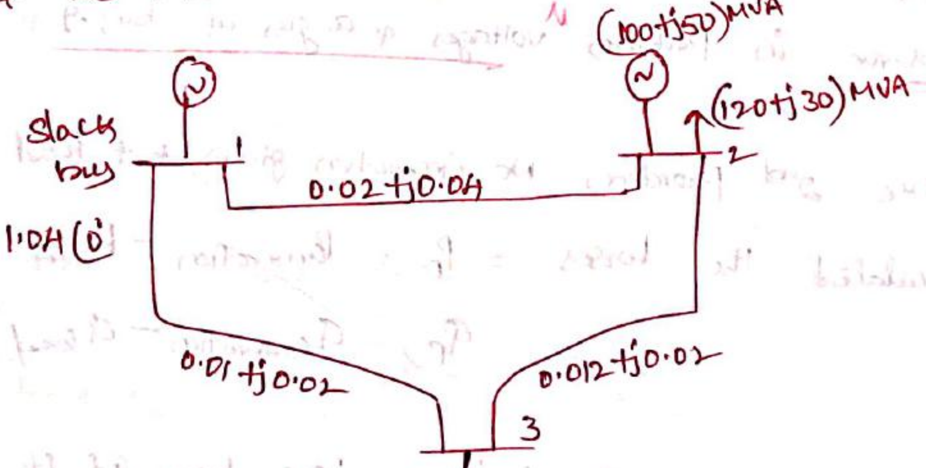
$$= \frac{2.1489 - j12.0725}{3-j12} = 0.9889 - j0.06817$$

Now, $E_{4acc}^1 = E_4^0 + \alpha\left(E_4^1 - E_4^0\right)$

$$= (1+j0) + 1.6\left[(0.9889 - j0.06817) - (1+j0)\right]$$

$$= 0.98224 - j0.1089.$$

3. The power System Network is shown below, the bus-1 is Considered as a Slack bus of voltage $1.04\angle 0^\circ$ pu. The line impedance are indicated in the networks on 100 MVA base and neglect the line shunt Admittance.



By using Gauss-Siedel Method at the end of 1st iteration, Determine the bus voltages and phase angles at bus 2 & 3.

Sol: given the values of between busses are impedances.

As there are 3 busses, so, the Y bus will be of order 3X3.

$$Y_{12} = \frac{1}{0.02+j0.04} = 10-j20, \quad Y_{13} = \frac{1}{0.01+j0.02} = 20-j40$$

$$Y_{23} = \frac{1}{0.012+j0.02} = 22.0588-j36.7647$$

$$\therefore Y_{Bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} \\ -Y_{21} & Y_{22} & Y_{23} \\ -Y_{31} & -Y_{32} & Y_{33} \end{bmatrix}$$

$$\therefore Y_{Bus} = \begin{bmatrix} 30-j60 & -10+j20 & -20+j40 \\ -10+j20 & 32.0588-j56.7647 & -22.0588+j36.764 \\ -20+j40 & -22.0588+j36.7647 & 42.0588-j76.7647 \end{bmatrix}$$

given the bus-1, is slack bus with the voltage $1.04\angle 0°$.

Now, At bus-2 there are the notations of load and generation, since, the voltage magnitude is not mentioned at bus-2 and also when we observe in problem voltages & angles at bus 2 & 3.

As like 2nd problem No Generation given, but load given we have calculated the losses $= P_P = P_{Generation} - P_{Load}$

$$Q_P = Q_{Generation} - Q_{load}$$

∴ The Bus-2 should be considered as PQ bus. If it is a PV bus there must be Q limits & voltage magnitude to be mentioned, but in above problem it does not. So, the Bus-2 is to be taken (or) considered as PQ bus.

Assuming the flat voltage start $V_2^0 = 1+j0$, $V_3^0 = 1+j0$.

Using Guass- siedel Method.

$$E_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^K)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^{K}\right]$$

$$P_p = P_2 = P_G - P_L = 100 - 120 = -20 \quad \bigg| \quad Q_p = Q_2 = Q_G - Q_L = 50 - 30 = 20$$

$\therefore P_2 = -20,\ Q_2 = 20$

$S_2 = -20+j20$

$$S_{2pu} = \frac{\text{Actual value}}{\text{Base value}} = \frac{-20+j20}{100} = \underset{P_2+jQ_2}{-0.2+j0.2}$$

Similarly,

$$P_3 = P_G - P_L = 0 - 30 = -30 \quad \bigg| \quad Q_3 = Q_G - Q_L = 0 - 50 = -50$$

$\therefore S_3 = -30+j(-50)$

$$S_{3pu} = \frac{\text{Actual value}}{\text{Base value}} = \frac{-30-j50}{100} = \underset{P_3+jQ_3}{-0.3-j0.5}$$

$$\therefore P_2 = -0.2,\quad Q_2 = 0.2 \quad \bigg| \quad P_3 = -0.3,\quad Q_3 = -0.5$$

$$\therefore E_2^1 = \frac{1}{Y_{22}} * \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1' - Y_{23}V_3^0\right]$$

$$= \frac{1}{32.0588 - j56.7647}\left[\frac{-0.2-j0.2}{1-j0} - (-10+j20)(1.04) - (-22.0588+j36.764)(1+j0)\right]$$

$$= \frac{1}{32.0588 - j56.7647}\left[-0.2 - j0.2 - (-10.4 + j20.8) - 22.0588 - j36.7647\right]$$

$$= \frac{1}{32.0588 - j56.7647}\left[-0.2 - j0.2 + 10.4 - j20.8 + 22.0588 - j36.7647\right]$$

$$= \frac{\cancel{11.8588}\ 32.2588 - j57.764}{32.0588 - j56.7647}$$

$$E_2^1 = 1.0148 - j4.8719 \times 10^{-3}$$

Similarly,

$$E_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$= \frac{1}{42.0588 - j76.7647}\left[\frac{-0.3 + j0.5}{1 - j0} - (-20 + j40)(1.04 + j0) - (-22.0588 + j36.7647)(1.0148 - j4.871\times10^{-3})\right]$$

$$= \frac{1}{42.0588 - j76.7647}\left[-0.3 + j0.5 - (-20.8 + j41.6) - (-22.2061 + j37.4162)\right]$$

$$= \frac{1}{42.0588 - j76.7647}\left[-0.3 + j0.5 + 20.8 - j41.6 + 22.2061 - j37.4162\right]$$

$$= \frac{42.7061 - j78.5162}{42.0588 - j76.7647}$$

$$= 1.0211 - j0.003129.$$

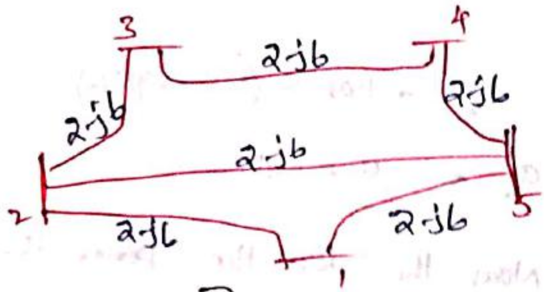
4. Each line has an impedance of 0.05+j0.15 for the network shown below. The bus data is given as,

Bus	P_L	Q_L	P_G	Q_G	V	Remarks.
1.	1.0	0.5	—	—	$1.02\angle 0^\circ$	Slack
2.	0	0	2	—	1.02	PV
3.	0.5	0.2	0	0	—	PQ
4.	0.5	0.2	0	0	—	PQ
5.	0.5	0.2	0	0	—	PQ.

Find $\delta_2, Q_2, V_3, V_4, V_5$ and $\delta_3, \delta_4, \delta_5$ using G.S. Method after the end of first iteration. Assume Reactive power limits $0.2 \le Q \le 0.6$.

Sol given are impedances of

$0.05+j0.15$



$$\therefore Y = \frac{1}{0.05+j0.15} = 2-6j$$

$$\text{The } Y_{Bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} & -Y_{14} & -Y_{15} \\ -Y_{21} & Y_{22} & -Y_{23} & -Y_{24} & -Y_{25} \\ -Y_{31} & -Y_{32} & Y_{33} & -Y_{34} & -Y_{35} \\ -Y_{41} & -Y_{42} & -Y_{43} & Y_{44} & -Y_{45} \\ -Y_{51} & -Y_{52} & -Y_{53} & -Y_{54} & Y_{55} \end{bmatrix}$$

$$\therefore Y_{Bus} = \begin{bmatrix} 4-j12 & -2+j6 & 0 & 0 & -2+j6 \\ -2+j6 & 6-j18 & -2+j6 & 0 & -2+j6 \\ 0 & -2+j6 & 4-j12 & -2+j6 & 0 \\ 0 & 0 & -2+j6 & 4-j12 & -2+j6 \\ -2+j6 & -2+j6 & 0 & -2+j6 & 6-j18 \end{bmatrix}$$

given 2nd bus as PV bus

to find out $Q_2 = -|V_i| \sum_{q=1}^{n} Y_{iq} V_q \sin(\delta_q + \theta_{iq} - \delta_i)$

$$\Rightarrow Q_2 = -|V_i|\left[Y_{21}V_1 \sin(\delta_1+\theta_{21}-\delta_2) + Y_{22}V_2 \sin(\delta_2+\theta_{22}-\delta_2) + Y_{23}V_3 \sin(\delta_3+\theta_{23}-\delta_2) + Y_{24}V_4 \sin(\delta_4+\theta_{24}-\delta_2) + Y_{25}V_5 \sin(\delta_5+\theta_{25}-\delta_2)\right]$$

$$\therefore Y_{Bus} = \begin{bmatrix} 12.64\angle{-71.565} & 6.324\angle{108.434} & 0 & 0 & 6.324\angle{108.324} \\ 6.324\angle{108.324} & 18.9736\angle{-71.565} & 6.324\angle{108.324} & 0 & 6.324\angle{108.324} \\ 0 & 6.324\angle{108.324} & 12.64\angle{-71.565} & 6.324\angle{108.324} & 0 \\ 0 & 0 & 6.324\angle{108.324} & 12.64\angle{-71.565} & 6.324\angle{108.324} \\ 6.324\angle{108.324} & 6.324\angle{108.324} & 0 & 6.324\angle{108.324} & 18.9736\angle{-71.565} \end{bmatrix}$$

Now,

$$Q_2 = -1.02\Big[1.02\times 6.324\sin(108.324) + 18.9736\times 1.02\times\sin(-71.565) + 6.324\times 1\times \sin(108.324) + 0 + 6.324\times 1\times\sin(108.324)\Big]$$

$$= -1.02\times(-0.22986)$$

$$Q_2 = 0.2391$$

$\because$ at busses 3,4,5 $|V|$ & δ are not Mentioned so, assumed flat voltage start i.e, $V_3 = V_4 = V_5 = 1+j0 \Rightarrow 1\angle 0^\circ$ $\therefore |V_3| = |V_4| = |V_5| = 1$, $\delta_3 = \delta_4 = \delta_5 = 0$

Now the Reactive power limits are $0.2 \le Q \le 0.6$

the Calculated value is with in the limits, $\therefore$ Considering the 2nd bus as PV bus itself and $Q_2 = 0.2391$.

$\therefore$ to Calculate δ_2

$$\Rightarrow E_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$P_2 = P_G - P_L$
$= 2 - 0 = 2$

$Q_2 = 0.2391$

Assuming flat voltage start
$V_3^0 = V_4^0 = V_5^0 = 1+j0$

Similarly,

$P_3 = P_G - P_L$
$= 0 - 0.5$
$= -0.5$

$Q_3 = Q_G - Q_L$
$= 0 - 0.2$
$= -0.2$

$P_5 = P_G - P_L$
$= 0 - 0.5 = -0.5$

$P_4 = P_G - P_L$
$= 0 - 0.5$
$= -0.5$

$Q_4 = Q_G - Q_L$
$= 0 - 0.2$
$= -0.2$

$Q_5 = Q_G - Q_L$
$= 0 - 0.2$
$= -0.2$

$$E_2' = \frac{1}{6-j18}\left[\frac{2-j0.2391}{1.02-j0} - (-2+j6)(1.02+j0) - (-2+j6)\cdot 1 - (-2+j6)\cdot 1\right]$$

$$= \frac{1}{6-j18}\left[1.9607 - j0.2344 - (-2.04+j6.12) + 2 - j6 + 2 - j6\right]$$

$$= \frac{8 - 18.3544j}{6-j18} = 1.051 + j0.094$$

$$= 1.055\angle 5.11 = \underline{1.02\angle 5.11}$$

$\because$ PV bus

$$E_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^0 - V_5^0 Y_{35}\right]$$

Should be $1.02\angle 5.11$

$$= \frac{1}{4-j12}\left[\frac{-0.5+j0.2}{1+j0} - 0 - (-2+j6)(1.051+j0.094) - (-2+j6)1 - 0\right]$$

$$= \frac{1}{4-j12}\left[-0.5+j0.2 - (-2.666+j6.118) + 2 - j6\right]$$

$$= \frac{4.166 - j11.918}{4-j12} = 0.998 + j0.0145 = 0.998\angle 0.86$$

$$E_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3' - Y_{45}V_5^0\right]$$

$$= \frac{1}{4-j12}\left[\frac{-0.5+j0.2}{1-j0} - 0 - 0 - (-2+j6)(0.998+j0.0145) - (-2+j6)(1+j0)\right]$$

$$= \frac{1}{4-j12}\left[-0.5+j0.2 - (-2.083+j5.939) + 2 - j6\right]$$

$$= \frac{3.583 - j11.739}{4-j12} = 0.97 - j0.02475$$

$$E_5' = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{(V_5^0)^*} - Y_{51}V_1' - Y_{52}V_2' - Y_{53}V_3' - Y_{5}V_4'\right]$$

$$= \frac{1}{6-j18}\left[\frac{-0.5+j0.2}{1-j0} - (-2+j6)(1.02) - (-2+j6)(1.051+j0.094) - (-2+j6)(0.97-j0.02475)\right]$$

$$= \frac{1}{6-j18}\left[-0.5+j0.2 + 2.04 - j6.12 + 2.666 - j6.118 + 1.791 - j5.8695\right)$$

$$\therefore E_5^1 = \frac{5.997 - j17.9075}{6 - j18} = 0.9953 + j\,1.3916 \times 10^{-3}$$

(15) If the Reactive power limits are $0.3 \le Q \le 0.6$ for the above problem of same data. Then Calculate the load flows.

Sol: from the above Problem,

we have Solved, $Q_2 = 0.2391$

limits are $0.3 \le Q \le 0.6$. Since Calculated value is not in the limits. So, bus-2 treating as PQ bus

$Q_2 = 0.3$. [$\because$ value nearer to the limit $Q_{min} \therefore Q_2 = Q_{min}$]

$$E_2^1 = \frac{1}{6 - j18}\left[\frac{2 - j0.3}{1.02 - j0} - (-2+j6)\,1.02 - (-2+j6).1 - (-2+j6).1\right]$$

$$= \frac{1}{6 - j18}\left[1.96 - j0.2941 + 2.04 - j6.12 + 2 - j6 + 2 - j6\right]$$

$$= \frac{8 - j18.4141}{6 - j18} = 1.054 + j0.0930 \quad (\because \text{PQ bus})$$

$$E_3^1 = \frac{1}{4 - j12}\left[\frac{-0.5 + j0.2}{1 - j0} - (-2+j6)(1.054 + j0.093) - (-2+j6).1\right]$$

$$= \frac{1}{4 - j12}\left[-0.5 + j0.2 + 2.666 - j6.138 + 2 - j6\right]$$

$$= \frac{4.166 - j11.938}{4 - j12} = 0.9995 + j0.014.$$

$$E_4^1 = \frac{1}{4 - j12}\left[\frac{-0.5 + j0.2}{1 - j0} - 0 - 0 - (-2+j6)(0.9995 + j0.014) - (-2+j6)\right]$$

$$= \frac{1}{4 - j12}\left(-0.5 + j0.2 + 2 - j6 + 2.083 - j5.969\right)$$

$$= \frac{3.583 - j11.769}{4 - j12} = 0.972 - j0.025$$

$$E_5' = \frac{1}{6-j18}\left[\frac{-0.5+j0.2}{1-j0} - (-2+j6)(1.02+j0) - (-2+j6)(1.054+j0.093) - (-2+j6)(0.972-j0.025)\right]$$

$$= \frac{1}{6-j18}\left[-0.5+j0.2+2.04-j6.12+2.666-j6.138+1.794-j5.882\right]$$

$$= \frac{6-j17.94}{6-j18} = 0.997+j1.\times10^{-3}.$$

Note :-

→ If the calculated 'Q' value is with in the limits. Then Considering that bus as PV bus it self And whatever may be the calculated voltage magnitude. we have to take the specified voltage magnitude and obtained angle.

→ If the calculated 'Q' values are not with in the limits. Then treating that bus as PQ bus. and calculating the voltage magnitude and voltage angle.

6. Form the Y-Bus for the 4-bus system, if the line impedances are

bus-to-bus	impedance
1-2	0.15+j0.6
1-3	0.1+j0.4
1-4	0.15+j0.6
2-3	0.05+j0.2
3-4	0.05+j0.2

the bus data is given,

Bus	P_L	Q_L	P_G	Q_G	V	Specification
1.	2	0.5	—	—	$1\angle 0°$	Slack
2.	0	0	1	—	1	PV $Q_2 min = 0$ & $Q_2 max = 0.6$
3.	0.7	0.3	0	0	—	PQ
4.	0.7	0.3	0	0	—	PQ

find $Q_2, \delta_2, V_3', V_4'$ after first iteration using G.S. Method.

Sol. The Admittances are

$1-2 \rightarrow 0.392 - j1.568$

$1-3 \rightarrow 0.588 - j2.352$

$1-4 \rightarrow 0.392 - j1.568$

$2-3 \rightarrow 1.176 - j4.705$

$3-4 \rightarrow 1.176 - j4.705$

$$\therefore Y_{Bus} = \begin{bmatrix} 1.372 - j5.491 & -0.392 + j1.568 & -0.588 + j\overset{2.353}{\cancel{2.352}} & -0.392 + j1.569 \\ -0.392 + j1.569 & +1.568 - j6.275 & -1.176 + j4.706 & 0 \\ -0.588 + j2.353 & -1.176 + j4.706 & 2.94 - j11.765 & -1.176 + j4.706 \\ -0.392 + j1.569 & 0 & -1.176 + j4.706 & 1.568 - j6.275 \end{bmatrix}$$

$$= \begin{bmatrix} 5.659\angle{-75.971} & 1.616\angle{104.036} & 2.425\angle{104.030} & 1.616\angle{104.036} \\ 1.616\angle{104.036} & 6.467\angle{-75.970} & 4.850\angle{104.030} & 0 \\ 2.425\angle{104.030} & 4.850\angle{104.030} & 12.126\angle{-75.969} & 4.850\angle{104.030} \\ 1.616\angle{104.036} & 0 & 4.850\angle{104.030} & 6.467\angle{-75.970} \end{bmatrix}$$

given bus-2 is PV bus

$V_2 = 1.$, assuming flat voltage start $V_3 = V_4 = 1 + j0$.

$$Q_i = -V_i \sum_{K=1}^{n} Y_{iK} V_K \sin(\delta_K + \theta_{iK} - \delta_i)$$

$$Q_2 = -V_2 \left[Y_{21} V_1 \sin(\delta_1 + \theta_{21} - \delta_2) + Y_{22} V_2 \sin(\delta_2 + \theta_{22} - \delta_2) + Y_{23} V_3 \sin(\delta_3 + \theta_{23} - \delta_2) + Y_{24} V_4 \sin(\delta_4 + \theta_{24} - \delta_2) \right]$$

$$= -1 \left[1.616 \times 1 \times \sin(104.036) + 6.467 \times 1 \times \sin(-75.970) + 4.850 \times 1 \times \sin(104.030) \right]$$

$$= 0.0010 \cong 0.$$

The obtained 'Q' value is within the limits $\therefore$ Considering the bus-2 as PV bus itself.

$$E_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$E_2^1 = \frac{1}{1.568-j6.275}\left[\frac{1-j0}{1-j0} - (-0.392+j1.569)(1+j0) - (-1.176+j4.706)\cdot 1\right]$$

$$= \frac{1}{1.568-j6.275}\left[1-j0+0.392-j1.569+1.176-j4.706\right]$$

$$= \frac{2.568-j6.275}{1.568-j6.275} = 1.0374+j0.14999$$

$$= 1.048 \angle 8.226^\circ$$

$\because$ bus-2 Considered as PV bus

$$\therefore E_2^1 = 1 \angle 8.226^\circ = 0.989+j0.1430$$

$$E_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3-jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{2.94-j11.765}\left[\frac{-0.7+j0.3}{1+j0} - (-0.588+j2.353)1 - (-1.176+j4.706)(0.989+j0.1430) - (-1.176+j4.706)\cdot 1\right]$$

$$= \frac{1}{2.94-j11.765}\left[-0.7+j0.3+0.588-j2.353+j1.836-j4.486+1.176-j4.706\right]$$

$$= \frac{2.9-j11.245}{2.94-j11.765} = 0.957+j7.195\times10^{-3}$$

$$= 0.957+j0.007195$$

$$= 0.957 \angle 0.43^\circ$$

$$E_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4-jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{1.568-j6.275}\left[\frac{-0.7+j0.3}{1-j0} - (-0.392+j1.569)(1+j0) - 0 - (-1.176+j4.706)(0.957+j0.007195)\right]$$

$$= \frac{1}{1.568-j6.275}\left(-0.7+j0.3+0.392-j1.569+1.1592-j4.4951\right)$$

$$= \frac{0.8512-j5.7641}{1.568-j6.275} = 0.896-j0.088$$

$$= 0.9 \angle -5.60^\circ$$

(7) The load flow data of a 4-bus System is given in tables. Assume the bus voltage of bus-3 as 1.04. Reactive power limits are $0 \le Q \le 0.3$. Taking bus-1 as Reference. Determine the bus voltages after 1st iteration using G-S- method.

Bus No	Voltages	Generation P	Generation Q	Load P	Load Q
1	1.06+j0	0	0	0	0
2	1.0+j0	0	0	0.2	0.1
3	1.04+j0	0.6	0.3	0.4	0.2
4	1.00+j0	0.0	0.0	0.4	0.05

Line data:

Bus Code	Impedance	Charging Admittance
1-2	0.02+j0.08	j0.04
1-3	0.06+j0.24	j0.03
2-3	0.04+j0.16	j0.025
2-4	0.04+j0.16	j0.025
3-4	0.01+j0.04	j0.015

Sol. given lines are impedances.

1-2 ⟶ 0.02+j0.08 ⟶ 2.941 -j11.764

1-3 ⟶ 0.06+j0.24 ⟶ 0.980 -j3.921

2-3 ⟶ 0.04+j0.16 ⟶ 1.470 -j5.882

2-4 ⟶ 0.04+j0.16 ⟶ 1.470 -j5.882

3-4 ⟶ 0.01+j0.04 ⟶ 5.882 -j23.529

Charging Admittance $y_1 = y_{12} + y_{13} = 0.04 + 0.03$
$= j0.07$

" " $y_2 = y_{23} + y_{24} + y_{12} = j0.025 + j0.025 = j0.05 + 0.04$
$= j0.09$

" " $y_3 = y_{13} + y_{23} + y_{34} = j0.03 + j0.025 + j0.015$
$= j0.07$

charging Admittance $y_4 = y_{24} + y_{34}$

$$= j0.025 + j0.015$$
$$= j0.04.$$

$$Y_{Bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} & -Y_{14} \\ -Y_{21} & Y_{22} & -Y_{23} & -Y_{24} \\ -Y_{31} & -Y_{32} & Y_{33} & -Y_{43} \\ -Y_{41} & -Y_{42} & -Y_{43} & Y_{44} \end{bmatrix}$$

$$\therefore Y_{Bus} = \begin{bmatrix} 3.921 - j15.615 & -2.941 + j11.764 & -0.980 + j3.921 & 0 \\ -2.941 + j11.764 & 5.881 - j23.438 & -1.470 + j5.882 & -1.470 + j5.882 \\ -0.980 + j3.921 & -1.490 + j5.882 & 8.332 - j33.262 & -5.882 + j23.529 \\ 0 & -1.470 + j5.882 & -5.882 + j23.529 & 7.352 - j29.371 \end{bmatrix}$$

$$= \begin{bmatrix} 16.099\angle -75.904 & 12.126\angle 104.036 & 4.041\angle 104.032 & 0 \\ 12.126\angle 104.036 & 24.164\angle -75.914 & 6.062\angle 104.031 & 6.062\angle 104.031 \\ 4.041\angle 104.032 & 6.062\angle 104.031 & 34.289\angle -75.937 & 24.253\angle 104.035 \\ 0 & 6.062\angle 104.031 & 24.253\angle 104.035 & 30.277\angle -75.946 \end{bmatrix}$$

Now Assuming the flat voltage start

i.e. $V_4^0 = 1 + j0$, $V_2^0 = 1 + j0$.

$$P_2 = P_G - P_L = 0 - 0.2 = -0.2$$

$$Q_2 = Q_G - Q_L = 0 - 0.1 = -0.1$$

$$\therefore E_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0 \right]$$

(it should be 1.06)

$$= \frac{1}{5.881 - j23.438} \left[\frac{-0.2 + j0.1}{1 - j0} - (-2.941 + j11.764)\cdot 1 - (-1.470 + j5.882)\cdot 1.04 - (-1.470 + j5.882)\, 1 \right]$$

$$= \frac{1}{5.881 - j23.438} \left[-0.2 + j0.1 + 2.941 - j11.764 + 1.470 - j5.882 + 1.470 + (-j5.882) \right]$$

$$\frac{5.681 - j23.428}{5.881 - j23.438} = 0.9975 - j7.927 \times 10^{-3}$$

$$= 0.9975 - j0.007927$$

$$= 0.9975 \angle -0.455$$

given bus-3 as PV bus.

$$Q_3 = -V_3\left[Y_{31}V_1\sin(\delta_1+\theta_{31}-\delta_3) + Y_{32}V_2\sin(\delta_2+\theta_{32}-\delta_3) + Y_{33}V_3\sin(\delta_3+\theta_{33}-\delta_3) + Y_{34}V_4\sin(\delta_4+\theta_{34}-\delta_3)\right]$$

$$= -1.04\left[4.041\times1\times\sin(104.032) + 6.062\times\overset{0.9975}{\cancel{1}}\times\sin(-0.455+104.031-0) + 34.289\times1.04\times\sin(-75.937) + 24.253\times1\times\sin(104.035)\right]$$

$$= -1.04\times-1.249 \quad = 1.299$$

given limits are $0 \le Q \le 0.3$

obtained value is not with in the limits. Therefore Considering bus-3 as PQ bus.

$Q_3 = 0.3$

$P_3 = P_G - P_L = 0.6 - 0.4 = 0.2$ | ~~$Q_3 = Q_G - Q_L = 0.3 - 0.2 = 0.1$~~ Limits

Q_3 value 0.3 considered

Should be 1.06

$$E_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{8.332-j33.262}\left[\frac{0.2-j0.3}{1.04+j0} - (-0.980+j3.921)(1+j0) - (-1.470+j5.882)(0.9975-j0.007927) - (-5.882+j23.529)1\right]$$

$$= \frac{1}{8.332-j33.262}\left[(0.192-j0.288)+0.980-j3.921+1.4196-j5.878+5.882-j23.529\right]$$

$$= \frac{1}{8.332-j33.262}\times 8.4736-j33.616 = 1.011+j1.497\times10^{-3}$$

$$= 1.011+j0.001497.$$

$$E_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4-jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$P_4 = P_G - P_L = -0.4$ | $Q_4 = Q_G - Q_L = 0 - 0.05 = -0.05$

$$= \frac{1}{7.352-j29.371}\left[\frac{-0.4+j0.05}{1j0} - (-1.470+j5.882)(0.9975-j0.007927) - (-5.882+j23.529)(1.011+j0.001497)\right]$$

$$\therefore E_4^1 = \frac{1}{7.352-j29.371}\left[-0.4+j0.05+1.4196-j5.878+5.981-j23.779\right)$$

$$= \frac{7-j29.607}{7.352-j29.371} = 1.004\ j0.013.$$

* Algorithm (when only PQ busses are present):-

1. The slack bus voltage magnitude and voltage angle are assumed usually 1+j0 pu. The load profile and the Generation profile are specified at each load busses and Generation busses Respectively.

2. Compute the YBus with the help of line data and the Shunt Admittances data (if any) using Direct Inspection method. If the mutual Admittances are given then, the computation of YBus will be done by using singular Transformation method.

3. Iterative computation of bus voltages To start the iterations set of flat voltage starts are assumed at the busses other than slack bus as 1+j0.

Since all are PQ busses the unspecified values (|V|, δ) have to be calculated By the Guass-Siedel Method the unspecified parameters at each and every busses will be calculated by the following formula

$$\boxed{E_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^K)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^K\right]}$$

And this Process will be continued for number of iterations at every bus till the change in magnitude of bus voltages, between two Consecutive iterations is less than a equal to to the Tolerance value (ε).

$\varepsilon = 0.0001$

$$\Delta V_i^{K+1} = V_i^{K+1} - V_i^{K} \le \varepsilon$$

4. After finding all the parameters at each & every bus, the Computation of P, Q at slack bus to be done by static load flow study equations.

$$P = |V_i| \sum_{K=1}^{n} Y_{iK} V_K \cos(\delta_K + \theta_{iK} - \delta_i)$$

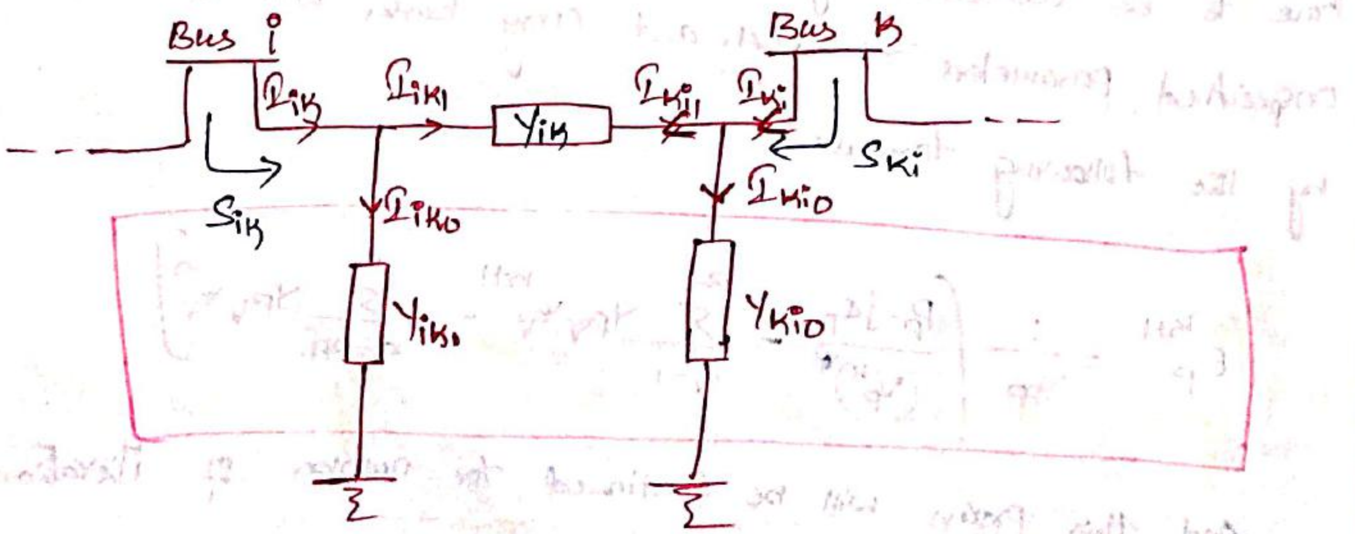
$$Q = -|V_i| \sum_{K=1}^{n} Y_{iK} V_K \sin(\delta_K + \theta_{iK} - \delta_i)$$

where K = number of busses.

5. Now, after finding all the parameters at every bus,

The Computation of power flows & load flows takes place

Consider the line connecting busses i & K. The line & transformers at each end can be represented by a circuit with series Admittance (Y_{iK}) and two shunt Admittances Y_{iK0} & Y_{Ki0}.



Now the current fed by bus i into the line can be expressed as,

$$I_{ik} = I_{ik_1} + I_{ik_0} = (V_i - V_k) Y_{ik} + V_i Y_{ik0}$$

Now, the power fed into the line from the bus i,

$$S_{ik} = P_{ik} + jQ_{ik} = V_i I_{ik}^*$$

$$= V_i \left((V_i - V_k) Y_{ik} + V_i Y_{ik0} \right)^*$$

$$\Rightarrow \boxed{S_{ik} = V_i (V_i^* - V_k^*) Y_{ik}^* + V_i V_i^* Y_{ik0}^*} \quad \text{— (A)}$$

Similarly, the power fed into the line from bus k.

$$\Rightarrow \boxed{S_{ki} = V_k (V_k^* - V_i^*) Y_{ki}^* + V_k V_k^* Y_{ki0}^*} \quad \text{— (B)}$$

Now, the overall power loss in the line $i-k$ is the sum of the equations (A) and (B).

$\therefore$ The total transmission load flows can be computed by summing all the line flows for 'n' number of busses.

*** Algorithm (when both PQ and PV busses are present):**

1. First repeat the iteration for PQ busses as in above algorithm then continue for the PV busses. At the PV bus the real power & voltage magnitude are specified and Q, δ are unknowns to be determined.

2. At PV bus the reactive power can be calculated by the static power flow equation.

$$\boxed{Q_i = -|V_i| \sum_{k=1}^{n} Y_{ik} V_k \sin(\delta_k + \theta_{ik} - \delta_i)}$$

For the PV bus to be as PV bus the calculated 'Q' value will be with in the specified limits $Q_{min} \le Q \le Q_{max}$.

If it is with in the limits then consider that bus as PV bus itself and Q = Calculated Value

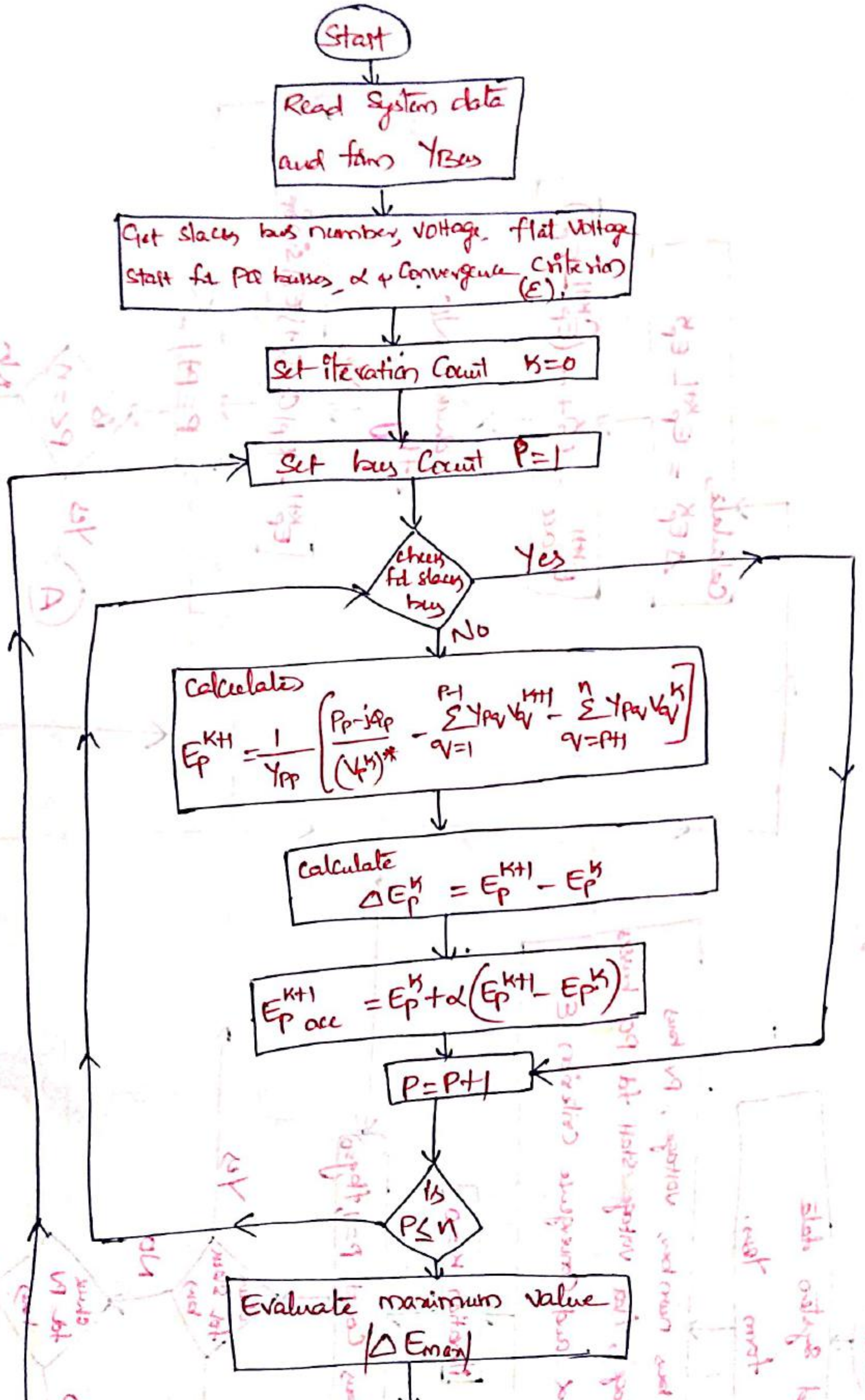
To find out δ again going to the main formula for G.S. Method. Using that formule the voltage magnitude & angle will be found. Since this PV bus the magnitude will be the specified value and not the calculated one and δ will be taken.

3. If the calculated 'Q' value is not with in the limits then treat it as PQ bus. Now, the Q value will be the limit nearer to the calculated value will be considered.

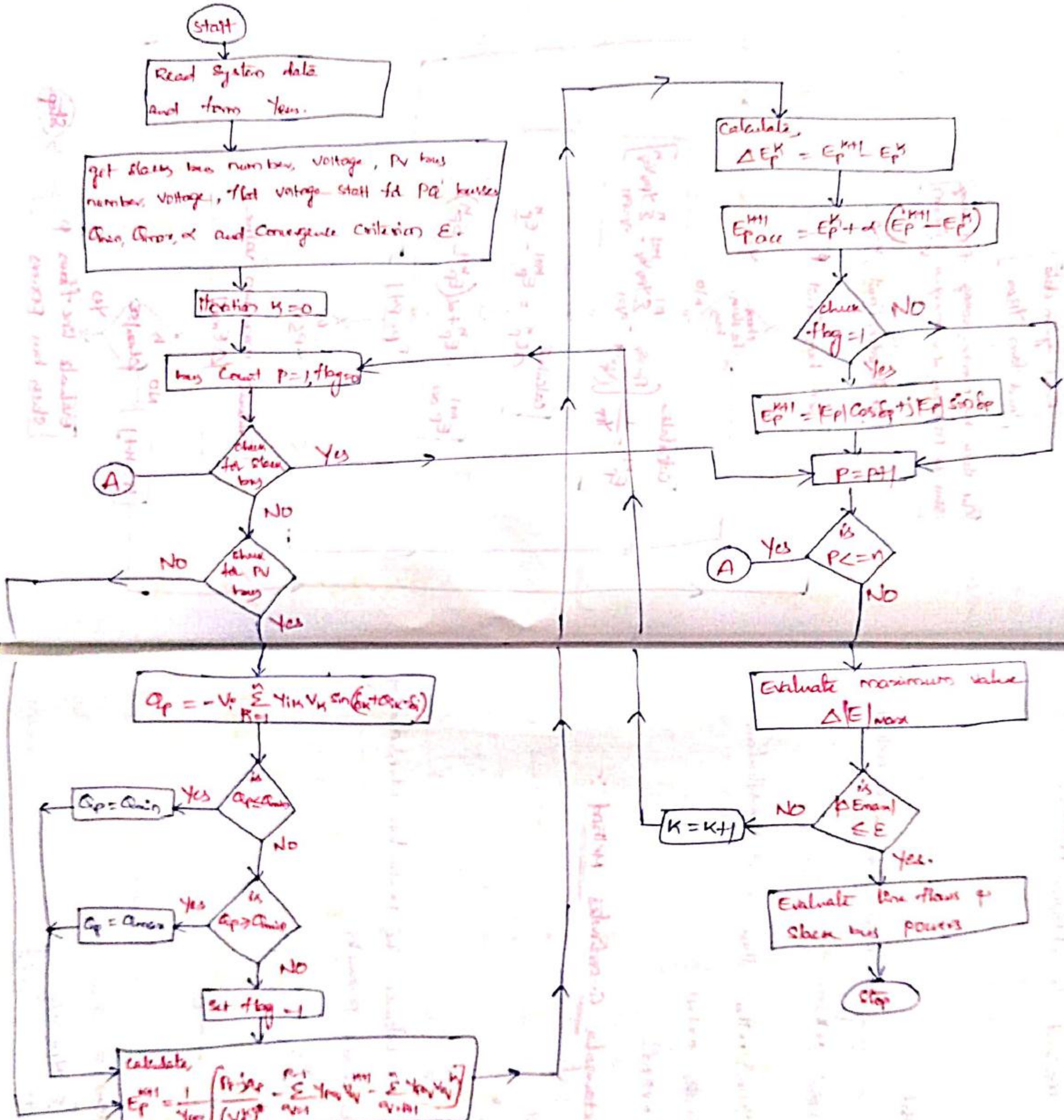
To find out δ going to main formula of G.S. Method. what ever may be the specified voltage magnitude, the calculated voltage magnitude & angle to be taken in this case.

Since treating as PQ busses ($\because$ Q not in limits)

* Flowchart for G.S. Method (only PQ busses are present) :-



* Flowchart for G.S. Method when PV busses also present :-



Date
11/4/19

Unit - IV

Load Flows - II

Load flows can be obtained by different methods,

1. G-S-Method.
2. N-R Method
3. Decoupled Method
4. Fast Decoupled Method.

In this chapter we are going to discuss about N-R Method, Decoupled and Fast Decoupled Methods.

In Newton-Raphson (N-R) Method there are 2 classifications:

1. Rectangular Co-ordinates method
2. Polar - Co-ordinates method.

* Newton Raphson Rectangular Co-ordinates Method :-

As we know that,

$$P + jQ = V I^*$$

$$\Rightarrow P_p + jQ_p = V_p I_p^*$$

It is quite difficult to calculate the parameters when dealing with the conjugate to the current parameter.

$$\therefore P_p - jQ_p = V_p^* I_p \quad \text{—— (A)}$$

$$\Rightarrow I_p = \frac{P_p - jQ_p}{V_p^*} \quad \text{—— (1)}$$

Also we know that,

$$I = V \cdot Y_{Bus}$$

$$\Rightarrow I_p = \sum_{q=1}^{n} Y_{pq} V_q \quad \text{—— (2)}$$

At Equilibrium Condition ① = ②

$$\Rightarrow \frac{P_p - jQ_p}{E_p^*} = \sum_{q=1}^{n} Y_{pq} V_q$$

$$\Rightarrow P_p - jQ_p = E_p^* \sum_{q=1}^{n} Y_{pq} V_q$$

$$\Rightarrow P_p - jQ_p = E_p^* \sum_{q=1}^{n} Y_{pq} E_q \quad \text{--- ③}$$

for Rectangular Co-ordinates, lets $E_p = e_p + jf_p$, $Y_{pq} = G_{pq} - jB_{pq}$

Substituting in above equation.

$$\Rightarrow P_p - jQ_p = (e_p + jf_p)^* \sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q)$$

* Hidden *

$$\therefore P_p - jQ_p = (e_p - jf_p) \sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q).$$

$$= (e_p - jf_p) \sum_{q=1}^{n} (e_q G_{pq} + jf_q G_{pq} - jB_{pq} e_q + f_q B_{pq})$$

$$= (e_p - jf_p) \sum_{q=1}^{n} (e_q G_{pq} + f_q B_{pq}) + j (f_q G_{pq} - e_q B_{pq})$$

Now, Seperating the Real and Imaginary parts we get

$$\boxed{\begin{aligned} P_p &= \sum_{q=1}^{n} e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \quad \text{--- (B)} \\ Q_p &= \sum_{q=1}^{n} f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \quad \text{--- (C)} \end{aligned}}$$

and $E_p = e_p + jf_p$

$$\Rightarrow |E_p| = \sqrt{e_p^2 + f_p^2}$$

$$\Rightarrow \boxed{|E_p|^2 = e_p^2 + f_p^2} \quad \text{--- (D)}$$

Now, the equations (B), (C) & (D) are known as the static load flow equations for Newton Raphson Rectangular Co-ordinates method.

The left hand side quantities P_p, Q_p and E_p are the specified quantities at the busses.

where, P_p, Q_p are the specified quantities at the load bus.

Similarly, $P_p, |E_p|^2$ are the specified quantities at the PV bus.

∴ The above equations (B), (C), (D) are the Non-linear equations interms of Real and imaginary terms.

* Newton Raphson Rectangular Co-ordinates Method:

If we consider $x_1, x_2, x_3 \ldots x_n$ are the unknown variables, $y_1, y_2, y_3 \ldots y_n$ are known variables then these are related with a set of non-linear equations are,

$$y_1 = f_1(x_1, x_2, x_3, \ldots x_n)$$

$$y_2 = f_2(x_1, x_2, x_3, \ldots x_n)$$

$$\vdots$$

(E) $$y_n = f_n(x_1, x_2, \ldots x_n)$$

Now to solve these equations, we have to start with approximate solution which is zeroth iteration, then these are represented as,

$$y_1 = f_1(x_1^0, x_2^0, x_3^0 \ldots x_n^0)$$

$$y_2 = f_2(x_1^0, x_2^0, x_3^0 \ldots x_n^0)$$

$$\vdots$$

$$y_n = f_n(x_1^0, x_2^0, x_3^0, \ldots x_n^0)$$

Now, Assuming $\Delta x_1^0, \Delta x_2^0 \ldots \Delta x_n^0$ are the corrections required for $x_1^0, x_2^0, \ldots x_n^0$ respectively for next better solution.

The equations are Represented as,

$$y_1 = f_1\left(x_1^0+\Delta x_1^0,\ x_2^0+\Delta x_2^0,\ x_3^0+\Delta x_3^0, \ldots\ldots x_n^0+\Delta x_n^0\right)$$

By the Taylor's series the above equation can be expanded as,

$$y_1 = f_1\left(x_1^0, x_2^0, x_3^0, \ldots x_n^0\right) + \Delta x_1^0 \frac{\partial f_1}{\partial x_1} + \Delta x_2^0 \frac{\partial f_1}{\partial x_2} + \ldots + \Delta x_n^0 \frac{\partial f_1}{\partial x_n} + \phi$$

Where, ϕ is the higher order differential terms and so neglecting in Newton Raphson Method.

Now, if we Represents all such equations in Matrix form as,

$$\begin{bmatrix} y_1 - f_1(x_1^0, x_2^0, \ldots x_n^0) \\ y_2 - f_2(x_1^0, x_2^0, \ldots x_n^0) \\ y_3 - f_3(x_1^0, x_3^0 \ldots x_n^0) \\ \vdots \\ y_n - f_n(x_1^0, x_2^0, x_3^0 \ldots x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \Delta x_3^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix}$$

$$\Rightarrow B = J \times C$$

where, B = Residual Column Vector

J = Jacobian Matrix

C = Corrections Required for the next better Solution.

for next iteration,

$$x_1^1 = x_1^0 + \Delta x_1^0$$

$$x_2^1 = x_2^0 + \Delta x_2^0 \quad \ldots\ldots \quad x_n^1 = x_n^0 + \Delta x_n^0$$

Rectangular Co-ordinates method when only PQ busses :-

when Reffered to the typical power system there present a load busses and Generation busses. Now, Similar to the above Equations the power system equations will be,

$$
\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix}
=
\left[\begin{array}{cccc|cccc}
\frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \frac{\partial P_2}{\partial e_4} \cdots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \cdots & \frac{\partial P_2}{\partial f_n} \\
\frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \cdots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \cdots & \frac{\partial P_3}{\partial f_n} \\
\vdots & & & & & & & \vdots \\
\frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \cdots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \cdots & \frac{\partial P_n}{\partial f_n} \\
\hline
\frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \cdots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \cdots & \frac{\partial Q_2}{\partial f_n} \\
\frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \cdots & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & \cdots & \frac{\partial Q_3}{\partial f_n} \\
\vdots & & & & & & & \vdots \\
\frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \cdots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \cdots & \frac{\partial Q_n}{\partial f_n}
\end{array}\right]
\begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \hline \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \end{bmatrix}
$$

In General,

$$
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}
$$

where the left hand side Coloumn matrix are known Residual Matrix,

And the Jacobian matrix elements are to be Calculated and $(\Delta e, \Delta f)$ Coloumn matrix are the required Corrections for the next better Solution.

where

$$\boxed{\begin{aligned}\Delta P &= P_{specified} - P^{0}_{calculated}\\ \Delta Q &= Q_{specified} - Q^{0}_{calculated}.\end{aligned}}$$

The superscript zero indicates that the value at the zeroth iteration. Now, the elements of Jacobian elements are calculated as,

from the static load flow equations,

$$P_p = \sum_{q=1}^{n} e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq})$$

$$\Rightarrow P_p = e_p (e_p G_{pp} + f_p B_{pp}) + f_p (f_p G_{pp} - e_p B_{pp}) +$$

$$\sum_{\substack{q=1\\ q \neq p}}^{n} e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}). \quad \text{— (A)}$$

Now, for J_1 Matrix.

the diagonal elements

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + \cancel{f_p B_{pp}} - \cancel{f_p B_{pp}} + \sum_{\substack{q=1\\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\therefore \boxed{\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + \sum_{\substack{q=1\\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq}).} \quad \text{— (1)}$$

The off-diagonal element.

$$\boxed{\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}} \quad \text{for } q \neq p \quad \text{— (2)}$$

for J_2 Matrix :

the diagonal and off-diagonal elements are

$$\frac{\partial P_p}{\partial f_p} = \cancel{e_p B_{pp}} + 2 f_p G_{pp} - \cancel{e_p B_{pp}} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\Rightarrow \boxed{\frac{\partial P_p}{\partial f_p} = 2 f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})} \quad \text{—— (3)}$$

Similarly.

$$\boxed{\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}} \quad \text{for } q \neq p \quad \text{—— (4)}$$

for (A) J_3 Matrix :

The static load flow equation,

$$Q_p = \sum_{q=1}^{n} f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})$$

$$Q_p = f_p (e_p G_{pp} + f_p B_{pp}) - e_p (f_p G_{pp} - e_p B_{pp}) + \sum_{\substack{q=1 \\ q \neq p}}^{n} f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \quad \text{—— (B)}$$

Now, The diagonal & off-diagonal elements are,

(1)

$$\frac{\partial Q_p}{\partial e_p} = \cancel{f_p G_{pp}} - \cancel{f_p G_{pp}} + 2 e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\therefore \boxed{\frac{\partial Q_p}{\partial e_p} = 2 e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})} \quad \text{—— (5)}$$

Similarly,

$$\boxed{\frac{\partial Q_p}{\partial e_q} = f_p G_{pq} + e_p B_{pq}} \quad \text{—— (6)}$$

for J_4 Matrix :-

The diagonal and off-diagonal elements are,

$$\frac{\partial Q_p}{\partial f_p} = e_p G_{pp} + 2f_p B_{pp} - e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\Rightarrow \boxed{\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})} \quad (7)$$

Similarly, $\boxed{\frac{\partial Q_p}{\partial f_q} = f_p B_{pq} - e_p G_{pq}}$ (8)

Now, with the help of above 8 formulas the Jacobian Matrix elements are to be calculated and then finally, the Required correction column matrix is going to be calculated.

$$e_1^1 = e_1^0 + \Delta e_1^0 \quad \text{for next iteration.}$$

$$e_2^1 = e_2^0 + \Delta e_2^0$$

$$\vdots$$

$$e_n^1 = e_n^0 + \Delta e_n^0$$

And this process will continues till the largest element in the column vector $(\Delta e_2, \Delta e_3 \ldots \Delta_n, \Delta f_2, \Delta f_3 \ldots \Delta f_n)$ is less than a prescribed value.

$$V_1^1 = e_1 = (e_1^0 + \Delta e_1^0) + j(f_1^0 + \Delta f_1^0)$$

Now, $\Delta \epsilon = V_1^1 - V_1^0 \leq \epsilon$ (tolerance value)

$$\epsilon = 0.00001$$

In General $\epsilon = \underline{0.0001}$

* Rectangular Co-ordinates Method when PV also Present :-

When PV busses also present in the Power System Networks then the Matrix equations are given by.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \hline \Delta |E_p|^2 \end{bmatrix} = \left[\begin{array}{cccc|cccc} \frac{\partial P_2}{\partial e_1} & \frac{\partial P_2}{\partial e_2} & \cdots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_1} & \frac{\partial P_2}{\partial f_2} & \cdots & \frac{\partial P_2}{\partial f_n} \\ \vdots & J_1 & & & & J_2 & & \\ \frac{\partial P_n}{\partial e_1} & \frac{\partial P_n}{\partial e_2} & \cdots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_1} & \frac{\partial P_n}{\partial f_2} & \cdots & \frac{\partial P_n}{\partial f_n} \\ \hline \frac{\partial Q_2}{\partial e_1} & \frac{\partial Q_2}{\partial e_2} & \cdots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_1} & \frac{\partial Q_2}{\partial f_2} & \cdots & \frac{\partial Q_2}{\partial f_n} \\ \vdots & J_3 & & & & & J_4 & \\ \hline \frac{\partial |E_p|^2}{\partial e_1} & \frac{\partial |E_p|^2}{\partial e_2} & \cdots & \frac{\partial |E_p|^2}{\partial e_n} & \frac{\partial |E_p|^2}{\partial f_1} & \frac{\partial |E_p|^2}{\partial f_2} & \cdots & \frac{\partial |E_p|^2}{\partial f_n} \\ & & J_5 & & & & J_6 & \end{array}\right] \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \hline \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \end{bmatrix}$$

In General,

$$\begin{bmatrix} \Delta P_p \\ \hline \Delta Q_p \\ \hline \Delta |E_p|^2 \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{array}\right] \begin{bmatrix} \Delta e \\ \hline \Delta f \end{bmatrix}$$

Since for PV buses P and voltage magnitude specified

as we know that, $E_p = e_p + j f_p$

$$|E_p| = \sqrt{e_p^2 + f_p^2}$$

$$\Rightarrow e_p^2 + f_p^2 = |E_p|^2$$

Now, for J_5 and J_6 Matrices.

$$\frac{\partial |E_p|^2}{\partial e_p} = 2e_p \qquad \Bigg| \qquad \frac{\partial |E_p|^2}{\partial e_q} = 0$$

Similarly, $\frac{\partial |E_p|^2}{\partial f_p} = 2f_p$ | $\frac{\partial |E_p|^2}{\partial f_q} = 0$

when PV is given then, Calculate the 'Q' value using static load flow solution. The calculated value is with in the prescribed limits then consider it as a PV bus itself and follow above method.

If 'Q' calculated value is not with in the limits then consider that bus as PQ bus and take 'Q' value equal to the limit which is nearer to the calculated value and proceed with the method when only PQ busses are given.

Similar to ΔP_p & ΔQ_p

$$\Rightarrow \boxed{\Delta |E_p|^2 = |E_{p\,specified}|^2 - |E_{p\,calculated}|^2}$$

The same corrections matrix has to be found here also.

The power losses will be found out same as explained in Guass - Siedel method.

* Problems:

PV bus కి Voltage Magnitude ఇస్తే, దాన్ని తీసుకోవాలి, mention చేయకపోతే flat voltage start తీసుకో

① Determine the load flows using N-R Rectangular Method for the given busdata and line data. (1a)

Bus code	Impedance	Admittances
1-2	0.08 + j 0.24	1.25 − j3.75
1-3	0.02 + j 0.06	5 − j15
2-3	0.06 + j0.18	1.666 − j5

~~Assuming the bus voltage at 2 as 1.04.~~

Bus code	Voltages	Generation P_G	Generation Q_G	Load P_L	Load Q_L
1	1.06+j0	0	0	0	0
2	1.0+j0	0.2	0	0	0
3	1.0+j0	0	0	0.6	0.25

$$Z = R + jX$$
$$Y = \frac{1}{Z} = \frac{1}{R+jX}$$
$$Y = G - jB$$

The Reactive power limits for bus-2 is $0 \le Q \le 0.3$.

Sol: Firstly calculate the Y_{Bus} for the given data.

$$Y_{Bus} = \begin{array}{ccc} G_{11} - jB_{11} & G_{12} - jB_{12} & G_{13} - jB_{13} \\ \end{array}$$
$$Y_{Bus} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.916 - j8.75 & -1.666 + j5 \\ -5 + j15 & -1.666 + j5 & 6.666 - j20 \end{bmatrix}$$

Assuming the flat voltage start for bus-2 & bus-3.

$\therefore V_2 = V_3 = 1 + j0 \Rightarrow e_2 + jf_2 = e_3 + jf_3 = 1 + j0$

$e_2 = e_3 = 1$, $f_2 = f_3 = 0$, and given, $e_1 + jf_1 = 1.06 + j0$

$\Rightarrow e_1 = 1.06$, $f_1 = 0$.

from the Y_{Bus},

$G_{11} = 6.25$ | $G_{12} = -1.25$ | $G_{13} = -5$ | $G_{22} = 2.916$

$B_{11} = 18.75$ | $B_{12} = -3.75$ | $B_{13} = -15$ | $B_{22} = 8.75$

$G_{23} = -1.666$ | $G_{33} = 6.666$

$B_{23} = -5$ | $B_{33} = 20.$

By the problem the bus-2 is PV bus. ∴ firstly calculate P_2 and Q_2

(1b)

$$\Rightarrow P_p = \sum_{q=1}^{n} e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq})$$

$$\Rightarrow P_2 = \left[e_2 (e_1 G_{21} + f_1 B_{21}) + f_2 (f_1 G_{21} - e_1 B_{21})\right] + \left[e_2 (e_2 G_{22} + f_2 B_{22}) + f_2 (f_2 G_{22} - e_2 B_{22})\right] + e_2 (e_3 G_{23} + f_3 B_{23}) + f_2 (f_3 G_{23} - e_3 B_{23})$$

$$= \left[1 \cdot (1.06 \times -1.25) + 0\right] + \left[1 (1 \times 2.916)\right] + \left[1 \times (1 \times -1.666)\right]$$

$$= -0.075$$

Similarly P_3.

$$\Rightarrow P_3 = e_3 \left((e_1 G_{31} + f_1 B_{31}) + f_3 (f_1 G_{31} - e_1 B_{31})\right) + \left[e_3 (e_2 G_{32} + f_2 B_{32}) + f_3 (f_2 G_{32} - e_2 B_{32})\right] + \left[e_3 (e_3 G_{33} + f_3 B_{33}) + f_3 (f_3 G_{33} - e_3 B_{33})\right]$$

$$= 1 \cdot (1.06 \times -5) + \left[1 \times -1.666\right] + (1 \times 6.666)$$

$$= -0.3.$$

$$\therefore \Delta P_2 = P_2 \text{ specified} - P_2 \text{ calculated}$$

$$= 0.2 + 0.075 = 0.275$$

$$\Delta P_3 = P_3 \text{ specified} - P_3 \text{ calculated}$$

$$= -0.6 - (-0.3) = -0.3.$$

[bus-2 దగ్గర voltage Magnitude ఇవ్వబడిన PV bus కావున్నప్పటికీ, అందుకే flat voltage start గా తీసుకున్నాము.]

Now, Q_2 & Q_3.

$$Q_p = \sum_{q=1}^{n} \left[f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})\right]$$

$$Q_2 = \left[f_2 (e_1 G_{21} + f_1 B_{21}) - e_2 (f_1 G_{21} - e_1 B_{21})\right] + \left[f_2 (e_2 G_{22} + f_2 B_{22}) - e_2 (f_2 G_{22} - e_2 B_{22})\right] + \left[f_2 (e_3 G_{23} + f_3 B_{23}) - (f_3 G_{23} - e_3 B_{23})\right]$$

$$\Rightarrow Q_2 = \left[-1\times(-1.06\times-3.25)\right] + \left[-1\times(-1\times 8.75)\right] + \left[-1\times(-1\times-5)\right]$$

1c

$$= -0.225$$

But the Reactive limits are $0 \leq Q \leq 0.3$

The obtained value is not in the limits. So, Assuming

3) Considering the bus-2 as PQ bus.

Now, $Q_2 = 0$ (nearer to the value obtained).

$$\left[\because Q_{2\,sp} = Q_{2\,Gen} - Q_{2\,load} = 0 - 0 = \underline{\underline{0}}\right]$$

$$\therefore \Delta Q_2 = Q_2 \text{ specified} - Q_2 \text{ calculated}$$

$$= 0 - (-0.225) = 0.225$$

Similarly,

$$Q_3 = \left[f_3(e_1G_{31} + f_1B_{31}) - e_3(f_1G_{31} - e_1B_{31})\right] + \left[f_3(e_2G_{32} + f_2B_{32}) - e_3(f_2G_{32} - e_2B_{32})\right] + \left[f_3(e_3G_{33} + f_3B_{33}) - (e_3f_3G_{33} - e_3B_{33})\right]$$

$$= \left[-1\times(-1.06\times-15)\right] + \left[-1\times(-1\times-5)\right] + \left[-1(-1\times20)\right]$$

$$= -0.9$$

$$\Delta Q_3 = Q_3 \text{ specified} - Q_3 \text{ calculated}$$

$$= -0.25 - (-0.9) = 0.65$$

Now, the Matrix equation is,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \left[\begin{array}{cc|cc} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} \\ & J_1 & & J_2 \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} \\ \hline \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} \\ & J_3 & & J_4 \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} \end{array}\right] \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

for J_1 Matrix.

1d

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial P_2}{\partial e_2} = 2e_2 G_{22} + (e_1 G_{21} + f_1 B_{21}) + (e_3 G_{23} + f_3 B_{23})$$

$$= 2\times1\times2.916 + (1.06\times -1.25) + (1\times -1.666)$$

$$= 2.848$$

$$\frac{\partial P_3}{\partial e_3} = 2e_3 G_{33} + (e_1 G_{31} + f_1 B_{31}) + (e_2 G_{32} + f_2 B_{32})$$

$$= 2\times1\times6.666 + (1.06\times -5) + (1\times -1.666)$$

$$= 6.3666$$

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}$$

$$\frac{\partial P_2}{\partial e_3} = e_2 G_{23} - f_2 B_{23} = 1\times -1.666 = -1.666$$

$$\frac{\partial P_3}{\partial e_2} = e_3 G_{32} - f_3 B_{32} = 1\times -1.666 = -1.666.$$

Note:- At any Method, at PV bus if Voltage Magnitude is Mentioned then Consider it in Theoritical Calculations. if at PV bus "(V)" not Mentioned then assume flat voltage start

for J_2 Matrix.

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial P_2}{\partial f_2} = 2f_2 G_{22} + (f_1 G_{21} - e_1 B_{21}) + (f_3 G_{23} - e_3 B_{23})$$

$$= 2\times 0 + (-1.06\times -3.75) + (-1\times -5)$$

$$= 3.975 + 5 = 8.975$$

$$\frac{\partial P_3}{\partial f_3} = 2f_3 G_{33} + (f_1 G_{31} - e_1 B_{31}) + (f_2 G_{32} - e_2 B_{32})$$

$$= 0 + (-1.06\times -15) + (-1\times -5) = 20.9$$

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}$$

$$\frac{\partial P_2}{\partial f_3} = e_2 B_{23} + f_3 G_{23} = 1\times 5 = -5 = \frac{\partial P_3}{\partial f_2}$$

1e

for J_3 Matrix

$$\frac{\partial q_p}{\partial e_p} = 2e_pB_{pp} - \sum_{\substack{q=1 \\ q\neq p}}^{n} (f_qG_{pq} - e_qB_{pq})$$

$$\frac{\partial q_2}{\partial e_2} = 2e_2B_{22} - \left[(f_1G_{21} - e_1B_{21}) + (f_3G_{23} - e_3B_{23})\right]$$

$$= 2\times1\times8.75 - (-1.06\times-3.75) - (-1\times-5)$$

$$= 8.525$$

$$\frac{\partial q_3}{\partial e_3} = 2e_3B_{33} - (f_1G_{31} - e_1B_{31}) - (f_2G_{32} - e_2B_{32})$$

$$= 2\times1\times20 - (-1.06\times-15) - (-1\times-5)$$

$$= 19.1$$

$$\frac{\partial q_p}{\partial e_q} = f_pG_{pq} + e_pB_{pq}$$

$$\frac{\partial q_2}{\partial e_3} = f_2G_{23} + e_2B_{23}$$

$$= 1\times-5 = -5$$

$$\frac{\partial q_3}{\partial e_2} = f_3G_{32} + e_3B_{32}$$

$$= 1\times-5 = -5$$

for J_4 Matrix

$$\frac{\partial q_p}{\partial f_p} = 2f_pB_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} (e_qG_{pq} + f_qB_{pq})$$

$$\frac{\partial q_3}{\partial f_3} = 2f_3B_{33} + (e_1G_{31} + f_1B_{31}) + (e_2G_{32} + f_2B_{32})$$

$$= 0 + (1.06\times-5) + (1\times-1.666)$$

$$= -6.966.$$

$$\frac{\partial q_2}{\partial f_2} = 2f_2B_{22} + (e_1G_{21} + f_1B_{21}) + (e_3G_{23} + f_3B_{23})$$

$$= 0 + (1.06\times-1.25) + (1\times-1.666) = -2.991$$

$$\frac{\partial q_p}{\partial f_q} = f_pB_{pq} - e_pG_{pq}$$

$$\frac{\partial q_2}{\partial f_3} = f_2B_{23} - e_2G_{23}$$

$$= -1\times-1.666$$

$$= 1.666$$

$$\frac{\partial q_3}{\partial f_2} = -1\times-1.666$$

$$= +1.666.$$

$$\therefore \begin{bmatrix} 0.275 \\ -0.3 \\ 0.225 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5 \\ -1.666 & 6.366 & -5 & 20.9 \\ 8.525 & -5 & -2.991 & 1.666 \\ -5 & 19.1 & 1.666 & -6.966 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

2. for the above Problem Consider the Reactive power limits are $-0.3 \le Q \le 0.3$. Determine the equations at the end of first iteration for $V_2 = 1.04$ $\boxed{IF}$

Sol From the above problem,

$Q_2 = -0.225$

The obtained value is with in the limits then Consider the 2nd bus as PV bus itself

$E_p = e_p + jf_p$

$|E_p| = \sqrt{e_p^2 + f_p^2}$

$\Rightarrow |E_p|^2 = e_p^2 + f_p^2$

The equations will be

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \\ \Delta |E_2|^2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} \\ \frac{\partial |E_2|^2}{\partial e_2} & \frac{\partial |E_2|^2}{\partial e_3} & \frac{\partial |E_2|^2}{\partial f_2} & \frac{\partial |E_2|^2}{\partial f_3} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

J_1, J_2, J_3, J_4, J_5, J_6

for $|E_p| = \sqrt{e_p^2 + f_p^2}$

$\Rightarrow |E_p|^2 = e_p^2 + \cancel{f_p^2}$ $\to 0$

$\Delta |E_2|^2 = |E_{2spe}|^2 - |E_{2cal}|^2$

$= (1.04)^2 - (1.0)^2$

$= 0.0816$

J_5 $\frac{\partial |E_p|^2}{\partial e_p} = 2e_p$ | $\frac{\partial |E_p|^2}{\partial e_q} = 0$

J_6 $\frac{\partial |E_p|^2}{\partial f_p} = 2f_p$ | $\frac{\partial |E_p|^2}{\partial f_q} = 0$

$\frac{\partial |E_2|^2}{\partial e_2} = 2e_2 = 2 \times 1.04 = 2.08$ | $\frac{\partial |E_2|^2}{\partial f_2} = 2f_p = 0$

$= 2f_2 = 0$

$\therefore$ The Equations are,

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.65 \\ 0.0816 \end{bmatrix} = \left[\begin{array}{cc|cc} 2.846 \quad J_1 & -1.666 & 8.975 \quad J_2 & -5 \\ -1.666 & 6.366 & -5 & 20.9 \\ \hline -5 \quad J_3 & 19.1 & 1.666 \quad J_4 & -6.966 \\ 2.08 \quad J_5 & 0 & 0 \quad J_6 & 0 \end{array}\right] \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

3. The Bus Admittance matrix of a 3-bus System is,

$$\begin{bmatrix} -j20 & j10 & j10 \\ j10 & -j15 & j5 \\ j10 & j5 & -j15 \end{bmatrix}$$

The latest solution is,

$V_1 = 1.05 \angle 4.696$

$V_2 = 0.9338 \angle -8.8$

$V_3 = 1.0 \angle 0°$.

Bus Data,

Bus No	Bus type	P_G	Q_G	P_L	Q_L	$\lvert E \rvert$
1.	P-V	2.9034	—	0	0	1.05
2.	P-Q	—	—	4.0089	1.7915	—
3.	~~P-V~~ Slack	—	—	—	—	1.0

Assume Q_1 are with in the limits. Do the load flow solution after one iteration using Rectangular Co-ordinates Method.

Sol. given $Y_{Bus} = \begin{bmatrix} -j20 & j10 & j10 \\ j10 & -j15 & j5 \\ j10 & j5 & -j15 \end{bmatrix}$

$\therefore B_{11} = 20$, $B_{12} = -10$, $B_{13} = -10$,

$B_{22} = 15$, $B_{23} = -5$, $B_{33} = 15$

Now, from the above problem the PV bus is with in the limits. So, assuming it as PV bus itself

$\therefore$ The Matrix Equations are,

(2)

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_2 \\ \Delta |E_1|^2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial e_1} & \frac{\partial P_1}{\partial e_2} & \frac{\partial P_1}{\partial f_1} & \frac{\partial P_1}{\partial f_2} \\ \frac{\partial P_2}{\partial e_1} & \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial f_1} & \frac{\partial P_2}{\partial f_2} \\ \frac{\partial Q_2}{\partial e_1} & \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial f_1} & \frac{\partial Q_2}{\partial f_2} \\ \frac{\partial |E_1|^2}{\partial e_1} & \frac{\partial |E_1|^2}{\partial e_2} & \frac{\partial |E_1|^2}{\partial f_1} & \frac{\partial |E_1|^2}{\partial f_2} \end{bmatrix} \begin{bmatrix} \Delta e_1 \\ \Delta e_2 \\ \Delta f_1 \\ \Delta f_2 \end{bmatrix}$$

Now, given,

$V_1 = 1.05 \angle 4.696 = 1.0465 + j0.08596 = e_1 + jf_1$

$V_2 = 0.9338 \angle -8.8 = 0.9228 - j0.14285 = e_2 + jf_2$

$V_3 = 1 \angle 0^0 = 1 + j0. = e_3 + jf_3.$

Now,

$$P_p = \sum_{q=1}^{n} e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}).$$

Since all conductances are zero.

$$\therefore P_p = \sum_{q=1}^{n} e_p B_{pq} f_q - f_p e_q B_{pq}$$

$$P_1 = (e_1 B_{11} f_1 - f_1 e_1 B_{11}) + (e_1 B_{12} f_2 - f_1 e_2 B_{12}) + (e_1 B_{13} f_3 - f_1 e_3 B_{13})$$

$= (1.0465 \times 20 \times 0.08596 - 0.08596 \times 1.0465 \times 20) +$
$(1.0465 \times -10 \times -0.14285 - 0.08596 \times 0.9228 \times -10) +$
$(1.0465 \times -10 \times 0 - 0.08596 \times 1 \times -10)$

$= 3.146.$

$$P_2 = (e_2 B_{21} f_1 - f_2 e_1 B_{21}) + (e_2 B_{22} f_2 - f_2 e_2 B_{22}) + (e_2 B_{23} f_3 - f_2 e_3 B_{23})$$

$= (0.9228 \times -10 \times 0.08596 - -0.14285 \times 1.0465 \times -10) +$
$(0.9228 \times 15 \times -0.14285 - -0.14285 \times 0.9228 \times 15) +$
$(0.9228 \times 0 - -0.14285 \times 1 \times -5)$

$= -3.0035$

$\therefore \Delta P_1 = P_1 \text{ speci} - P_1 \text{ calcul}$

$= 2.9034 - 3.146 = -0.2426.$

$\Delta P_2 = P_2 \text{ speci} - P_2 \text{ calcul}$

$= -4.0089 + 3.0035 = -1.0054.$

Similarly,

$$Q_p = \sum_{q=1}^{n} f_p (e_q G_{pq} + f_q B_{pq}) + e_p (e_q B_{pq} - f_q G_{pq})$$

All Conductances are zero.

$$\Rightarrow Q_p = \sum_{q=1}^{n} f_p B_{pq} f_q + e_p e_q B_{pq}$$

Similarly, $Q_2 = -1.0688$

$\Delta Q_2 = -1.7915 - (-1.0688)$

$= -0.72268.$

and $\Delta |E_1|^2 = |E_1 \text{ speci}|^2 - |E_1 \text{ calcul}|^2$

$= (1.05)^2 - (1.05)^2 = 0$

Now,

J Matrix:-

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial P_1}{\partial e_1} = 2e_1 G_{11} + (\cancel{f_1 B_{11}} + f_2 B_{12} + f_3 B_{13})$$

$= [\cancel{0.08596 \times 20} + (-0.14286 \times -10)] = \cancel{3.1477}$ 1.4286

$$\frac{\partial P_2}{\partial e_2} = 2e_2 G_{22} + (f_1 B_{21} + \cancel{f_2 B_{22}} + f_3 B_{23})$$

$= [0 + 0.08596 \times -10 + (\cancel{-0.14265}) 2615 + 0]$

$= \cancel{-3.00235}$ 0.8596

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}$$

$$\frac{\partial P_1}{\partial e_2} = e_1 G_{12} - f_1 B_{12} = -0.08596 \times -10 = 0.8596$$

$$\frac{\partial P_2}{\partial e_1} = e_2 G_{21} - f_2 B_{21} = -(-0.14285 \times -10) = -1.4285$$

(4)

for J_2 Matrix,

$$\frac{\partial P_p}{\partial f_p} = 2 f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} \left(f_q G_{pq} - e_q B_{pq} \right)$$

$$\frac{\partial P_1}{\partial f_1} = 0 + \left(f_2 G_{12} - e_2 B_{12} + f_3 G_{13} - e_3 B_{13} \right)$$
$$= (-0.9228 \times -10 - 1 \times -10) = 19.228$$

$$\frac{\partial P_2}{\partial f_2} = f_1 G_{21} - e_1 B_{21} + f_3 G_{23} - e_3 B_{23}$$
$$= -1.0465 \times -10 + -1 \times -5 = 15.465$$

$$\frac{\partial P_p}{\partial f_q} = f_p G_{pq} - e_p B_{pq}$$

$$\frac{\partial P_1}{\partial f_2} = f_1 G_{12} - e_1 B_{12}$$
$$= \cancel{0.08596 \times -10}$$
$$= \cancel{-0.08596}$$
$$= 1.0465 \times -10$$
$$= -10.465$$

$$\frac{\partial P_2}{\partial f_1} = f_2 G_{21} - e_2 B_{21}$$
$$= -0.9228 \times -10$$
$$= 9.228$$

for J_3 Matrix

$$\frac{\partial Q_p}{\partial e_p} = 2 e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} \left(f_q G_{pq} - e_q B_{pq} \right)$$

$$\cancel{\frac{\partial Q_2}{\partial e_2} = 2 e_2 B_{22} - \left[(f_2 G_{12} - e_2 B_{12}) + (f_3 G_{13} - e_3 B_{13}) \right]}$$
$$\cancel{= (0.9228 \times -10) + (1 \times -10)}$$
$$\cancel{= 2 \times 1.0465 \times 2.0}$$
$$=$$

(5)

$$\frac{\partial Q_2}{\partial e_2} = 2e_2 B_{22} - \left[(f_1 G_{21} - e_1 B_{21}) + (f_3 G_{23} - e_3 B_{23})\right]$$

$$= 2 \times 0.9228 \times 15 - \left[(-1.0465 \times -10) + (-1 \times -5)\right]$$

$$= 12.219$$

$$\frac{\partial Q_2}{\partial e_1} = f_p G_{pq} + e_p B_{pq}$$

$$= e_2 B_{21} = \cancel{1.0465}\ 0.9228 \times -10$$

$$= 9.228$$

for J_4 Matrix:-

$$\frac{\partial Q_p}{\partial f_p} = 2 f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial Q_2}{\partial f_2} = 2 \times f_2 \times B_{22} + \left[(e_1 G_{21} + f_1 B_{21}) + (e_3 G_{23} + f_3 B_{23})\right]$$

$$= -2 \times 0.14285 \times 15 + (0.08596 \times -10) + 0$$

$$= -5.1451$$

$$\frac{\partial Q_p}{\partial f_q} = f_p B_{pq} - e_p G_{pq}$$

$$\frac{\partial Q_2}{\partial f_1} = f_2 B_{21}$$

$$= -0.14285 \times -10 = 1.4285$$

for J_5 Matrix:-

$$\frac{\partial |E_1|^2}{\partial e_1} = 2e_1$$

$$= 2 \times 1.0465$$

$$= 2.093$$

for J_6 Matrix:-

$$\frac{\partial |E_1|^2}{\partial f_1} = 2f_1$$

$$= 2 \times 0.08596$$

$$= 0.1719.$$

$$\therefore \begin{bmatrix} -0.2426 \\ -1.0054 \\ -0.72268 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.429 & 0.86 & 19.228 & -10.465 \\ -1.429 & -0.86 & -9.228 & 15.465 \\ -9.228 & 12.219 & 1.429 & \cancel{15.4\ldots}\ -5.1451 \\ 2.093 & 0 & 0.172 & 0 \end{bmatrix} \begin{bmatrix} \Delta e_1 \\ \Delta e_2 \\ \Delta f_1 \\ \Delta f_2 \end{bmatrix}$$

* Newton - Raphson Polar Co-ordinates Method :-

As we know that,

$$S = P + jQ$$

$$S_i = P_i + jQ_i = V_i I_i^*$$

$$\Rightarrow S_i^* = P_i - jQ_i = V_i^* I_i \quad \text{—— (A)}$$

from the performance Equation

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \quad \text{—— (1)}$$

Substituting (1) in (A) we get,

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} V_j Y_{ij} \quad \text{—— (2)}$$

As this is the polar coordinates method, assuming

$$V_i = |V_i| \angle \delta_i \, , \quad V_j = |V_j| \angle \delta_j \, , \quad Y_{ij} = |Y_{ij}| \angle \theta_{ij} .$$

Substitute all the assumptions in Eq (2)

$$\Rightarrow P_i - jQ_i = |V_i| \angle -\delta_i \sum_{j=1}^{n} |V_j| \angle \delta_j |Y_{ij}| \angle \theta_{ij} .$$

$$= |V_i| \sum_{j=1}^{n} |V_j| |Y_{ij}| \angle (\delta_j + \theta_{ij} - \delta_i)$$

∴ The Real power term can be written as,

$$\boxed{P_i = |V_i| \sum_{j=1}^{n} |V_j| |Y_{ij}| \cos(\delta_j + \theta_{ij} - \delta_i).} \quad \text{—— (3)}$$

Similarly,

$$\boxed{Q_i = -|V_i| \sum_{j=1}^{n} |V_j| |Y_{ij}| \sin(\delta_j + \theta_{ij} - \delta_i).} \quad \text{—— (4)}$$

The above (3) and (4) equations are known as static load flow equations for N-R polar Co-ordinates method.

As like the NR Rectangular Coordinates method. The (7)*
Matrix equations for the above Non-linear static load flow equations will be written as,

*** Polar Coordinates method when only PQ busses are Present:**

When only PQ busses are present in a typical power system Networks then, the static load flows can be written as,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \left[\begin{array}{cccc|cccc} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cdots & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} & \cdots & \frac{\partial P_2}{\partial |V_n|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & \frac{\partial P_3}{\partial \delta_n} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} & \cdots & \frac{\partial P_3}{\partial |V_n|} \\ \vdots & & J_1 & & \vdots & & J_2 & \\ \frac{\partial P_n}{\partial \delta_2} & \frac{\partial P_n}{\partial \delta_3} & \cdots & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial |V_2|} & \frac{\partial P_n}{\partial |V_3|} & \cdots & \frac{\partial P_n}{\partial |V_n|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \cdots & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \cdots & \frac{\partial Q_2}{\partial |V_n|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & J_3 \cdots & \frac{\partial Q_3}{\partial \delta_n} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} & J_4 \cdots & \frac{\partial Q_3}{\partial |V_n|} \\ \vdots & & & & \vdots & & & \\ \frac{\partial Q_n}{\partial \delta_2} & \frac{\partial Q_n}{\partial \delta_3} & \cdots & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial |V_2|} & \frac{\partial Q_n}{\partial |V_3|} & \cdots & \frac{\partial Q_n}{\partial |V_n|} \end{array}\right] \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \vdots \\ \Delta \delta_n \\ \hline \Delta |V_2| \\ \Delta |V_3| \\ \vdots \\ \Delta |V_n| \end{bmatrix}$$

→ In General

Residual Column Matrix   Jacobian Matrix   Corrections Matrix

$$\begin{bmatrix} \Delta P \\ \hline \Delta Q \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \end{array}\right] \begin{bmatrix} \Delta \delta \\ \hline \Delta |V| \end{bmatrix}$$

The left hand side Matrix is known as Residual Column Matrix. and the J_1, J_2, J_3, J_4 are sub-matrices of the Jacobian Matrix.

And the Right most Column matrix is known as Corrections Required for the next better Solution for next iterations.

$$\boxed{\Delta P = P_{specified} - P^0_{calculated}}$$

$$\boxed{\Delta Q = Q_{specified} - Q^0_{calculated}}$$

The Superscript zero indicates that the value at the zeroth iteration. Calculated value can be computed by the static load flow equations.

And the Jacobian elements can be computed from the static load flow equations.

$$P_i = V_i V_i Y_{ii} \cos(\delta_i - \delta_i + \theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||V_j||Y_{ij}| \cos(\delta_j + \theta_{ij} - \delta_i)$$

$$Q_i = -\left[V_i V_i Y_{ii} \sin(\delta_i - \delta_i + \theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||V_j||Y_{ij}| \sin(\delta_j - \delta_i + \theta_{ij})\right]$$

$$\left[\begin{aligned} \frac{d}{d\theta}\sin\theta &= \cos\theta \\ \frac{d}{d\theta}\cos\theta &= -\sin\theta \end{aligned}\right]$$

for J_1 Matrix :-

$$\frac{\partial P_i}{\partial \delta_i} = 0 + \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||V_j||Y_{ij}| \sin(\delta_j - \delta_i + \theta_{ij}) \quad \text{--- (1)}$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\delta_j - \delta_i + \theta_{ij}) \quad (\text{for } j \neq i). \quad \text{--- (2)}$$

for J_2 Matrix :-

$$\frac{\partial P_i}{\partial |V_i|} = 2V_i Y_{ii} \cos(\theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_j||Y_{ij}| \cos(\delta_j - \delta_i + \theta_{ij}) \quad \text{--- (3)}$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad \text{for } j \neq i \quad \text{--- (4)}$$

for J_3 Matrix :-

$$\frac{\partial Q_i}{\partial \delta_i} = 0 + \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||V_j||Y_{ij}| \cos(\delta_j - \delta_i + \theta_{ij}). \quad \text{--- (5)}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\delta_j + \theta_{ij} - \delta_i). \quad \text{--- (6)}$$

fol J_4 Matrix :-

(9)

$$\frac{\partial Q_i}{\partial |V_i|} = -2V_i Y_{ii} \sin(\theta_{ii}) - \sum_{\substack{j=1 \\ j\neq i}}^{n} |V_j||Y_{ij}| \sin(\delta_j - \delta_i + \theta_{ij}) \quad \text{—(7)}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \quad \text{—(8)}$$

from the above 8 formulas the Jacobian Matrix can be calculated. Then finally, by inversing the Jacobian Matrix with the Residual column matrix the Error Corrections Required fol the Next Iteration will be computed.

fol next iteration,

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2^0 \qquad |V_2^1| = |V_2^0| + \Delta|V_2^0|$$

$$\delta_3^1 = \delta_3^0 + \Delta\delta_3^0 \qquad |V_3^1| = |V_3^0| + \Delta|V_3^0|$$

$$\vdots$$

(1) $$\delta_n^1 = \delta_n^0 + \Delta\delta_n^0 \qquad |V_n^1| = |V_n^0| + \Delta|V_n^0|$$

And the Process will be continues till the largest element in the column vector $(\Delta\delta_2, \Delta\delta_3 \ldots \Delta\delta_n, \Delta|V_2|, \Delta|V_3|, \ldots \Delta|V_n|)$ is less than a prescribed value (or) a tolerance value.

$$\Delta V_2 = |V_2^1|\angle\delta_2^1 - |V_2^0|\angle\delta_2^0 \le \varepsilon$$

$$\Delta V_3 = |V_3^1|\angle\delta_3^1 - |V_3^0|\angle\delta_3^0 \le \varepsilon$$

$$\varepsilon = 0.00005$$

(2) In general we take $\varepsilon = 0.0005$

(3)

* Polar Co-ordinates method when PV busses also Present:

When PV busses also Present in the Power System Network then the Procedure will be,

By eliminating the Particular PV bus Reactive Power entire Row and the particular PV bus entire Voltage magnitude column, the Remaining the matrix equations can be computed for the Polar Co-ordinates method when PV busses also Present

for Example:-

Assuming a 3 bus System, which contains a 2nd bus as PV bus, then

Eliminating the 2nd bus entire voltage magnitude column.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \boxed{\Delta Q_2} \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial (\delta_2)} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \boxed{\Delta |V_2|} \\ \Delta |V_3| \end{bmatrix}$$

eliminating the 2nd bus entire Reactive power Row.

The Remaining matrix Equations will be,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \hline \Delta Q_3 \end{bmatrix} = \left[\begin{array}{cc|c} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_3|} \\ & J_1 & J_2 \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_3|} \\ \hline \frac{\partial Q_3}{\partial \delta_2}\ J_3 & \frac{\partial Q_3}{\partial \delta_3} & J_4\ \frac{\partial Q_3}{\partial |V_3|} \end{array}\right] \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \hline \Delta |V_3| \end{bmatrix}$$

And continue with the Normal Procedure. (11)

Note :-

Before eliminating the Row and Coloumn, firstly we have to check whether the Reactive power value lies with in the limits or not

If the calculated value is not with in the limits then the particular PV bus can be treated as PQ bus and continue with the Normal Procedure.

~~If~~ ~~the~~ The Reactive power can be taken the limit which is the nearer to the obtained value

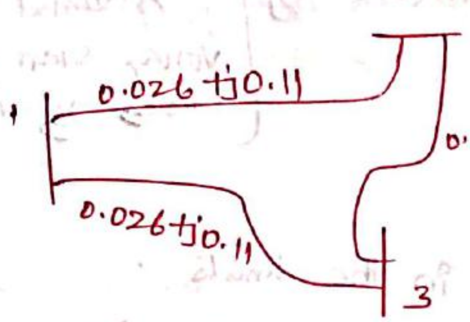
If the calculated value is with in the limits then treat the bus as PV bus itself, then eliminate the entire Row and entire Coloumn of Reactive power and Voltage magnitudes Respectively of a particular bus then continue with the Normal Procedure.

⇒ The Q value will be calculated from the static load flow equation..

* problems :-

1) Do the load flows for the given bus data and line data.

Bus	P_G	Q_G	P_L	Q_L	Bus voltage	$\lvert V_1 \rvert \angle \delta_1$
1.	—	—	1.0	0.5	1.03+j0 (slack)	$= 1.03\angle 0^\circ$
2.	1.5	—	0	0	1.03 (PV).	$= 1.03 = \lvert V_2 \rvert$
3.	0	0	1.2	0.5	— (PQ). =	flat voltage start $V_3 = 1+j0 = 1\angle 0^\circ = \lvert V_3 \rvert = 1$ & $\delta_3 = 0^\circ$



with a shunt Admittance j0.04 at every bus.

The Reactive power limits for bus-2 is $0 \le Q \le 0.8$

Sol: Admittance $= \dfrac{1}{0.026+j0.11} = 2.035 - j8.609$

Now, all impedances are equal between the lines.

$\therefore Y_{11} = Y_{12} + Y_{13} = 2(2.035 - j8.0609) + j0.04$

$= 4.07 - j17.219 + j0.04$

$= 4.07 - j17.179 = Y_{22} = Y_{33}$

$= 17.655\angle -76.67$

Similarly,

$Y_{12} = Y_{21} = Y_{13} = Y_{31} = Y_{23} = Y_{32} = -2.035 + j8.609$

$= 8.846\angle 103.29$.

$$\therefore Y_{Bus} = \begin{bmatrix} 4.07 - j17.179 & -2.035+j8.609 & -2.035+j8.609 \\ -2.035+j8.609 & 4.07-j17.179 & -2.035+j8.609 \\ -2.035+j8.609 & -2.035+j8.609 & 4.07-j17.179 \end{bmatrix}$$

$$\therefore Y_{Bus} = \begin{bmatrix} 17.655\angle -76.67 & 8.846\angle 103.29 & 8.846\angle 103.29 \\ 8.846\angle 103.29 & 17.655\angle -76.67 & 8.846\angle 103.29 \\ 8.846\angle 103.29 & 8.843\angle 103.29 & 17.655\angle -76.67 \end{bmatrix}$$

Now, given 2nd bus as PV bus, calculating Q_2 (13)

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\delta_j - \delta_i + \theta_{ij})$$

$$Q_2 = -|V_2| \left[V_1 |Y_{21}| \sin(\theta_{21} + \delta_2 + \delta_1) + V_2 |Y_{22}| \sin(\theta_{22} + \delta_2 + \delta_2) + V_3 |Y_{23}| \sin(\theta_{23} + \delta_3 - \delta_2) \right]$$

$$= -1.03 \left[1.03 \times 8.846 \times \sin(103.29) + 1.03 \times 17.655 \sin(-76.67) + 1 \times 8.846 \sin(103.29) \right]$$

$\left[\because \text{Assumed that voltage start for bus-3 } V_3 = 1 + j0 \right]$

$$= 0.2248$$

The limits are $0 \le Q \le 0.8$

$\therefore$ The obtained value is with in the limits.

Now, Considering the bus-2 as PV bus itself, The matrix equations are

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \left[\begin{array}{cc|c} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_3|} \\ \hline \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_3|} \end{array}\right] \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_3| \end{bmatrix}$$

J_1, J_2, J_3, J_4

$$P_i = \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\Rightarrow P_2 = V_2 \left[V_1 Y_{21} \cos(\theta_{21}) + V_2 Y_{22} \cos(\theta_{22}) + V_3 Y_{23} \cos(\theta_{23}) \right]$$

$$= 1.03 \left[1.03 \times 8.846 \times \cos(103.29) + 1.03 \times 17.655 \cos(-76.79) + 1 \times 8.846 \cos(103.29) \right]$$

$$= 0.0645$$

Similarly,

$$P_3 = V_3 \left[V_1 Y_{31} \cos(\theta_{31}) + V_2 Y_{32} \cos(\theta_{32}) + V_3 Y_{33} \cos(\theta_{33}) \right]$$

$$= 1 \left[1.03 \times 8.846 \times \cos(103.29) + 1.03 \times 8.846 \times \cos(103.29) + 1 \times 17.655 \times \cos(-76.67) \right]$$

$$= -0.1185$$

(14)

$\therefore \Delta P_2 = P_{spec} - P_{2cal}$

$= 1.5 - 0.0645 = 1.4355$

$\Delta P_3 = P_{3sp} - P_{3cal}$

$= -1.2 + 0.1185 = -1.0815$

$\Delta Q_3 = Q_{3sp} - Q_{3cal}$

$Q_3 = -|V_3| \left[V_1 Y_{31} \sin(\theta_{31}) + V_2 Y_{32} \sin(\theta_{32}) + V_3 Y_{33} \sin(\theta_{33}) \right]$

$= -1 \times \left[1.03 \times 8.846 \times \sin(103.29) + 1.03 \times 8.846 \times \sin(103.29) + 17.655 \times \sin(-76.67) \right]$

$= -0.557$

$\Delta Q_3 = Q_{3sp} - Q_{3cal}$

$= -0.5 + 0.557$

$= 0.057.$

Now, for J_1 Matrix

$\frac{\partial P_2}{\partial \delta_2} = V_2 \left[V_1 Y_{21} \sin(\theta_{21}) + V_3 Y_{23} \sin(\theta_{23}) \right]$

$= 1.03 \left[1.03 \times 8.846 \times \sin(103.29) + 1 \times 8.846 \times \sin(103.29) \right]$

$= 17.994$

$\frac{\partial P_2}{\partial \delta_3} = -|V_2| |V_3| Y_{23} \sin(\theta_{23})$

$= -1.03 \times 1 \times 8.846 \times \sin(103.29)$

$= -8.867 = \frac{\partial P_3}{\partial \delta_2}$

$\frac{\partial P_3}{\partial \delta_3} = V_3 \left[V_1 Y_{31} \sin(\theta_{31}) + V_2 Y_{32} \sin(\theta_{32}) \right]$

$= 1 \left[1.03 \times 8.846 \times \sin(103.29) + 1.03 \times 8.846 \times \sin(103.29) \right]$

$= 17.734.$

for J_2 Matrix :-

$\frac{\partial P_i}{\partial |V_j|} = |V_i| |Y_{ij}| \cos(\theta_{ij})$

$\frac{\partial P_2}{\partial |V_3|} = |V_2| |Y_{23}| \cos(\theta_{23})$

$= 1.03 \times 8.846 \times \cos(103.29)$

$= -\cancel{2.0335}$

$$\frac{\partial P_3}{\partial |V_3|} = 2|V_3|\, Y_{33} \cos(\theta_{33}) + \left[V_1 Y_{31} \cos(\theta_{31}) + V_2 Y_{32} \cos(\theta_{32})\right]$$

$$= 2 \times 17.655 \times \cos(-76.67) + \left[1.03 \times 8.846 \times \cos(103.29) + 1.03 \times 8.846 \times \cos(103.29)\right]$$

15

$$= \underline{3.952}$$

for $\underline{J_3 \text{ Matrix}}$:-

$$\frac{\partial Q_3}{\partial \delta_2} = -|V_3|\,|V_2|\,|Y_{32}| \cos(\theta_{32})$$

$$= -1 \times 1.03 \times 8.846 \times \cos(103.29)$$

$$= \underline{2.0945}$$

$$\frac{\partial Q_3}{\partial \delta_3} = V_3 \left[V_1 Y_{31} \cos(\theta_{31}) + V_2 Y_{32} \cos(\theta_{32})\right]$$

$$= 1 \left[1.03 \times 8.846 \times \cos(103.29) + 1.03 \times 8.846 \times \cos(103.29)\right]$$

$$= \underline{-4.189};$$

for $\underline{J_4 \text{ Matrix}}$:-

$$\frac{\partial Q_3}{\partial |V_3|} = -2|V_3|\, Y_{33} \sin(\theta_{33}) - \left[V_1 Y_{31} \sin(\theta_{31}) + V_2 Y_{32} \sin(\theta_{32})\right]$$

$$= -2 \times 1 \times 17.655 \times \sin(-76.67) - \left[1.03 \times 8.846 \times \sin(103.29) + 1.03 \times 8.846 \times \sin(103.29)\right]$$

$$= \underline{16.623}.$$

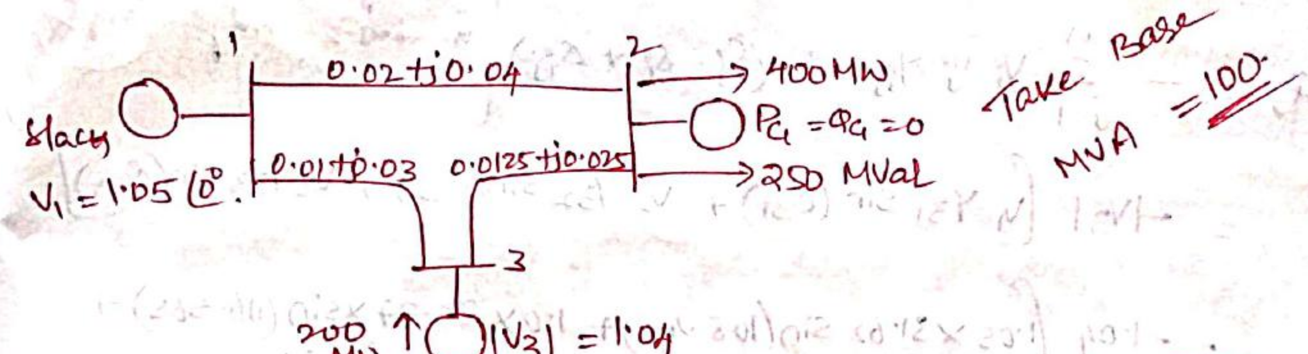
$$\therefore \begin{bmatrix} 1.4355 \\ -1.0815 \\ 0.057 \end{bmatrix}_{3\times1} = \begin{bmatrix} 17.994 & -8.867 & -2.0335 \\ -8.887 & 17.734 & 3.952 \\ 2.0945 & -4.189 & 16.623 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix}$$

$(3\times3)^{-1}(3\times1)$

$\underline{3\times1}$

$$\Rightarrow \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} 0.0659 \\ -0.0254 \\ -0.0113 \end{bmatrix}$$

2. Do the load flows for the given networks using Polar co-ordinates method.



i, Consider the Reactive power is does not lies with in limits

ii, " " " " lies with in the limits

Sol. $y_{12} = \dfrac{1}{0.02+j0.04} = 10-j20$

$y_{13} = \dfrac{1}{0.01+j0.03} = 10-j30$

$y_{23} = \dfrac{1}{0.0125+j0.025} = 16-j32$

$$\therefore Y_{Bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

$$= \begin{bmatrix} 53.85\angle -68.198 & 22.36\angle 116.565 & 31.62\angle 108.43 \\ 22.36\angle 116.565 & 58.13\angle -63.434 & 35.77\angle 116.565 \\ 31.62\angle 108.43 & 35.77\angle 116.565 & 67.23\angle -67.249 \end{bmatrix}$$

For 2nd bus:

$S_{2\,pu} = \dfrac{(0-400)+j(0-250)}{100}$ ($\because$ Gener − Load = Losses)

$= \dfrac{-400-j250}{100} = -4-j2.5$

Similarly, $S_{3\,pu} = \dfrac{200+j0}{100}$ $\Rightarrow$ $P_2 = -4$ | $Q_2 = -2.5$

$= 2+j0$ $\quad P_3 = 2$ | $Q_3 = 0$

Assuming the flat voltage start for bus-2 $\Rightarrow V_2 = 1+j0$

given, $V_1 = 1.05+j0$, $|V_3| = 1.04$

∵ Here $|V_3|$ is mentioned, so assuming flat voltage start for bus-2 only

17

i) $Q_3 = -\sum_{j=1}^{n} V_i V_j Y_{ij} \sin(\delta_j - \delta_i + \theta_{ij})$

$= -|V_3| \left[V_1 Y_{31} \sin(\theta_{31}) + V_2 Y_{32} \sin(\theta_{32}) + V_3 Y_{33} \sin(\theta_{33})\right]$

$= -1.04 \left[1.05 \times 31.62 \sin(108.43) + 1.0 \times 35.77 \times \sin(116.565) + 1.04 \times 67.23 \times \sin(-67.249)\right]$

$= 1.026$

Assuming the obtained value not in the limits

assuming the limits $2 \le Q \le 4$

∴ $Q_3 = 2$, (limit nearer to the value).

∴ Treating the bus-3 as PQ bus and the matrix equations are,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \\ \Delta |V_3| \end{bmatrix}$$

J_1 J_2 J_3 J_4

$Q_2 = -V_2 \left[V_1 Y_{21} \sin(\theta_{21}) + V_2 Y_{22} \sin(\theta_{22}) + V_3 Y_{23} \sin(\theta_{23})\right]$

$= -1 \left[1.05 \times 22.36 \sin(116.565) + 1 \times 58.13 \times \sin(-63.434) + 1.04 \times 35.77 \times \sin(116.565)\right]$

$= -2.280$

18

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\delta_j - \delta_i + \theta_{ij})$$

$$P_2 = |V_2| \left[V_1 Y_{21} \cos(\theta_{21}) + V_2 Y_{22} \cos(\theta_{22}) + V_3 Y_{23} \cos(\theta_{23})\right]$$

$$= 1 \left[1.05 \times 22.36 \cos(116.565) + 1 \times 58.13 \cos(-63.434) + 1.04 \times 35.77 \cos(116.565)\right]$$

$$= -1.138$$

$$P_3 = |V_3| \left[V_1 Y_{31} \cos(\theta_{31}) + V_2 Y_{32} \cos(\theta_{32}) + V_3 Y_{33} \cos(\theta_{33})\right]$$

$$= 1.04 \left[1.05 \times 31.62 \cos(108.43) + 1 \times 35.77 \cos(116.565) + 1.04 \times 67.23 \times \cos(-67.249)\right]$$

$$= 0.568$$

$$\therefore \Delta P_2 = P_{2sp} - P_{2cal}$$
$$= -4 + 1.138 = -2.862$$

$$\Delta P_3 = P_{3sp} - P_{3cal}$$
$$= 2 - 0.568 = 1.432$$

$$\Delta Q_2 = Q_{2sp} - Q_{2cal}$$
$$= -2.5 + 2.280 = -0.22$$

$$\Delta Q_3 = Q_{3sp} - Q_{3cal} = -1.026$$
$$= 0 - 1.026$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||V_j||Y_{ij}| \sin(\delta_j + \theta_{ij} - \delta_i)$$

for J_1 Matrix :

$$\frac{\partial P_2}{\partial \delta_2} = V_2 \left[V_1 Y_{21} \sin(\theta_{21}) + V_3 Y_{23} \sin(\theta_{23})\right]$$

$$= 1 \left[1.05 \times 22.36 \sin(116.565) + 1.04 \times 35.77 \sin(116.565)\right]$$

$$= 54.272$$

$$\frac{\partial P_3}{\partial \delta_3} = V_3 \left[V_1 Y_{31} \sin(\theta_{31}) + V_2 Y_{32} \sin(\theta_{32})\right]$$

$$= 1.04 \left[1.05 \times 31.62 \sin(108.43) + 1 \times 35.77 \times \sin(116.565)\right]$$

$$= 66.03$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2|\,|V_3|\,|Y_{23}|\,\sin(\theta_{23})$$

$$= -1 \times 1.04 \times 35.77 \sin(116.565)$$

$$= -33.273 = \frac{\partial P_3}{\partial \delta_2} \qquad \frac{\partial P_i}{\partial \delta_j} = -V_i V_j Y_{ij} \sin(\delta_j + \theta_{ij} - \delta_i)$$

for J_2 Matrix:

$$\frac{\partial P_2}{\partial |V_2|} = 2 V_2 Y_{22} \cos(\theta_{22}) + \left[V_1 Y_{21} \cos(\theta_{21}) + V_3 Y_{23} \cos(\theta_{23})\right]$$

$$= 2 \times 1 \times 58.13 \cos(-63.434) + \left[1.05 \times 22.36 \cos(116.565) + 1.04 \times 35.77 \times \cos(116.565)\right]$$

$$= 24.865.$$

$$\frac{\partial P_3}{\partial |V_3|} = 2 V_3 Y_{33} \cos(\theta_{33}) + \left[V_1 Y_{31} \cos(\theta_{31}) + V_2 Y_{32} \cos(\theta_{32})\right]$$

$$= 2 \times 1.04 \times 67.23 \cos(-67.249) + \left[1.05 \times 31.62 \cos(108.43) + 1 \times 35.77 \cos(116.565)\right]$$

$$= 27.586.$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3|\,|Y_{32}|\,\cos(\theta_{32})$$

$$= 1.04 \times 35.77 \cos(116.565)$$

$$= \cancel{-15.99} = \cancel{\frac{\partial P_2}{\partial |V_3|}}$$

$$= -16.63$$

$$\frac{\partial P_2}{\partial |V_3|} = V_2 Y_{23} \cos(\theta_{23})$$

$$= 1 \times 35.77 \cos(116.565)$$

$$= -15.99$$

for J_3 Matrix:

$$\frac{\partial Q_2}{\partial \delta_2} = V_2 \left[V_1 Y_{21} \cos(\theta_{21}) + V_3 Y_{23} \cos(\theta_{23})\right]$$

$$= 1 \left[1.05 \times \overset{22.36}{\cancel{31.62}} \times \cos\left(\overset{116.565}{\cancel{108.43}}\right) + 1.04 \times 35.77 \cos(116.565)\right]$$

$$= -27.136.$$

$$\frac{\partial Q_3}{\partial \delta_3} = V_3 \left[V_1 Y_{31} \cos(\theta_{31}) + V_2 Y_{32} \cos(\theta_{32})\right]$$

$$= 1.04 \left[1.05 \times 31.62 \cos(108.43) + 1 \times 35.77 \times \cos(116.565)\right]$$

$$= -26.493$$

$$\frac{\partial Q_2}{\partial \delta_3} = -V_2 V_3 Y_{23} \cos(\theta_{23})$$

$$= -1 \times 1.04 \times 35.77 \cos(116.565)$$

$$= 16.6366 = \frac{\partial Q_3}{\partial \delta_2}$$

for J_4 Matrix:

$$\frac{\partial Q_2}{\partial |V_2|} = -2 V_2 Y_{22} \sin(\theta_{22}) + \left[V_1 Y_{21} \sin(\theta_{21}) + V_3 Y_{23} \sin(\theta_{23})\right]$$

$$= -2 \times 1 \times 58.13 \times \sin(-63.434) + \left[1.05 \times 22.36 \sin(116.565) + 1.04 \times 35.77 \sin(116.565)\right]$$

$$= 49.712$$

$$\frac{\partial Q_3}{\partial |V_3|} = -2V_3 Y_{33} \sin(\theta_{33}) - \left[V_1 Y_{31} \sin(\theta_{31}) + V_2 Y_{32} \sin(\theta_{32})\right]$$

$$= -2 \times 1.04 \times 67.23 \sin(-67.249) - \left[1.05 \times 31.62 \sin(108.43) + 1 \times 35.77 \sin(116.565)\right]$$

$$= 65.466.$$

$$\frac{\partial Q_2}{\partial |V_3|} = -|V_2| |Y_{23}| \sin(\theta_{23})$$

$$= -1 \times 35.77 \sin(116.565)$$

$$= -31.99. = \frac{\partial Q_3}{\partial |V_2|}.$$

$$\therefore \begin{bmatrix} -2.862 \\ 1.432 \\ -0.22 \\ -1.026 \end{bmatrix} = \begin{bmatrix} 54.272 & -33.273 & 24.865 & \cancel{-15.99}\ -15.99 \\ -33.273 & 66.03 & -16.63 & 27.586 \\ -27.136 & 16.636 & 49.712 & -31.99 \\ 16.636 & -26.493 & \cancel{-31.99}\ -33.27 & 65.466 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \\ \Delta|V_3| \end{bmatrix}$$

ii, If the Q_2 is with in the limits then the bus-3 can be treated as PV bus itself then the Matrix Equation will become,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

from the above Problem we have already calculated the values Regarding to above equation

$$\therefore \begin{bmatrix} -2.862 \\ 1.432 \\ -0.220 \end{bmatrix} = \begin{bmatrix} 54.272 & -33.273 & 24.865 \\ -33.273 & 66.03 & -16.63 \\ -27.136 & 16.636 & 49.712 \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta \delta_2^0 \\ \Delta \delta_3^0 \\ \Delta |V_2^0| \end{bmatrix} = \begin{bmatrix} -0.0453 \\ -7.8772\times10^{-3} \\ -0.02656 \end{bmatrix}$$

for next iteration,

$$\Delta \delta_2^1 = \delta_2^0 + \Delta \delta_2^0$$

$$\Delta \delta_3^1 = \delta_3^0 + \Delta \delta_3^0$$

$$\Delta |V_2^1| = |V_2^0| + \Delta |V_2^0|$$

$$\circledcirc$$

...upled Power Flow Solution :- Decoupled.

In general Power Transmission lines are mostly Reactive with Resistance value quite small, that is, the transmission lines have high X/R Ratio. Further during the steady-state operation of Electrical power Systems, the difference in bus voltage angle between two adjacent buses is . Reasonably small, typically a few degrees. The effect of these factors on the Sub-Matrix J_2 will now be examined.

The diagonal and off-diagonal terms of Sub-Matrix J_2 are given as,

$$\frac{\partial P_i}{\partial |V_i|} = 2V_i|Y_{ij}| \cos \theta_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad (1)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad \text{for } (i \neq j). \quad (2)$$

Since, the transmission lines have high X/R Ratio, the Admittance angle θ_{ij} is close to 90°. Thus, the off diagonal & diagonal elements of J_2 is neglected or equal to zero.

On this same Reason, the diagonal and off-diagonal elements of J_3 also equal to zero.

Thus, it can be said that the Real power flow is strongly dependent on the bus voltage angle δ_i, and negligibly on voltage magnitude $|V_i|$.

Similarly, the Reactive power flow is largely dependent on voltage magnitude $|V_i|$, where δ_i has no significant value.

Now the decoupling characteristic of Jacobian Matrix by equating the sub-matrices J_2 and J_3 to zero. The Resulting equations will be as follows,

$$\because \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

$$\Rightarrow [\Delta P] = [J_1][\Delta\delta] \text{ and }$$

$$[\Delta Q] = [J_4][\Delta|V|]$$

$$[\Delta P] = [J_1][\Delta\delta]$$

$$[\Delta Q] = [J_4][\Delta|V|]$$

$$\Rightarrow [\Delta\delta_i^K] = [J_1^K]^{-1}[\Delta P_i^K] \quad \text{---} (3)$$

$$\Rightarrow [\Delta|V_i|^K] = [J_4^K]^{-1}[\Delta Q_i^K]; \quad \text{---} (4)$$

The above stated (3) and (4) equations are the load flow equations of Decoupled power flow solution.

iii) Decoupled power flow solution :-

Fast Decoupled

It is observed that at the end of 1st iteration the elements of Jacobian Matrix have to be Re-computed. Considerable time can be saved if simplifications are introduced to avoid the re-computation of Jacobian Matrix at the end of iteration. Such modifications were introduced by Stott and Alsac in 1974 leading to Fast Decoupled power flow solution

The diagonal elements of J_1 are to be Re-written as,

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij}+\delta_j-\delta_i) - |V_i|^2|Y_{ii}| \sin(\theta_{ii}). \quad \text{①}$$

Now, Replacing the first term of above equation with Reactive power of static power flow equation.

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2|Y_{ii}| \sin(\theta_{ii}). \quad \text{②}$$

$$= -Q_i - |V_i|^2 B_{ii} \quad \text{③}$$

where, $|B_{ii}| = |Y_{ii}| \sin \theta_{ii}$ is the imaginary part of diagonal elements of YBus. In a typical power system, the self susceptance $B_{ii} >> Q_i$, and we may neglect Q_i.

And assuming $|V_i|^2 \approx |V_i|$ which yields,

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii}. \quad \text{④}$$

under normal operating conditions, $\delta_j - \delta_i$ is quite small.

Thus, assuming $\theta_{ii} + \delta_j - \delta_i \approx \theta_{ii}$, the off diagonal elements of J_1 becomes,

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i|\,|V_j|\,B_{ij}.$$

Now, by assuming $|V_j| = 1$, the further symplification will be,

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i|\,B_{ij} \quad \text{(A)}$$

Similarly, the diagonal elements of J_4 described by,

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,|Y_{ii}|\sin\theta_{ii} - \sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i)$$

Replacing 2nd term with $-Q_i$

$$\Rightarrow \frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,|Y_{ii}|\sin\theta_{ii} + Q_i$$

Again, since $B_{ii} = Y_{ii}\sin\theta_{ii} \gg Q_i$, so, Q_i may be neglected

$$\Rightarrow \frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,B_{ii}$$

Similarly, $$\frac{\partial Q_i}{\partial |V_j|} = -|V_i|\,|V_j|\,B_{ij} \quad \text{(A)}$$

by assuming $|V_j| = 1$ $\therefore$ $$\frac{\partial Q_i}{\partial |V_j|} = -|V_i|\,B_{ij} \quad \text{(C)}$$

By the Equations (A) and (C) we can write in general,

$$\left.\begin{aligned} \frac{\Delta P}{|V_i|} &= -B'\,\Delta\delta \\ \frac{\Delta Q}{|V_i|} &= -B''\,\Delta|V|. \end{aligned}\right\} \quad \text{— D}$$

This Equations are the load flow studies of Fast Decoupled load flow Solution.

Appendix A
DC Load Flow

A.1 The Load Flow Problem

Formulation of classic load flow problem requires considering four variables at each bus i of power system. These variables are

1. P_i (Net active power injection)
2. Q_i (Net reactive power injection)
3. V_i (Voltage magnitude)
4. θ_i (Voltage angle)

The active and reactive power injections are calculated as follows

$$P_i = P_{Gi} - P_{Di} \tag{A.1}$$

$$Q_i = Q_{Gi} - Q_{Di} \tag{A.2}$$

in which P_{Gi} and Q_{Gi} are active and reactive power generations at bus i, respectively, whereas P_{Di} and Q_{Di} are active and reactive power demands at this bus, respectively.

Based on the application of Kirchhoff's laws to each bus

$$\mathbf{I} = \mathbf{YV} \tag{A.3}$$

$$I_i = \frac{(P_i - jQ_i)}{|V_i|} e^{j\theta_i} \tag{A.4}$$

where

I_i Net injected current at bus i
$\mathbf{V}$ Vector of bus voltages
$\mathbf{I}$ Vector of injected currents at the buses
$\mathbf{Y}$ Bus admittance matrix of the system

I, **V** and **Y** are complex. $V_i = |V_i|e^{j\theta_i}$ is the ith element of vector **V**. The **Y** matrix is symmetrical. The diagonal element Y_{ii} (self admittance of bus i) contains the sum of admittances of all the branches connected to bus i. The off diagonal element Y_{ij} (mutual admittance) is equal to the negative sum of the admittances between buses i and j. $Y_{ij} = |Y_{ij}|e^{j\delta_{ij}} = G_{ij} + jB_{ij}$ lies in the ith row and the jth column of matrix **Y**. G and B are subsequently called conductance and susceptance, respectively..

Using (A.4) to replace **I** in (A.3) results in (A.5) and (A.6).

$$P_i = \sum_{j=1}^{N} |Y_{ij}||V_i||V_j| \cos(\theta_i - \theta_j - \delta_{ij}) \tag{A.5}$$

$$Q_i = \sum_{j=1}^{N} |Y_{ij}||V_i||V_j| \sin(\theta_i - \theta_j - \delta_{ij}) \tag{A.6}$$

where N is the number of system buses.

To solve full load flow equations, two of four variables must be known in advance at each bus. This formulation results in a non-linear system of equations which requires iterative solution methods. In this formulation, convergence is not guaranteed.

A.2 DC Load Flow Solution

Direct Current Load Flow (DCLF) gives estimations of lines power flows on AC power systems. DCLF looks only at active power flows and neglects reactive power flows. This method is non-iterative and absolutely convergent but less accurate than AC Load Flow (ACLF) solutions. DCLF is used wherever repetitive and fast load flow estimations are required.

In DCLF, nonlinear model of the AC system is simplified to a linear form through these assumptions

- Line resistances (active power losses) are negligible i.e. $R \ll X$.
- Voltage angle differences are assumed to be small i.e. $\sin(\theta) = \theta$ and $\cos(\theta) = 1$.
- Magnitudes of bus voltages are set to 1.0 per unit (flat voltage profile).
- Tap settings are ignored.

Based on the above assumptions, voltage angles and active power injections are the variables of DCLF. Active power injections are known in advance. Therefore for each bus i in the system, (A.5) is converted to

$$P_i = \sum_{j=1}^{N} B_{ij}(\theta_i - \theta_j) \tag{A.7}$$

in which B_{ij} is the reciprocal of the reactance between bus i and bus j. As mentioned earlier, B_{ij} is the imaginary part of Y_{ij}.

As a result, active power flow through transmission line i, between buses s and r, can be calculated from (A.8).

$$P_{Li} = \frac{1}{X_{Li}}(\theta_s - \theta_r) \tag{A.8}$$

where X_{Li} is the reactance of line i.

DC power flow equations in the matrix form and the corresponding matrix relation for flows through branches are represented in (A.9) and (A.10).

$$\theta = [\mathbf{B}]^{-1}\mathbf{P} \tag{A.9}$$

$$\mathbf{P_L} = (\mathbf{b} \times \mathbf{A})\theta \tag{A.10}$$

where

P	N × 1 vector of bus active power injections for buses 1, …, N
B	N × N admittance matrix with R = 0
$\boldsymbol{\theta}$	N × 1 vector of bus voltage angles for buses 1, …, N
$\mathbf{P_L}$	M × 1 vector of branch flows (M is the number of branches)
b	M × M matrix (b_{kk} is equal to the susceptance of line k and non-diagonal elements are zero)
A	M × N bus-branch incidence matrix

Each diagonal element of **B** (i.e. B_{ii}) is the sum of the reciprocal of the lines reactances connected to bus i. The off-diagonal element (i.e. B_{ij}) is the negative sum of the reciprocal of the lines reactances between bus i and bus j.

A is a connection matrix in which a_{ij} is 1, if a line exists from bus i to bus j; otherwise zero. Moreover, for the starting and the ending buses, the elements are 1 and −1, respectively.

Example A.1 A simple example is used to illustrate the points discussed above. A three-bus system is considered. This system is shown in Fig. A.1, with the details given in Tables A.1 and A.2.

With base apparent power equal to 100 MVA, **B** and **P** are calculated as follows

$$\mathbf{B} = \begin{bmatrix} 23.2435 & -17.3611 & -5.8824 \\ -17.3611 & 28.2307 & -10.8696 \\ -5.8824 & -10.8696 & 16.7519 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} \text{Unknown} \\ 0.53 \\ -0.9 \end{bmatrix}$$

As bus 1 is considered as slack,[1] the first row of **P** and the first row and column of **B** are disregarded. θ_2 and θ_3 are then calculated using (A.9) as follows.

[1] With angle = 0.

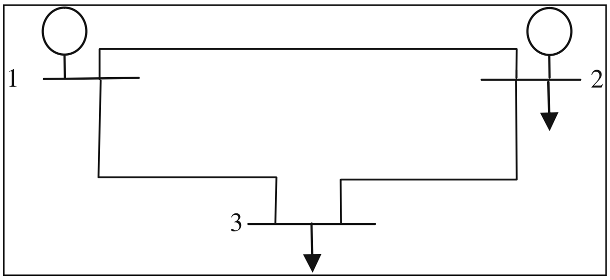


Fig. A.1 Three-bus system

Table A.1 Loads and generations

Bus number	Bus type	P_D (MW)	Q_D (MVAr)	P_G (MW)
1	Slack	0	0	Unknown
2	PV	10	5	63
3	PQ	90	30	0

Table A.2 Branches

Line number	From bus	To bus	X (p.u.)	Rating (MVA)
1	1	2	0.0576	250
2	2	3	0.092	250
3	1	3	0.17	150

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 28.2307 & -10.8696 \\ -10.8696 & 16.7519 \end{bmatrix}^{-1} \begin{bmatrix} 0.53 \\ -0.9 \end{bmatrix} = \begin{bmatrix} -0.0025 \\ -0.0554 \end{bmatrix} \text{Radian} = \begin{bmatrix} -0.1460^\circ \\ -3.1730^\circ \end{bmatrix}$$

A and **b** are then calculated as

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 17.3611 & 0 & 0 \\ 0 & 10.8696 & 0 \\ 0 & 0 & 5.8824 \end{bmatrix}$$

Therefore, the transmission flows are calculated using (A.10) as follows

$$\begin{bmatrix} P_{L1} \\ P_{L2} \\ P_{L3} \end{bmatrix} = \text{BaseMVA} \times \mathbf{b} \times \mathbf{A} \times \theta$$

$$= 100 \times \begin{bmatrix} 17.3611 & 0 & 0 \\ 0 & 10.8696 & 0 \\ 0 & 0 & 5.8824 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ -0.0025 \\ -0.0554 \end{bmatrix}$$

$$= \begin{bmatrix} 4.4243 \\ 57.4243 \\ 32.5757 \end{bmatrix} \text{MW}$$

Symmetrical Faults :-

Generally, the faults are classified as two types.

1. Symmetrical faults and
2. un symmetrical faults

Symmetrical fault Analysis from V.K.Mehtha & Rohit Mehtha

→ Symmetrical faults are 3ϕ faults → Most Severe fault

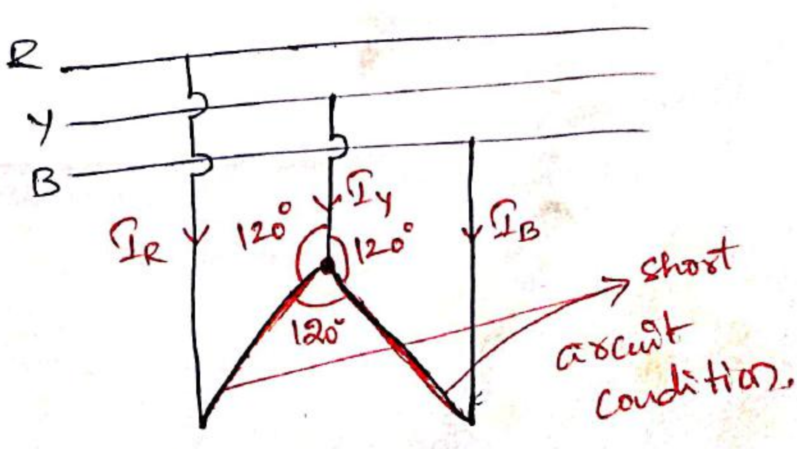
→ un symmetrical faults are → L-L-fault
→ L-G-fault
→ L-L-G-fault

Most frequently occuring fault

Symmetrical fault :-

The fault that occurs on the power system which gives rise to symmetrical fault currents (i.e, equal fault currents in the lines with 120° displacement) is known as symmetrical fault

The symmetrical fault occurs when all the three conductors of a 3ϕ lines are brought together simultaneously into a short circuit condition. This type of faults gives rise to symmetrical currents i.e, equal fault currents with 120° displacement.



Because of Balanced nature of fault, only one phase need to be considered for the system calculations, since the condition for all other phases are similar.

→ Symmetrical fault rarely occurs in practice as the majority of faults are of unsymmetrical in nature.

→ Symmetrical fault is the most severe fault.

* Percentage Reactance :-

The Reactance of Generators, transformers etc is usually specified in percentage Reactance to permit rapid short circuit calculations. The percentage reactance of a circuit is defined as

It is the percentage of total phase voltage dropped in the circuit when full load current is flowing.

$$\Rightarrow \quad \% X = \frac{IX}{V} \times 100 \quad \text{——} \quad (1)$$

where I = full load current

V = phase voltage

X = Reactance in ohms.

Alternatively, percentage Reactance ($\% X$) can also be expressed in terms of KVA and KV as,

$$\% X = \frac{X\,(KVA)}{10\,(KV)^2} \quad \text{——} \quad (2)$$

Base KVA and Base KV

If 'x' is the only Reactance in the circuit then the short circuit current is given by

$$I_{sc} = \frac{V}{X} \quad (A)$$

By substituting x from equation ① we get

$$① \Rightarrow \%X = \frac{IX}{V} \times 100$$

$$\Rightarrow X = \frac{\%X V}{I \times 100}$$

$$\therefore I_{sc} = \frac{V}{\frac{\%X V}{I \times 100}} = \frac{\cancel{V} \times I \times 100}{\%X \times \cancel{V}}$$

$$= \frac{I}{\%X} \times 100.$$

$$I_{sc} = \frac{V}{Z} = \frac{V}{R+jX}$$

$$\text{or} \quad I_{sc} = \frac{V}{\sqrt{R^2+X^2}}$$

$$= \frac{V}{\sqrt{X^2}}$$

$$I_{sc} = \frac{V}{X}$$

$$\therefore \boxed{I_{sc} = \frac{100}{\%X} \times I}$$

where, I = full load current

$$I = \frac{\text{Base KVA}}{\sqrt{3} \times \text{KV at Point of fault occurs.}}$$

Eg: Percentage Reactance of an element is 20%. and Full load current is 50A then I_{sc} = ?

Sol. given $\%X = 20\%$

$$I = 50\,A$$

$$\therefore I_{sc} = \frac{100}{\%X} \times I$$

$$= \frac{100}{2\cancel{0}} \times 5\cancel{0}$$

$$= 250\,A.$$

* Percentage Reactance and Base KVA :-

From the equation (2) we can say that the percentage reactance of an element depends up on the KVA rating. As we know that different elements in power systems having different KVA ratings. So, it is necessary to find the percentage ratings of all elements on common KVA rating. This common KVA rating is known as Base KVA.

$$\boxed{\%\ \text{reactance at Base KVA} = \frac{\text{Base KVA}}{\text{Rated KVA}} \times \%\ \text{reactance at rated KVA}} \quad \text{--- (3)}$$

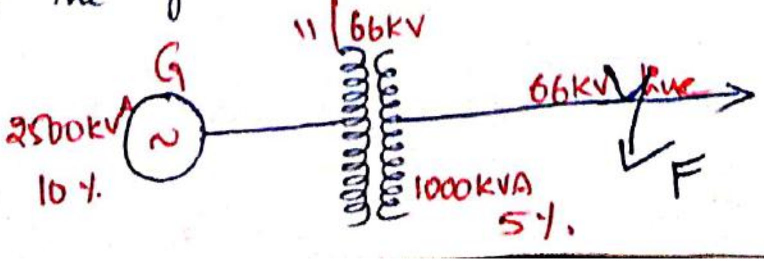
NOTE :-

Percentage Reactance depends up on Base KVA, but the short circuit current does not have any dependence on Base KVA. That means, the Base KVA does not affect the short circuit current.

problem :-

A 3ϕ transmission line operating at 66KV and connected through a 1000KVA x^{mer} with 5% reactance. The generator is of 2500 KVA with 10% reactance. Suppose a fault between 3ϕ occurs at high voltage terminals of x^{mer}.

Sol. from the given data, the single line diagram will be,



Suppose if we choose 2500 KVA as base value then,

Reactance of X^{mer} on 2500 KVA $= \frac{2500}{1000} \times 5$

$\% X_{T/F} = 12.5\%$

Reactance of generator at 2500 KVA $= \frac{2500}{2500} \times 10$

$\% X_G = 10\%$

Now, total % Reactance on 2500 KVA $= \% X_T + \% X_G$

$= 12.5 + 10$

$\underline{\% X_T} = 22.5\%$

Short circuit Current $= I \times \frac{100}{\% X}$

The full load Current $I = \frac{\text{Base KVA}}{\sqrt{3} \times \text{KV at which fault occurs}}$

$= \frac{2500 \text{ KVA}}{\sqrt{3} \times 66 \text{ KV}} = 21.87 \text{ A}$

Now, S/C Current $= 21.87 \times \frac{100}{22.5}$

$\boxed{I_{sc} = 97.2 \text{ A}}$

If we assume Base KVA as 3000 KVA then,

Reactance of X^{mer} at 3000KVA $= \frac{3000}{1000} \times 5$

$= 15\%$

Reactance of generator on 3000 KVA $= \frac{3000}{2500} \times 10$

$= 12\%$

Now, Total % Reactance on 3000 KVA $= X_T + X_G$

$= 15 + 12 = \underline{27\%}$

full load Current on 3000 KVA $= \dfrac{\text{Base KVA}}{\sqrt{3} \times \text{fault occured KV}}$

$$= \frac{3000\ \cancel{K}VA}{\sqrt{3} \times 66\ \cancel{KV}}$$

$$= 26.24\ A$$

$\therefore$ Short circuit Current $I_{sc} = 26.24 \times \dfrac{100}{27}$

$$= 97.185\ A$$

$$\cong 97.2\ A$$

NOTE :-

From the above illustration we can say that the short circuit current will be same for any Base KVA value

* Short circuit KVA :-

Generally, the short circuit current expressed in terms of KVA.

The product of normal system voltage and short circuit current at the point of fault expressed in KVA is known as short circuit KVA.

we know that, $\boxed{I_{sc} = I \times \dfrac{100}{\%X}}$ A

Short circuit KVA will be calculated for 3ϕ systems

Short circuit KVA for 3ϕ circuit will be,

$$\boxed{I_{sc}\ (3\phi) = \text{Base KVA} \times \frac{100}{\%X}} \text{ KVA}$$

* Steps for Symmetrical fault Calculations :-

1. Draw a single line diagram of the complete network indicating the ratings of voltage and percentage Reactances of each element.

2. Choose a Convinient value of base KVA and convert all percentage reactances on this base value.

3. Corresponding to single line diagram of the network, draw the reactance diagram showing one phase of the system and Neutral. Indicate the % reactances on base KVA in Reactance diagram.

4. Find the total % reactance of the network upto the point of fault. let it be X %.

5. Find the full-load current corresponding to the selected KVA and the normal system voltage at the fault point 'F'. let it be I.

6. Then, the various short circuit calculations are

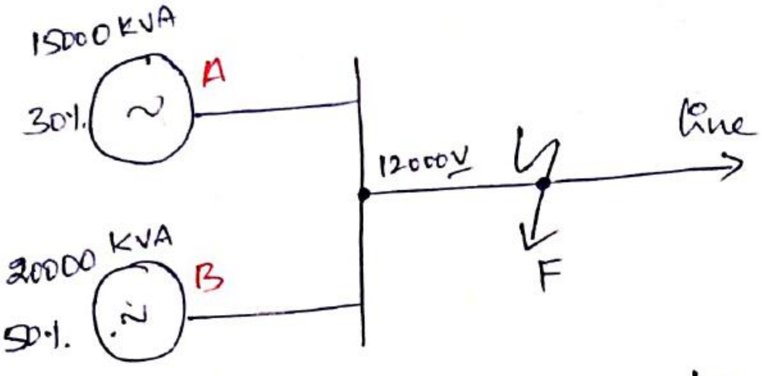
Short circuit Current $\boxed{I_{sc} = I \times \frac{100}{\% X}}$

$$\boxed{\text{Short circuit KVA} = \text{Base KVA} \times \frac{100}{\% X}}$$

Problems:-

1. For a single line diagram of 3φ system. Calculate the short circuit current that flow into a 3φ circuit at Fault F.

Sol:



Let us assume the Base KVA be 35000 KVA

% Reactance of alternator A at 35000 KVA $= \frac{35000}{15000} \times 30$

$= 70\%$

% Reactance of alternator B at 35000 KVA $= \frac{35000}{20000} \times 50$

$= 87.5\%$

Now, full load current $I = \frac{\text{Base KVA}}{\sqrt{3} \times \text{Voltage (KV) at fault occured}}$

$= \frac{35000}{\sqrt{3} \times 12} = 1684\ A$

Now, Total Reactance from generator to the fault will be,

$\%X = \%X_A \parallel \%X_B$

$= \frac{X_A \cdot X_B}{X_A + X_B} = \frac{70 \times 87.5}{70 + 87.5}$

$= 38.89\%$

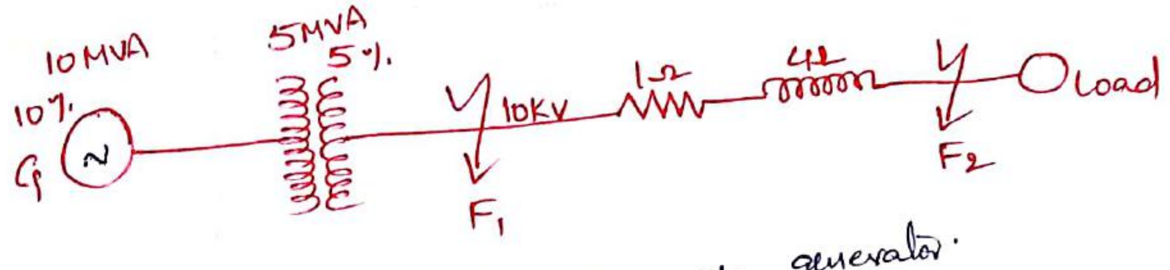
∴ Short circuit current $I_{sc} = I \times \frac{100}{\%X} = 1684 \times \frac{100}{38.89}$

$= 4330\ A$

(Q) 3φ tr. line operating at 10kV having a Resistance of 1Ω and Reactance of 4Ω. Calculate the short circuit kVA fed to symmetrical fault between phases if it occurs

i, at the end of transmission line

ii, at the high voltage terminals of transformer.



Sol. let assume Base values are at generator.

∴ % Reactance of alternator at 10MVA $= \frac{10000\,kVA}{10000\,kVA} \times 10\%$

$= 10\%$

% Reactance of transformer at 10,000 kVA $= \frac{10,000\,kVA}{5000\,kVA} \times 5$

$= 10\%$

The transmission line having Resistance and Reactance. So converting these also into percentage Reactances & Resistances.

% Reactance of transmission line $= \frac{(\text{Base kVA}) \times \text{Reactance in } \Omega}{10 \times (\text{Base kV})^2_{\text{line}}}$

$= \frac{10,000 \times 4}{10 \times (10)^2}$

$= 40\%$

% Resistance of line at 10,000 KVA $= \frac{10,000 \times 1}{10 \times (10)^2}$

$= 10\%$

i, Total % Reactance $= \% X_A + \% X_T + \% X_L$

$= 10 + 10 + 40 = 60\%$

% Resistance $= 10\%$

∴ % impedance from generator to line of fault F_2

$= \sqrt{(60)^2 + (10)^2} = 60.83\%$

Short circuit KVA $= 10,000 \times \frac{100}{60.83}$

$\left[\text{Base KVA} \times \frac{100}{\% Z} = I_{sc} \right]$

∴ $\boxed{I_{sc} = 16440 \text{ KVA}}$

ii, Total Reactance from generator to fault F_1,

$= \% X_A + \% X_T = 10 + 10$

$= 20\%$

Short circuit KVA $= 10000 \times \frac{100}{20}$

∴ $\boxed{I_{sc} = 50,000 \text{ KVA}}$

(5) A 3ϕ, 20MVA, 10kV Alternator has internal Reactance of 5% and negligible Resistance. Find the external Reactance per phase to be connected in Series with the Alternator, so that steady state Current on s/c does not exceed 8 times the full load Current.

Sol. let Base MVA = 20 MVA

Base KV = 10KV

when the s/c fault occurs, then the total Reactance from the generator neutral to the point of fault is,

$(x+5)\%$.

Short circuit KVA should not exceed 8 times = 8 × 20MVA

$$\Rightarrow \text{Fault KVA} = \text{Base KVA} \times \frac{100}{\%X}$$

$$\Rightarrow 20\text{MVA} \times 8 = 20,000 \times \frac{100}{x+5}$$

$$\rightarrow \cancel{20,000} \times 8 = \cancel{20000} \times \frac{100}{x+5}$$

$$\Rightarrow 8x + 40 = 100$$

$$\Rightarrow x = \frac{60}{8} = 7.5\%$$

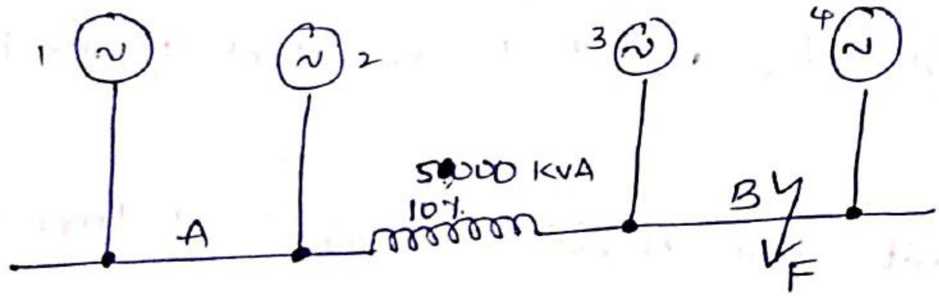
It can be converted to Ω.

$$\%X = \frac{(\text{KVA}) \times \text{Reactance in } \Omega}{10 \times (\text{KV})^2}$$

$$7.5 = \frac{20,000 \times (X)}{10 \times (10)^2}$$

$$\Rightarrow X = 0.375 \text{ in } \Omega.$$

(4) The Section bus-bars A and B are linked by a bus-bar Reactor rated 5000 KVA with 10% Reactance. on the bus bar A there are two generators of each 10,000 KVA with 10% Reactance and on B two generators each 8000 KVA with 12% Reactance. Find Short circuit KVA when the fault occurs on Bus-bar B between two generators.



Sol. let, Base KVA = 10,000 KVA.

% Reactance of generators on bus-bar A $= \dfrac{\text{Base KVA}}{\text{Rated KVA}} \times X_{\text{Rated}}$

$$\Rightarrow \% X = \frac{10,000}{10,000} \times 10\%$$

$$= 10\%$$

% Reactance of generators on bus-bar B $= \dfrac{\text{Base KVA}}{\text{Rated KVA}} \times X_{\text{Rated}}$

$$= \frac{10,000}{8,000} \times 12\%$$

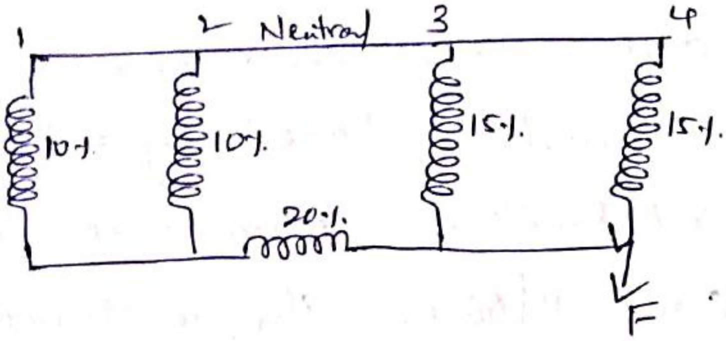
$$= 15\%.$$

% Reactance of Reactor between bus-bars A and B is,

$$\Rightarrow \% X = \frac{\text{Base KVA}}{\text{Rated KVA}} \times X_{\text{Rated}}$$

$$= \frac{10,000}{8,000} \times 10 = 20\%$$

Now, the circuit will be,



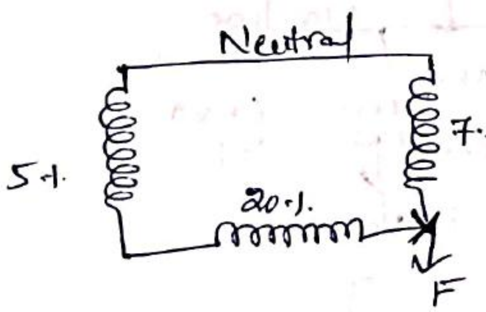
Two Sections కలిపినప్పుడు, ఏ section of Fault occur అయితే, ఆ section's Reactor ని consider చేయకూడదు.

Since 1,2 and 3,4 are in parallel

$\Rightarrow$ Equivalent Reactance of 1 & 2 $= \dfrac{10 \times 10}{10+10} = 5\%$.

Equivalent Reactance of 3 & 4 $= \dfrac{15 \times 15}{15+15} = 7.5\%$.

Modified circuit will be,



$\therefore$ from the Modified ckt. diagram it is clear that 5% & 20% are in series and the total in parallel with 7.5%.

$\therefore$ Total % Reactance $= (20+5) \parallel 7.5$

$= \dfrac{25 \times 7.5}{25+7.5}$

$\%X_{Total} = 5.769\%$.

Now, Short circuit KVA $= \text{Base KVA} \times \dfrac{100}{\%X}$

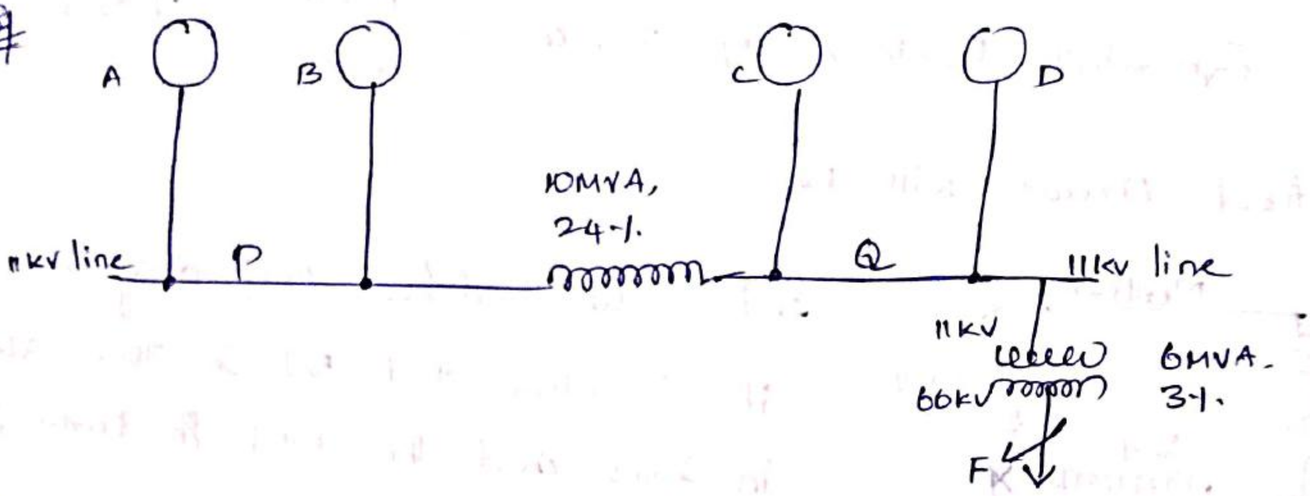
$= 10{,}000 \times \dfrac{100}{5.769}$

$= 173.34$ ~~MVA~~ MVA

(300a)

(5) An 11KV generating station has four identical 3φ Alternators A, B, C, D each of 10MVA and 12% Reactance. There are two Sections of bus-bars P and Q linked by a Reactor rated at 10MVA with 24% Reactance. From each Section load is taken through 6MVA, 11/66 KV step up transformers having Reactance of 3%. Calculate the s/c KVA when fault occurs at high voltage terminals of transformer.

Sol



Sol. let Base KVA = 10,000 KVA

$$\% \text{ Reactance of each generator} = \frac{\text{Base KVA}}{\text{Rated KVA}} \times X_{Rated}$$

$$= \frac{10,000 \text{ KVA}}{10,000 \text{ KVA}} \times 12\%$$

$$= 12\%.$$

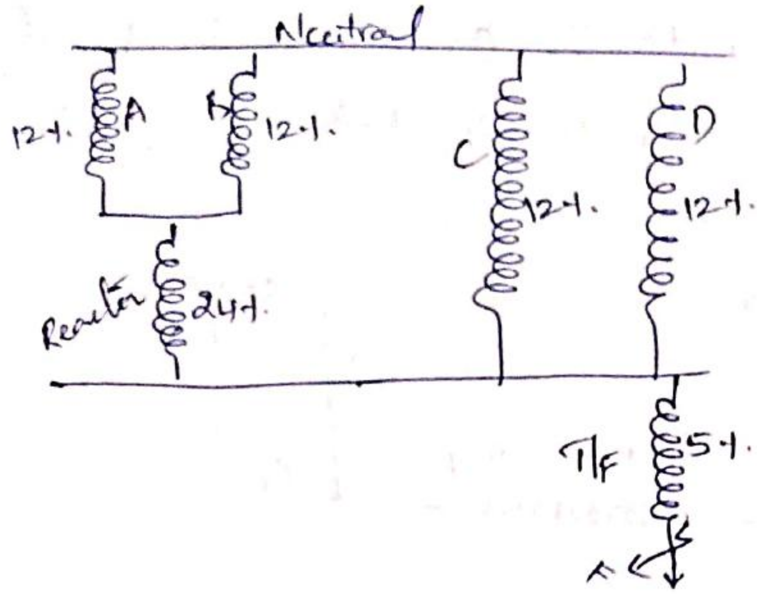
$$\% \text{ Reactance of Reactor} = \frac{10,000}{10,000} \times 24\%.$$

$$= 24\%.$$

$$\% \text{ Reactance of transformer} = \frac{10,000}{6,000} \times 3\%.$$

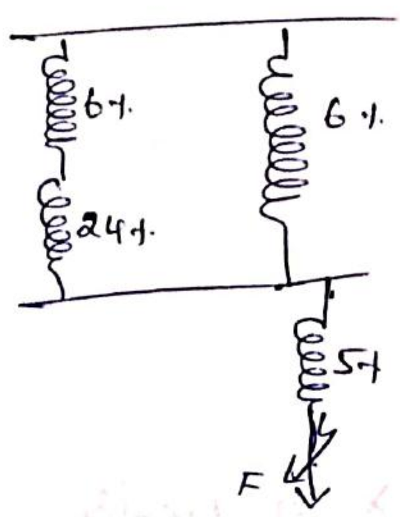
$$= 5\%.$$

Now the circuit will be,



A, B and C, D are in parallel then Equivalent Reactances will be, $\frac{12 \times 12}{12+12} = \frac{144}{24} = 6\%$.

Modified diagram will be,



$\therefore$ Overall % Reactance will be,

$$\% X = [(24+6) \parallel 6] + 5$$

$$= \frac{30 \times 6}{30+6} + 5 = \frac{180}{36} + 5$$

$$\% X_{Total} = 10\%.$$

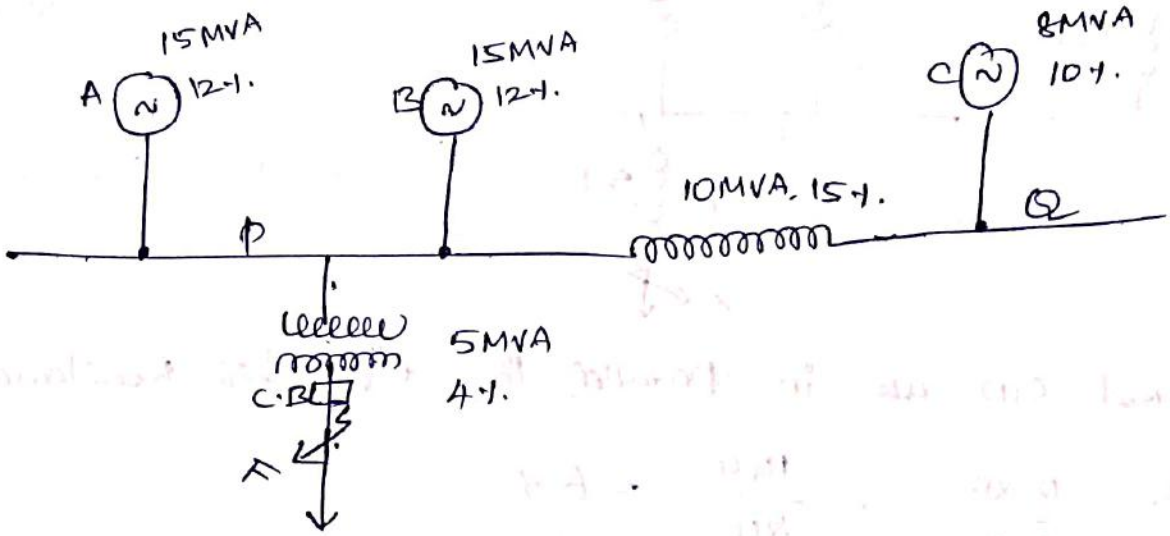
Now, Faulted KVA (or) Short circuit KVA will be,

$$s/c \text{ KVA} = \text{Base KVA} \times \frac{100}{\% X_{Total}}$$

$$= 10,000 \times \frac{100}{10\%}$$

$$= 100 \text{ MVA}$$

(6) The bus-bars of a power station are in two sections P & Q seperated by a Reactor as shown in the fig. Determine the Maximum Short circuit KVA. 3(a)



Sol: let the Base MVA = 10 MVA.

$\%X$ for A and B $= \cancel{\frac{10}{10} \times 12}\ \frac{10}{15} \times 12$

$= \cancel{12\%}\ .\ 8\%.$

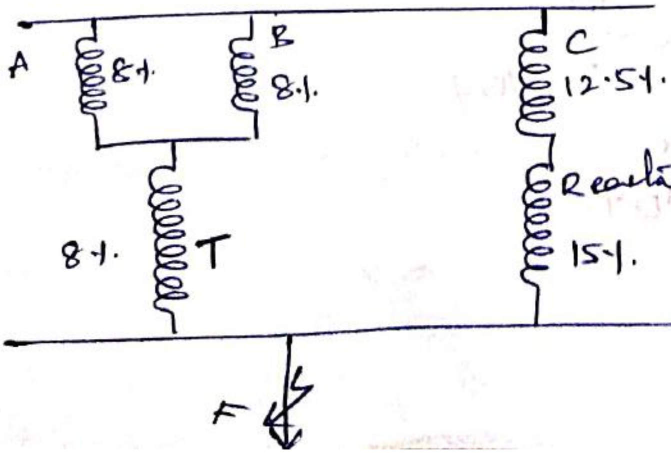
$\%X$ for C $= \frac{10}{8} \times 10$

$= 12.5\%.$

$\%X$ for transformer $= \frac{10}{5} \times 4$

$= 8\%.$

Now, the circuit diagram will be



$\%X$ for Reactor

$= \frac{\text{Base KVA}}{\text{Rated KVA}} \times \%X_{\text{Rated}}$

$\%X_r = \frac{10\,\cancel{MVA}}{10\,\cancel{MVA}} \times 15\%.$

$= \underline{15\%.}$

A, B are in parallel and Resultant in series with 8% (T/F)

$$\therefore \%X = \frac{8\times 8}{8+8} + 8$$

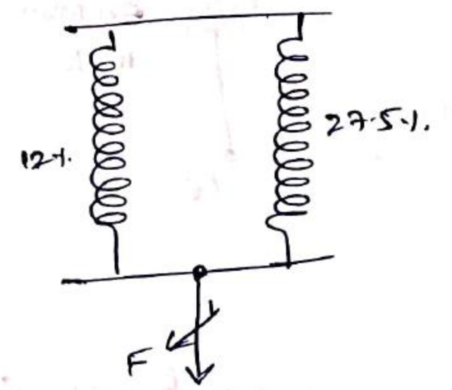
$$= \frac{64}{16} + 8 = 12\%$$

generator C and Reactor are in series

$$\%X = 12.5 + 15$$

$$= 27.5$$

$\therefore$ Modified circuit diagram



Now, the total % Reactance at the point of fault is,

$$\%X_{Total} = 12 \parallel 27.5$$

$$= \frac{12\times 27.5}{12+27.5}$$

$$= 8.35\%$$

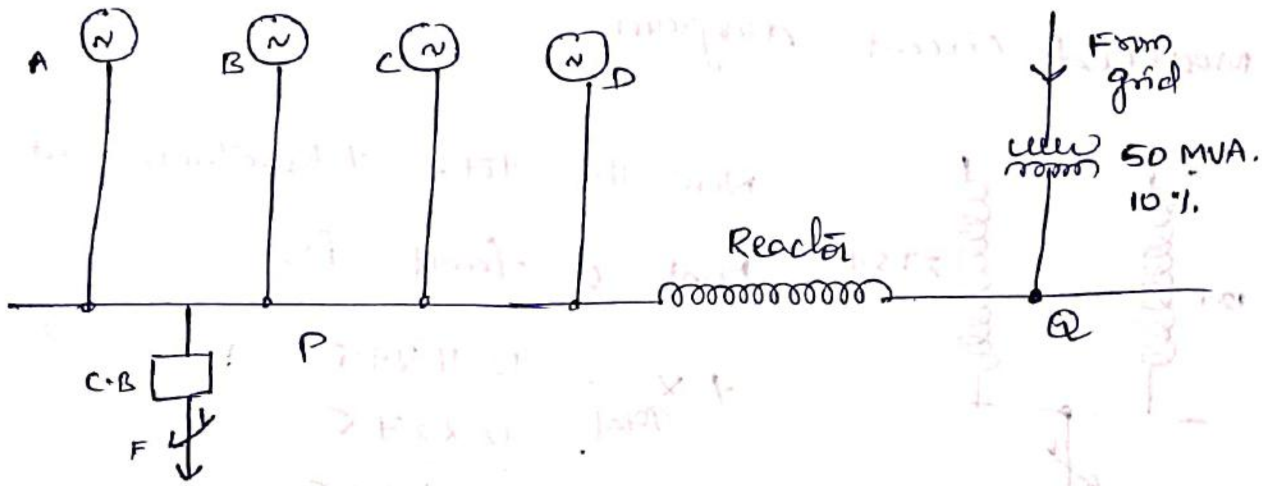
$$\therefore \text{Fault MVA (or) S/C MVA} = \text{Base MVA} \times \frac{100}{\%X}$$

$$= 10{,}000 \text{ KVA} \times \frac{100}{8.35}$$

$$= 119.76 \text{ MVA}$$

(7) 33 KV bus bars of a station are in two sections P and Q seperated by a reactor. The section P is fed from four 10 MVA generators having a Reactance of 20%. The Q is fed from grid through a 50 MVA transformer of 10% Reactance. The circuit breaker have a rupturing capacity of 500 MVA. Determine the Reactance of Reactor to prevent the C.B from the fault. Take a base of 50 MVA

320



Sol. From the above problem the rupturing capacity of C.B is 500 MVA i.e, short circuit MVA should not exceed 500 MVA.

Now, the % Reactance of each generator is,

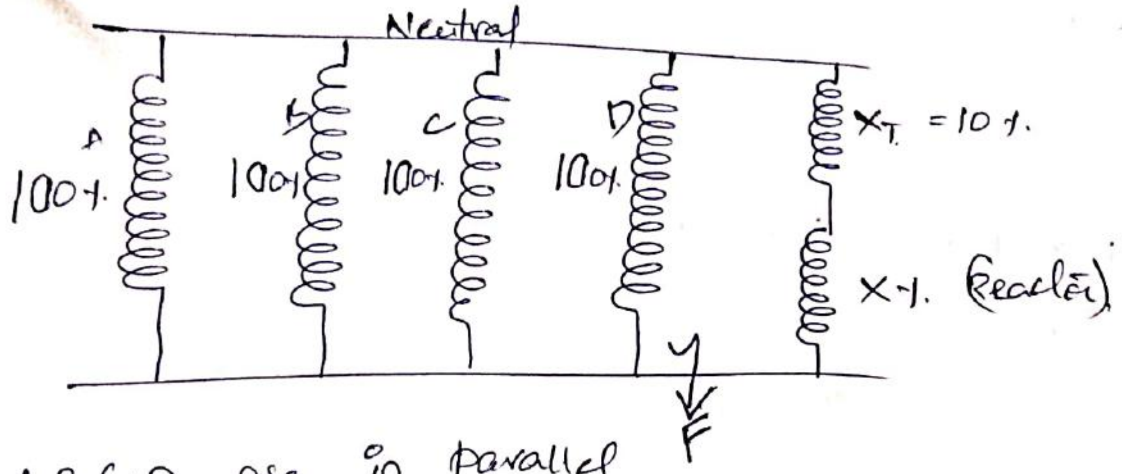
$$\% X = \frac{50}{10} \times 20$$

$$= 100\%.$$

$$\% X \text{ of transformer} = \frac{50}{50} \times 10$$

$$= 10\%.$$

Now, the circuit will be,



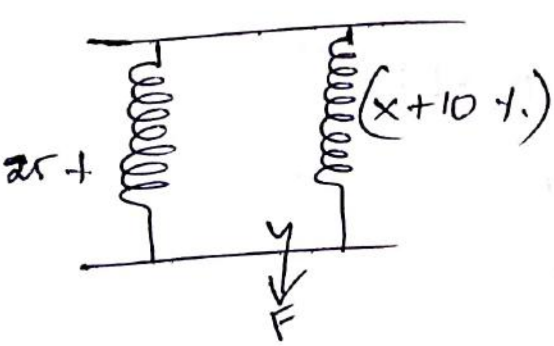
A, B, C, D are in parallel

∴ Equivalent Reactance is $\frac{100 \times 100}{100+100} \parallel \frac{100 \times 100}{100+100}$

$\Rightarrow 50 \parallel 50$

$\Rightarrow \frac{50 \times 50}{50+50} = 25\%$.

⇒ Modified diagram will be,



∴ Total % Reactance is,

$$\%X = \frac{25(x+10)}{25+(x+10)}$$

Now, the fault MVA should not exceed 500 MVA,

$$\Rightarrow \text{Short circuit MVA} = \text{Base MVA} \times \frac{100}{\% X}$$

$$\rightarrow 500 = 50 \times \frac{100}{\frac{25(x+10)}{25+(x+10)}}$$

$$\Rightarrow \frac{25x + 250}{x + 35} = 10$$

$$\Rightarrow 25x + 250 = 10x + 350$$

$$\Rightarrow 25x - 10x = 350 - 250$$

$$\Rightarrow 15x = 100$$

$$\Rightarrow x = \frac{100}{15} \%$$

Now,

$$\% \text{ Reactance} = \frac{(\text{Base KVA}) \times \text{Reactance in } \Omega}{10 \times (\text{KV})^2}$$

$$\frac{100}{15} = \frac{50\text{M} \times \text{Reactance in } \Omega}{10 \times (33)^2}$$

$$\Rightarrow \text{Reactance in } \Omega = \frac{\frac{\cancel{100}^{2}}{15} \times 10 (33)^2}{5\cancel{0} \times 10^3 \text{ KVA}}$$

$$= \frac{\frac{2}{15} \times 10 \times (33)^2}{1000}$$

$$= 1.452 \, \Omega$$

$\therefore$ Reactance of Reactor $= 1.452 \, \Omega$

Note: Importance of Reactors, & Location of Reactors' topic

Refer to Principles of Power Systems by V.K. Mehtha and Rohith Mehtha Page No. 400 & 401.

i, Generator Reactors ii, Feeder Reactors iii, Bus-bar Reactors.

Date 8/3/13 C.L. Wadhwa

Unit-6

V.K. Mehtha & Rohit Mehtha

Un Symmetrical faults

The great Majority of faults on the power System are of un symmetrical in nature. The most Common type being a short circuit from one line to ground. when such a fault occurs, it gives rise to un symmetrical currents i.e, Magnitude of fault currents in the three lines are different having unequal phase displacement.

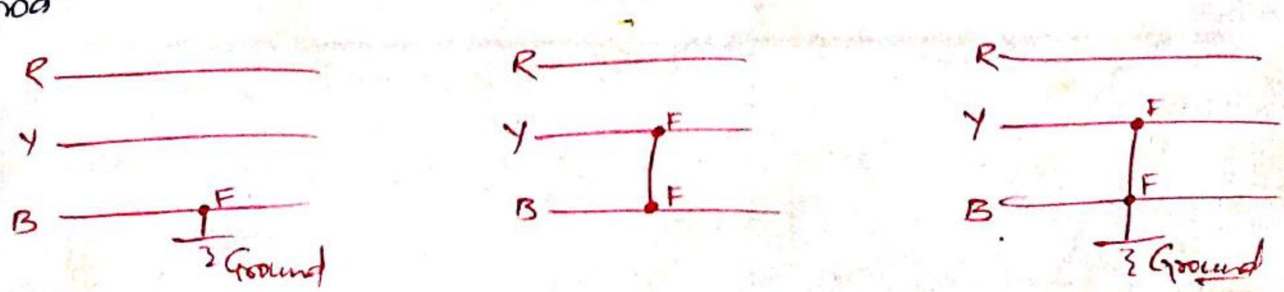
* Unsymmetrical faults :-

The fault on the power system which gives rise to un symmetrical fault currents (i.e, unequal fault Currents in the line with unequal phase displacement) are known as unsymmetrical fault. on the occurance of unsymmetrical fault, the currents in the three lines become unequal and so is the phase displacement.

There are 3 types of unsymmetrical faults may occur.

1. line to ground (L-G) fault
2. line to line (L-L) fault
3. Double line to ground (L-L-G) fault

The solution of unsymmetrical fault current problems can be obtained by either (a) Kirchoff's laws (or) (b) symmetrical components method.



Any three Coplanar Vectors of 3 phase unbalanced Voltages V_a, V_b & V_c (2)

Can be expressed as,

$$V_a = a_{11}V_1 + a_{12}V_2 + a_{13}V_3 \quad \text{--- (1)}$$
$$V_b = a_{21}V_1 + a_{22}V_2 + a_{23}V_3 \quad \text{--- (2)}$$
$$V_c = a_{31}V_1 + a_{32}V_2 + a_{33}V_3 \quad \text{--- (3)}$$

unbalanced phase voltages of a, b, c

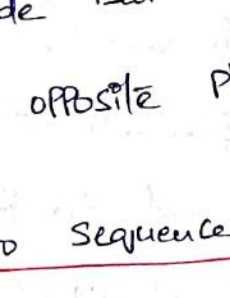
In 1918 Dr. C.L. Fortescue, an American Scientist, showed that any unbalanced system of 3-ϕ Currents (or) Voltages may be Regarded as being Composed of three seperate set of balanced vectors like; The three Symmetrical Components Vectors Replacing V_a, V_b and V_c are

1. Positive sequence Component which has 3 vectors of equal Magnitude but displaced in phase from each other by 120° and has the same phase sequence as the original vectors

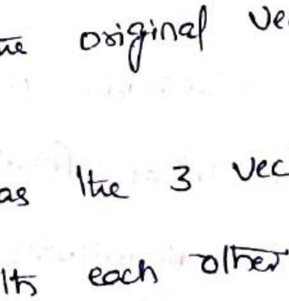
2. Negative sequence Component which has 3 vectors of equal Magnitude but displaced in phase from each other by 120° and has the opposite phase sequence as the original vectors.

3. Zero Sequence Component which has the 3 vectors of equal Magnitude and also are in phase with each other

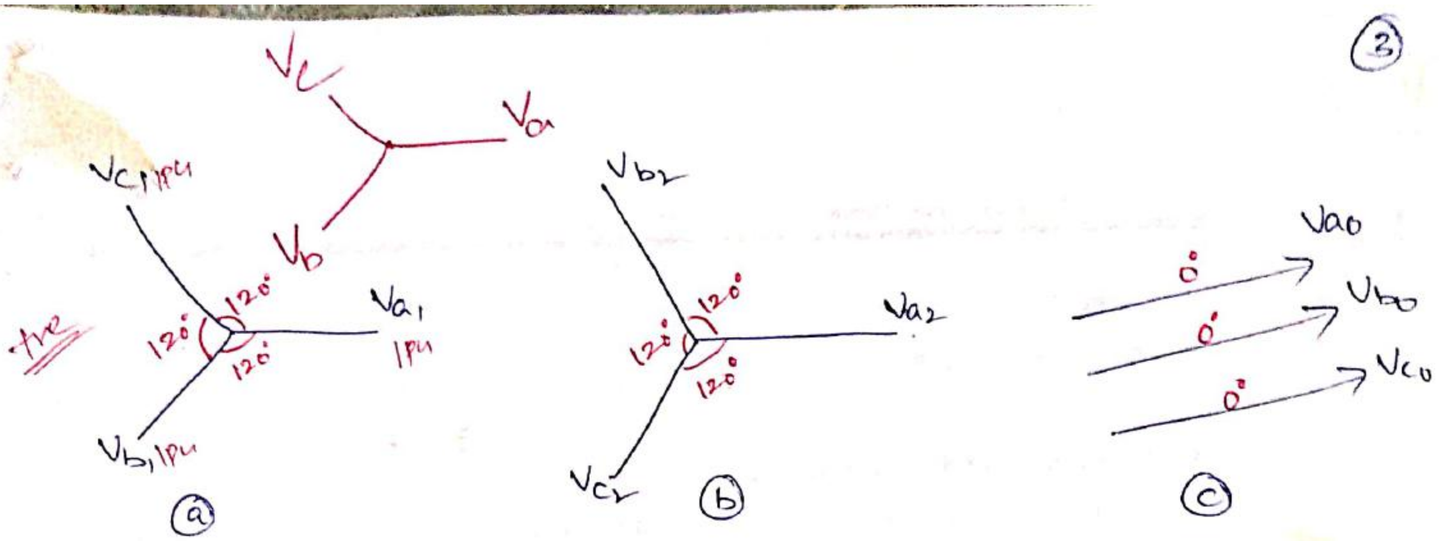
RYB Sequence



Vector Notation in clock-wise direction

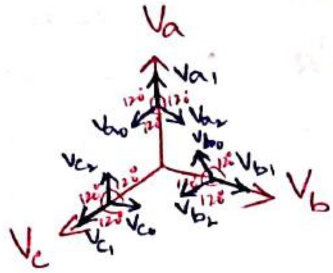


Phase Notation in Anti-clock wise direction



(a) positive (b) Negative (c) zero Sequence symmetrical Components.

From the above three phasors, the following Relations between the original unbalanced vectors and their corresponding symmetrical Components can be written as,



$$V_a = V_{a_1} + V_{a_2} + V_{a0} \quad (4)$$

$$V_b = V_{b_1} + V_{b_2} + V_{b0} \quad (5)$$

$$V_c = V_{c_1} + V_{c_2} + V_{c0} \quad (6)$$

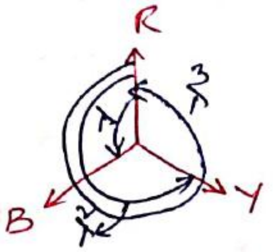
The following points to be kept in mind are,

1. The positive sequence currents $(\vec{I}_{R_1}, \vec{I}_{Y_1}, \vec{I}_{B_1})$. Negative $(\vec{I}_{R_2}, \vec{I}_{Y_2}, \vec{I}_{B_2})$. zero $(\vec{I}_{R_0}, \vec{I}_{Y_0}, \vec{I}_{B_0})$ seperately form balanced system of currents. Hence, they are called symmetrical components of the unbalanced system.

2. A balanced 3φ system consists of positive sequence components only, the negative and zero sequence components are being zero.

from the equations (4), (5), (6), assuming the phase "a" as the Reference. (A)

Here use is made of operator 'λ', which has a magnitude of unity and rotation through 120° i.e, when any vector is multiplied by λ, then vector magnitude remains same but is rotated anticlockwise through 120°,



$$\lambda = 1 \angle 120^\circ$$

$$= \cos(120) + j \sin 120^\circ$$

$$\therefore \boxed{\lambda = -0.5 + j0.866}$$

vectors → clockwise
phasors → Anti-clockwise.

Similarly, $\lambda^2 = 1 \angle 240^\circ$ (or) $1 \angle -120^\circ$

$$= \cos(240^\circ) + j\sin(240^\circ)$$

$$\therefore \boxed{\lambda^2 = -0.5 - j0.866}$$

$$\lambda^3 = 1 \angle 360^\circ \text{ (or) } 1 \angle 0^\circ$$

$$\therefore \boxed{\lambda^3 = 1}$$

Similarly, $\boxed{\lambda^4 = \lambda}$

$\therefore$ from the above illustration, we can write

$$\left.\begin{aligned} \lambda &= -0.5 + j0.866 \\ \lambda^2 &= -0.5 - j0.866 \\ \lambda^3 &= 1. \end{aligned}\right\} \text{—— (7)}$$

from the above equation, we can write,

$$\lambda^3 = 1 \Rightarrow \lambda^3 - 1 = 0$$

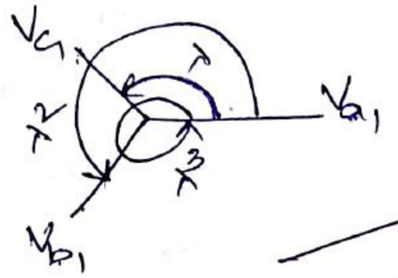
$$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda + 1) = 0 \qquad [\because \lambda \neq 1]$$

$$\Rightarrow \boxed{1 + \lambda + \lambda^2 = 0} \text{ —— (8)}$$

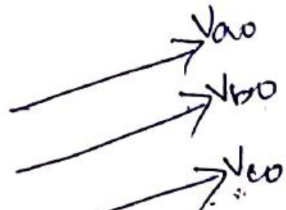
Now deriving relations between the symmetrical components of phase b and c in terms of symmetrical components of phase a.

from the above fig. $\boxed{V_{b_1} = \lambda^2 V_{a_1}}$

This means, in order to express V_{b_1} interms of V_{a_1}, V_{a_1} should be rotated anti-clockwise through $240°$.

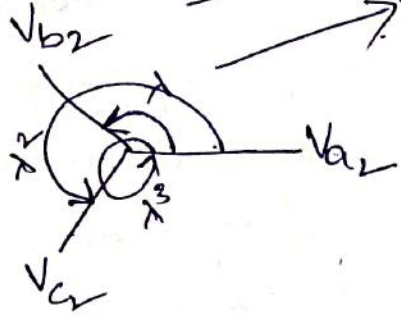


Similarly, $\boxed{V_{c_1} = \lambda V_{a_1}}$



for Negative sequence vectors,

$\boxed{V_{b_2} = \lambda V_{a_2}}$, $\boxed{V_{c_2} = \lambda^2 V_{a_2}}$



for zero sequence vectors,

$\boxed{V_{a0} = V_{b0} = V_{c0}.}$

Now, substituting above relations in equations (4), (5), (6) we can get,

$$V_a = V_{a_1} + V_{a_2} + V_{a_0} \quad —— (7)$$

'a' as Reference

$$V_b = \lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0} \quad —— (8)$$

$$V_c = \lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0}. \quad —— (9)$$

Comparing eqt. (7), (8), (9) to (1), (2), (3) we have,

$$a_{11} = a_{12} = a_{13} = 1$$

$$a_{21} = \lambda^2, \quad a_{22} = \lambda, \quad a_{23} = 1$$

$$a_{31} = \lambda, \quad a_{32} = \lambda^2, \quad a_{33} = 1$$

Similarly, the currents in the 3 phases a, b, c by taking 'a' as Reference

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$I_b = \lambda^2 I_{a1} + \lambda I_{a2} + I_{a0}$$

$$I_c = \lambda I_{a1} + \lambda^2 I_{a2} + I_{a0}$$

Normally, the unbalanced phase voltages and currents are known in system, and it is required to find out the symmetrical components.

Now, To find out the positive sequence component V_{a1}, multiply eq. (7), (8), (9) by $1, \lambda, \lambda^2$ respectively and Adding them,

$$\Rightarrow V_a + \lambda V_b + \lambda^2 V_c = V_{a1}(1+\lambda^3+\lambda^3) + V_{a2}(1+\lambda^2+\lambda^4) + V_{a0}(1+\lambda+\lambda^2)$$

$$= V_{a1}(3) + 0 + 0$$

$$= V_{a1} \times 3$$

$$\Rightarrow \boxed{V_{a1} = \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)} \quad \text{—— (10)}$$

For Negative sequence component V_{a2} Multiply (7), (8), (9) by $1, \lambda^2, \lambda$ respectively and adding them,

$$\Rightarrow V_a + \lambda^2 V_b + \lambda V_c = V_{a1}(1+\lambda^4+\lambda^2) + V_{a2}(1+\lambda^3+\lambda^3) + V_{a0}(1+\lambda+\lambda^2)$$

$$= 0 + V_{a2} \times 3 + 0$$

$$\Rightarrow \boxed{V_{a2} = \frac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)} \quad \text{—— (11)}$$

For zero sequence component add (7), (8), (9),

$$\Rightarrow V_a + V_b + V_c = V_{a1}(1+\lambda^2+\lambda) + V_{a2}(1+\lambda+\lambda^2) + 3V_{a0}$$

$$= 0 + 0 + 3V_{a0}$$

$$\Rightarrow \boxed{V_{a0} = \frac{1}{3}(V_a + V_b + V_c)} \quad \text{— (12)}$$

$$\therefore \boxed{\begin{aligned} V_{a1} &= \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c) \\ V_{a2} &= \frac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c) \\ V_{a0} &= \frac{1}{3}(V_a + V_b + V_c) \end{aligned}}$$

Similarly, the currents are also,

$$\boxed{\begin{aligned} I_{a1} &= \frac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c) \\ I_{a2} &= \frac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c) \\ I_{a0} &= \frac{1}{3}(I_a + I_b + I_c) \end{aligned}} \quad \text{— (13)}$$

In the equations above V_a, V_b and V_c may be the line to ground voltages, line to neutral voltages, line to line voltages.

Note:- In general, The zero sequence impedances are much larger than positive & Negative sequence impedances in a practical Transmission lines.

Problems:

1. The line to ground voltages on the high voltage side of step-up xmer are 100kV, 33kV and 38kV on phases a, b and c respectively. The voltage of phase a leads that of b by 100° and lags that of phase c by 176.5°. Determine symmetrical components of voltage.

Sol. Taking phase 'a' as Reference.

$$V_a = 100\angle 0^\circ$$

$$V_b = 33\angle -100^\circ$$

$$V_c = 38\angle 176.5^\circ$$

∴ positive component of voltage $V_{a_1} = \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$

$$\Rightarrow V_{a_1} = \frac{1}{3}\left[100\angle 0^\circ + 33\angle -100^\circ \times 1\angle 120^\circ + 38\angle 176.5^\circ \cdot 1\angle -120^\circ\right)$$

$$= \frac{1}{3}\left(100 + j0 + 33\angle 20^\circ + 38\angle 56.5^\circ\right)$$

$$V_{a_1} = \frac{1}{3}\left(151.97 + j42.97\right) = 50.65 + j14.32 \text{ V.}$$

Similarly, Negative sequence Component

$$V_{a_2} = \frac{1}{3}\left(V_a + \lambda^2 V_b + \lambda V_c\right)$$

$$= \frac{1}{3}\left(100 + j0 + 33\angle 220^\circ + 38\angle 296.5^\circ\right)$$

$$V_{a_2} = 30.55 - j4.26 \text{ V.}$$

Zero Sequence Component $V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$

$$\Rightarrow V_{a0} = \frac{1}{3}\left(100 + j0 + 33\angle -100^\circ + 38\angle 176.5^\circ\right)$$

$$V_{a0} = \frac{1}{3}\left(56.37 - j30.18\right) = 18.79 - j10.06 \text{ V.}$$

In a 3φ-4 wire system, the currents in R, Y and B lines under abnormal conditions of loading are as under,

$\vec{I_R} = 100 \angle 30^\circ$ A, $\vec{I_Y} = 50 \angle 300^\circ$ A $\vec{I_B} = 30 \angle 180^\circ$ A.

Calculate the positive, negative & zero sequence currents in the R-line and return current in Neutral wire.

Sol. let I_0, I_1, I_2 be the zero, positive and Negative sequence currents respectively of line currents in Red line.

$$I_0 = \frac{1}{3}(I_R + I_Y + I_B)$$

$$= \frac{1}{3}\left(100\angle 30^\circ + 50\angle 300^\circ + 30\angle 180^\circ\right)$$

$$= \frac{1}{3}\left[(86.60 + j50) + (25 - j43.3) + (-30 + j0)\right]$$

$$= \frac{1}{3}(81.6 + j6.7) = 27.2 + j2.23 \text{ A.}$$

$$I_1 = \frac{1}{3}(I_R + \lambda I_Y + \lambda^2 I_B)$$

$$= \frac{1}{3}\left[100\angle 30^\circ + (1\angle 120^\circ \times 50\angle 300^\circ) + (1\angle -120^\circ \times 30\angle 180^\circ)\right]$$

$$= \frac{1}{3}\left[(86.6 + j50) + (25 + j43.3) + (15 + j25.98)\right]$$

$$= 42.2 + j39.76 \text{ A.}$$

$$I_2 = \frac{1}{3}(I_R + \lambda^2 I_Y + \lambda I_B)$$

$$= \frac{1}{3}\left[100\angle 30^\circ + (1\angle -120^\circ \times 50\angle 300^\circ) + (1\angle 120^\circ \times 30\angle 180^\circ)\right]$$

$$= \frac{1}{3}(51.6 + j24.02) = 17.2 + j8 \text{ A}$$

Current in the Neutral wire $= I_R + I_Y + I_B$

$= (81.6 + j6.7)$

$= 81.87 \angle 4.7^\circ$ A.

NOTE :-

1. The vector sum of positive and Negative sequence Currents of an unbalanced 3ϕ system is zero. The Resultant solely consists of an three zero sequence currents i.e

Vector sum of all sequence currents in 3ϕ unbalanced system,

$$= \vec{I}_{R0} + \vec{I}_{Y0} + \vec{I}_{B0}$$

2. In a 3ϕ- 4 wire unbalanced system, the magnitude of zero sequence currents is one-third of the current in the neutral wire i.e,

$$\boxed{\text{Zero Sequence Current} = \frac{1}{3}(\text{Current in Neutral Wire})}$$

∴ A/c to Problem

Current in neutral wire $= 3$ (zero sequence current)

$= 3 \times (27.2 + j2.23)$

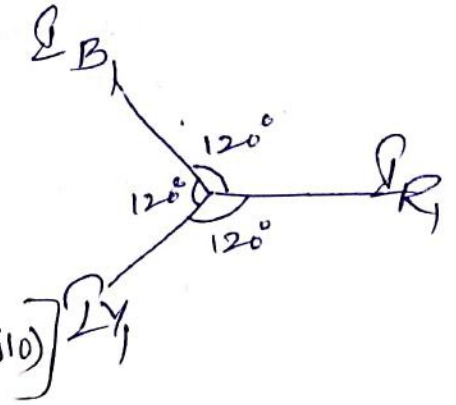
$= 81.6 + j6.69$

$= 81.87 \angle 4.67^\circ$ A

(2) The Currents in 3φ unbalanced System are:

$\vec{I_R} = (12+j6)\,A$, $\vec{I_y} = (12-j12)\,A$, $\vec{I_B} = (-15+j10)\,A$

The phase sequence in RYB, Calculate the positive, Negative, zero sequence components of Currents.



Sol: for R. phase :

$$\vec{I_{R_0}} = \frac{1}{3}(I_R + I_y + I_B)$$
$$= \frac{1}{3}\left[(12+j6) + (12-j12) + (-15+j10)\right]$$
$$= \frac{1}{3}(9+j4) = (3+j1.33)\ A$$

$$\vec{I_{R_1}} = \frac{1}{3}\left(\vec{I_R} + \lambda\vec{I_y} + \lambda^2\vec{I_B}\right)$$
$$= \frac{1}{3}\left[(12+j6) + \left[(-0.5+j0.866)(12-j12)\right] + (-0.5-j0.866)(-15+j10)\right]$$
$$= \frac{1}{3}(32.55+j30.39) = (10.85+j10.13)\ A$$

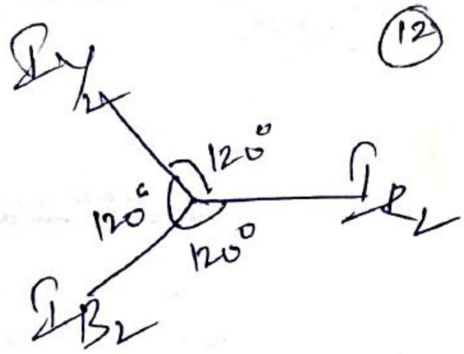
$$\vec{I_{R_2}} = \frac{1}{3}\left(\vec{I_R} + \lambda^2\vec{I_y} + \lambda\vec{I_B}\right)$$
$$= \frac{1}{3}\left[(12+j6) + (-0.5-j0.866)(12-j12) + (-0.5+j0.866)(-15+j10)\right]$$
$$= \frac{1}{3}\left[(-5.55-j6.41)\right] = (-1.85-j5.47)\ A$$

for Y-phase :

$$\vec{I_{y_0}} = \vec{I_{R_0}}$$
$$= (3+j1.33)\ A$$

$$\vec{I_{y_1}} = \lambda^2\vec{I_{R_1}} = (-0.5-j0.866)(10.85+j10.13)$$
$$= (3.35-j14.4)\ A$$

$$\vec{I}_{Y_2} = a\lambda\,\vec{I}_{R_2}$$
$$= (-0.5 + j0.866)\,(-1.85 - j5.47)$$
$$= (5.7 + j1.13)\ A$$



Similarly, for B-phase,

$$\vec{I}_{B_0} = \vec{I}_{R_0} = (3 + j1.33)\ A$$

$$\vec{I}_{B_1} = \lambda\,\vec{I}_{R_1} = (-0.5 + j0.866)\,(10.85 + j10.13)$$
$$= (-14.2 + j4.31)\ A$$

$$\vec{I}_{B_2} = \lambda^2\,\vec{I}_{R_2} = (-0.5 - j0.866)\,(-1.85 - j5.47)$$
$$= (-3.82 + j4.34)\ A$$

* Sequence Impedances :-

The Sequence impedances of an equipment (or) Component of a power System are the positive, Negative and zero Sequence impedances. They are defined as follows,

(a) positive Sequence impedance :-

The positive Sequence Impedance of an equipment is the impedance offered by the equipment to the flow of positive sequence currents.

Unsymmetrical fault Currents కనుక్కోవాలి అంటే Symmetrical Component Networks గురించి తెలుసుకోవడం తప్పనిసరి

(b) Negative Sequence Impedance :-

The Negative Sequence impedance of an equipment is the impedance offered by the equipment to the flow of Negative Sequence Currents.

(c) Zero Sequence Impedance :-

Similarly, the zero sequence impedance of an equipment is the impedance offered by the equipment to the flow of zero sequence currents.

For a 3φ - symmetrical static circuit without internal voltages like, transformers, transmission lines & Synchronous Machines, the impedances to the currents of any sequence are the same in the three phases. Also, the currents of a particular sequence will produce a drop of same sequence, (OR) A voltage of a particular sequence will produce current of the same sequence only.

Since, for a 3φ static symmetrical device, the positive and negative sequence impedances are equal, the zero sequence impedance which includes the impedance of the return path through the ground.

*** NOTE:- Since we are going to assume a balanced mode operation of Synchronous M/c, It induces emf's of only +ve seq. only. when ever a Machine is said to be balanced, it should induces a positive sequence emf's only but not negative & zero sequence Emf's ***

Voltage at the Neutral :-

For a 3ϕ circuit the potential of the neutral when it is grounded through some impedance (or) is isolated, will not be at ground potential under unbalanced conditions. The potential of the Neutral is given as, $V_n = -I_n Z_n$, where, Z_n is the Neutral grounding impedance and I_n is the Neutral Current.

Here, the Negative sign is used as the current flows from the ground to the neutral of the System

Now, for a 3ϕ System,

$$\boxed{I_n = I_a + I_b + I_c}$$

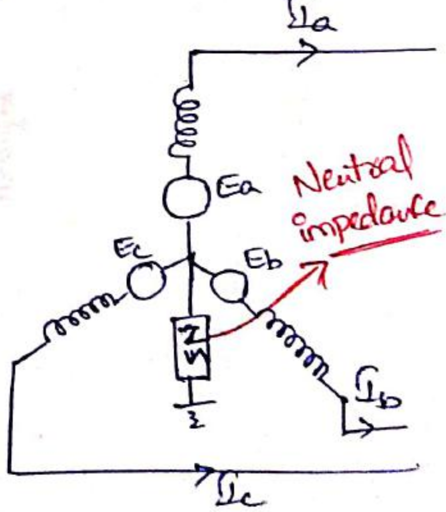
$$= (I_{a_1} + I_{a_2} + I_{a_0}) + (\lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0}) + (\lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0})$$

$$= I_{a_1}(1+\lambda+\lambda^2) + I_{a_2}(1+\lambda^2+\lambda) + 3I_{a_0}$$

$$= 3I_{a_0}$$

$$\therefore V_n = -I_n Z_n$$

$$= -I_n \cdot Z_n = -3I_{a_0} Z_n$$



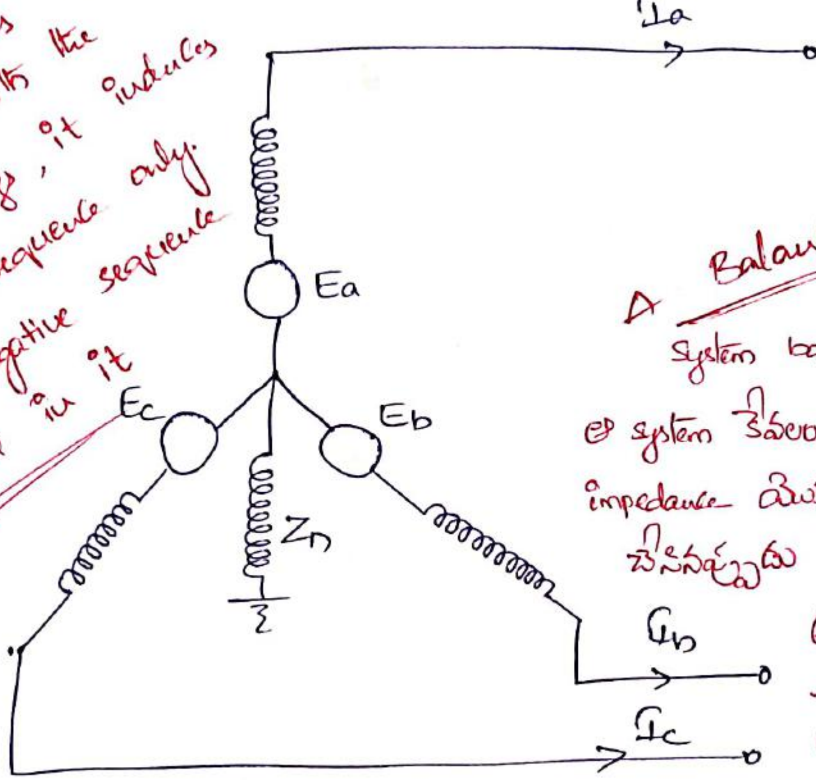
$$\therefore \boxed{V_n = -3I_{a_0} Z_n}$$

$\therefore$ From the above Equation we can say that the positive & negative sequence currents through the Neutral are absent. And the Voltage drops due to these Currents are also zero.

Sequence Network Equations :-

The Network equations will be derived from an unloaded (Synchronous M/c) alternator with neutral ~~Solidly~~ Grounded through some impedance. Assuming the system is balanced, i.e., the generated voltages are of equal magnitude and displaced by 120°.

Since, a Synchronous M/c is designed with the symmetrical windings, it induces emf's of positive sequence only. i.e., NO zero & negative sequence emf's are induced in it



A Balanced 3φ system

System balanced అని చెప్పాలి అంటే, ఆ system కేవలం positive sequence impedance వలన emf's ని induce చేసినప్పుడు మాత్రమే చెప్పగలం. అంటే balanced system లో -ve & zero sequence emf's ఉండవు

Since, the sequence impedances per phase are same for all the three phases and we are considering initially a balanced system the analysis will be done on single phase basis.

The positive sequence component of voltage at the point of fault is the positive sequence generated voltage minus drop due to positive sequence current in positive sequence impedance.

[as positive sequence current does not produce drop in both negative and zero sequence impedances]

$$\therefore \boxed{V_{a1} = E_a - I_{a1} Z_1}$$

Similarly, the negative sequence component of voltage at the fault point is the generated negative sequence voltage minus the drop due to negative sequence current in negative sequence impedance

$\therefore \quad \underline{V_{a2} = E_{a2} - I_{a2} Z_2}$

Since, the negative sequence voltage generated is zero

$\therefore \quad \boxed{V_{a2} = -I_{a2} Z_2}$

[$\because$ For a 3φ balanced system no negative & zero sequence ~~impedance~~ & EMF's will be produced.]

Similarly, for zero sequence voltage

$V_{a0} = E_{a0} - I_{a0} Z_0$

$\Rightarrow \quad V_{a0} = -I_{a0} Z_0$

But, actually, the zero sequence voltage is the difference between the neutral voltage and the drop of zero sequence current in zero sequence impedance which is connected to ~~ground~~ generator

$\therefore \quad V_{a0} = V_n - I_{a0} Z_{g0}$

$= -3 I_{a0} Z_n - I_{a0} Z_{g0}$

$\therefore \quad \boxed{V_{a0} = -I_{a0} (3Z_n + Z_{g0})} = \boxed{V_{a0} = -I_{a0} Z_0}$

where, Z_{g0} is the zero sequence impedance of the generator and

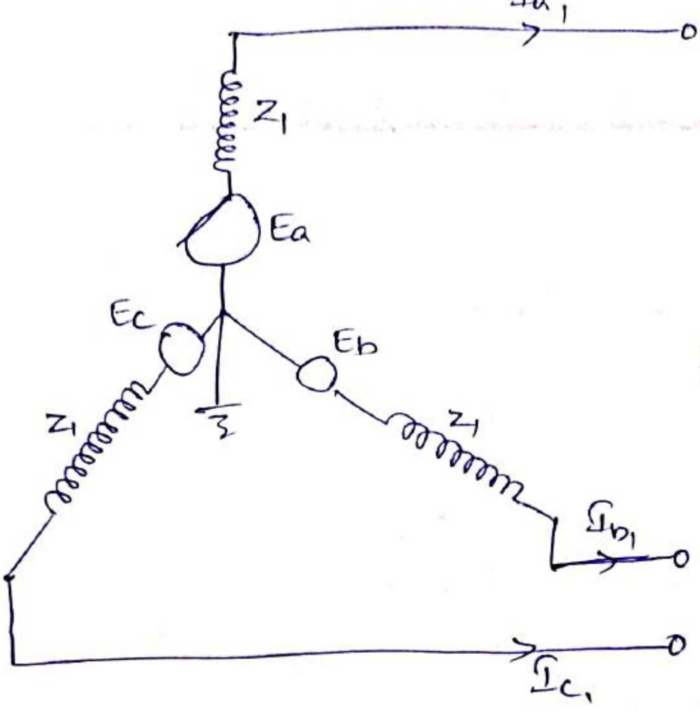
Z_n is the neutral impedance

$\therefore$ Now, 3 sequence equations are

$\Rightarrow \quad \underline{V_{a1} = E_a - I_{a1} Z_1}$

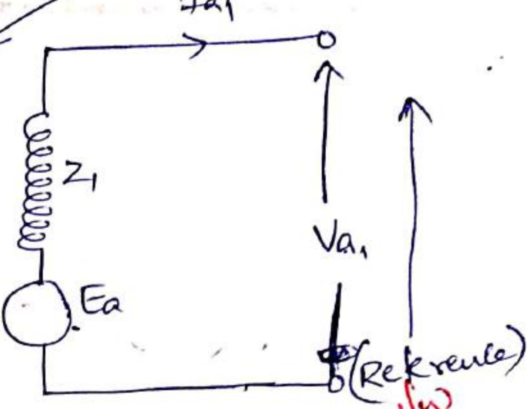
$\underline{V_{a2} = -I_{a2} Z_2}$ and $\underline{V_{a0} = -I_{a0} Z_0}$.

The corresponding sequence networks as shown,

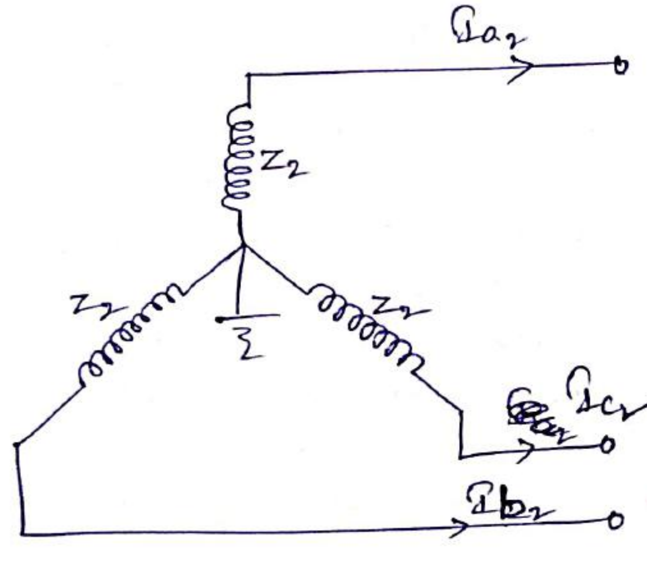


$\equiv$

$V_{a_1} = E_a - I_{a_1} Z_1$

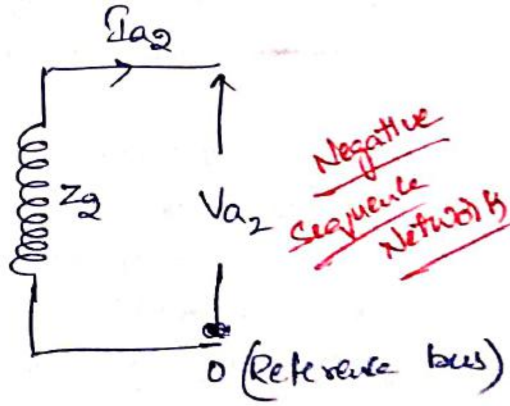


Positive sequence N/W

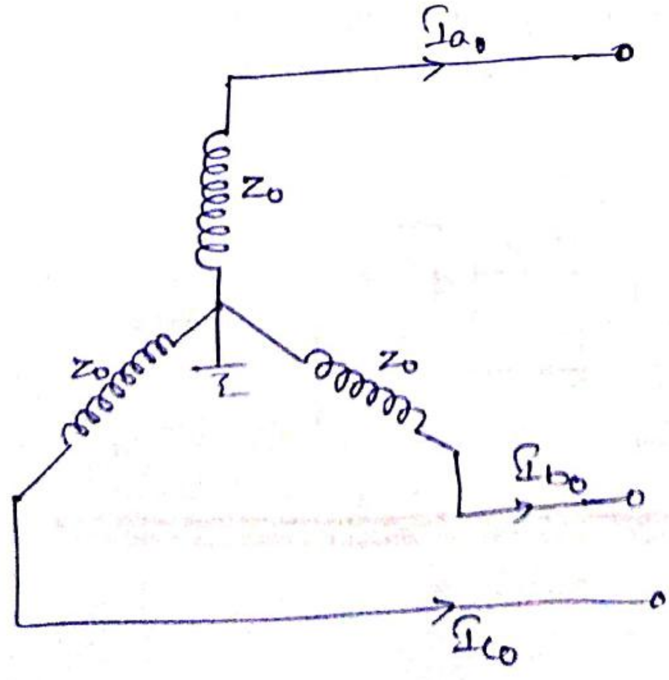


$\equiv$

$V_{a_2} = -I_{a_2} Z_2$

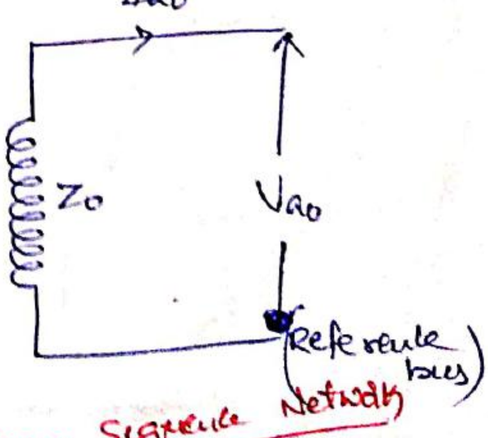


Negative sequence Network



$\equiv$

$V_{a_0} = -I_{a_0} Z_0$

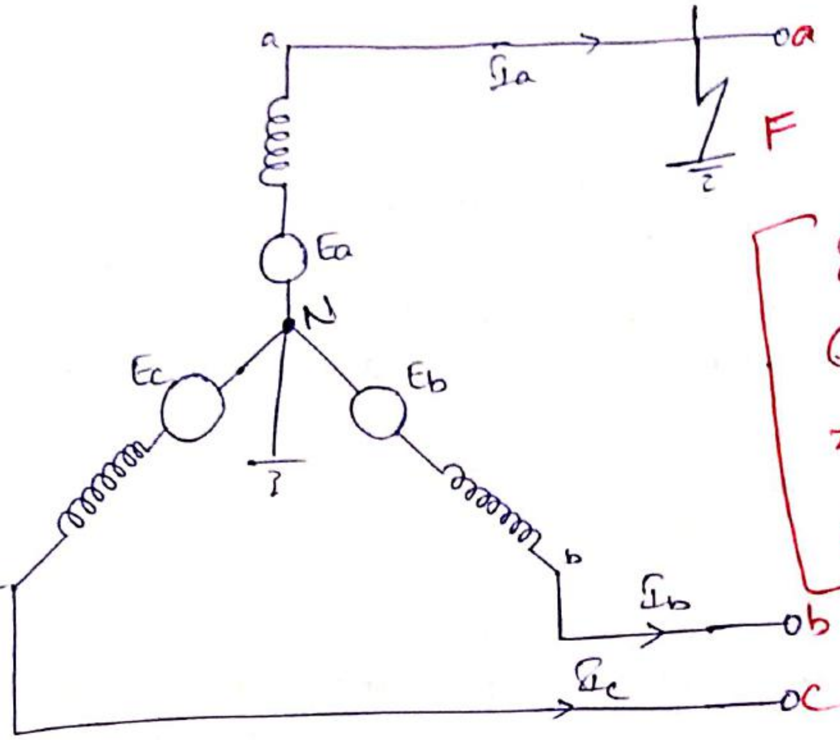


Zero sequence Network

Single-line to ground fault :-

The analysis for this fault can be done by following diagram.

let the fault takes place on phase "a".



[ఏ line లో fault occur అవుతుందో ఆ line voltage zero అవుతుంది. Other phases currents zero అవుతాయి]

The Boundary Conditions are

$$\left.\begin{aligned} V_a &= 0 \\ I_b &= 0 \\ I_c &= 0 \end{aligned}\right\} \longleftarrow (A)$$

NOTE:-

[fault earth కి connect అయింది కాబట్టి 'a' phase voltage zero అవుతుంది. Other phases లో currents zero అవుతాయి]

and the sequence Network equations are

$$V_{a0} = -I_{a0} Z_0$$

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = -I_{a2} Z_2$$

Now, the Symmetrical Component Currents are,

$$I_{a1} = \frac{1}{3}\left(I_a + \lambda I_b + \lambda^2 I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + \lambda^2 I_b + \lambda I_c\right)$$

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

Now, Substituting the values of I_b and I_c from eq. (A)

we get, $I_{a1} = \frac{1}{3}(I_a + 0 + 0) = I_a/3$

$I_{a2} = \frac{1}{3}(I_a + 0 + 0) = I_a/3$ $\Rightarrow \therefore \boxed{I_{a1} = I_{a2} = I_{a0}}$

$I_{a0} = \frac{1}{3}(I_a + 0 + 0) = I_a/3.$ $\Rightarrow I_f = I_a = 3 I_{a0}$

from the equation (A) we can say that,

$\therefore I_f = 3 I_{a0}$

$V_a = V_{a0} + V_{a1} + V_{a2} = 0$

Now, Substituting the network equations in above equation we get,

$\Rightarrow E_a - I_{a1} Z_1 - I_{a2} Z_2 - I_{a0} Z_0 = 0$ $[\because I_{a1} = I_{a2} = I_{a0}]$

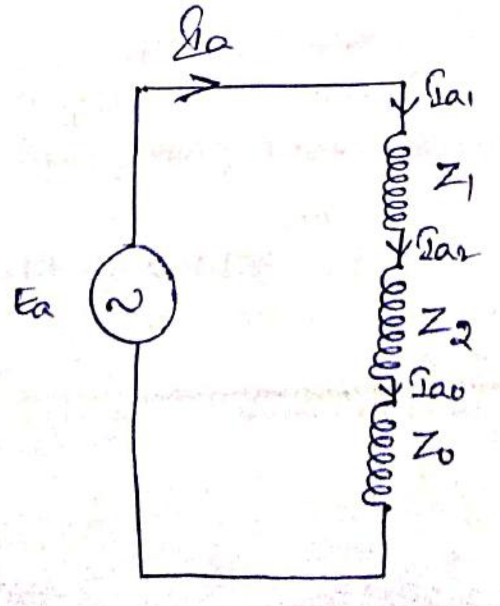
$\Rightarrow E_a - I_{a0} Z_1 - I_{a0} Z_2 - I_{a0} Z_0 = 0$

$$\Rightarrow \boxed{I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0}} \quad \text{(B)}$$

From the above equation it is clear that to calculate the L-G fault current all the three sequence ~~networks~~ impedances are required.

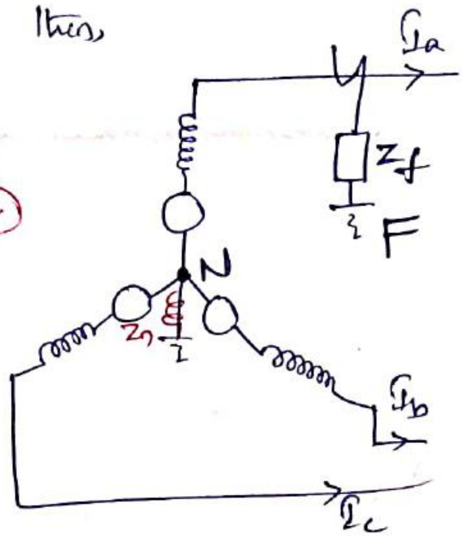
Since, the currents are all equal in magnitude and phase angle and are connected in series.

The interconnection of sequence network is shown below,



$I_{a1} = I_{a2} = I_{a0}$ in case of single line to ground fault

When a single line to ground fault occurs from the line to the ground through a fault impedance then, (20)



$$\boxed{I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0 + 3Z_f}} \quad \text{—— (C)}$$

where, Z_f = fault impedance

From the equation (B) we can say

$$I_{a_1} = I_{a_2} = I_{a_0} = \frac{I_a}{3}$$

$$\Rightarrow I_a = 3I_{a_1}$$

∴ The fault current at the phase "a" is

$$\boxed{I_a = \frac{3E_a}{Z_1 + Z_2 + Z_0}}$$

⟹ Fault current for L-G fault when fault occurs at phase 'a'

where, $I_b = I_c = 0$

Similarly, from (C)

$$\boxed{I_a = \frac{3E_a}{Z_1 + Z_2 + Z_0 + 3Z_f}}$$

Boundary conditions when fault impedance is there, from $I_{a_1} = \frac{(I_a + \lambda I_b + \lambda^2 I_c)}{3}$

$V_a = I_a Z_f$, $I_b = I_c = 0$

$I_{a_1} = I_{a_2} = I_{a_0} = \frac{I_a}{3}$

$\therefore V_{a_0} = -I_{a_0}(Z_{g_0} + 3Z_n)$

$V_{a_1} = E_a - I_{a_1}Z_1$

$V_{a_2} = -I_{a_2}Z_2$

$\Rightarrow V_a = V_{a_1} + V_{a_2} + V_{a_0} = I_a Z_f$

$\Rightarrow -I_{a_0}(Z_{g_0} + 3Z_n) + E_a - V_{a_1}Z_1 - I_{a_2}Z_2 = I_a Z_f$

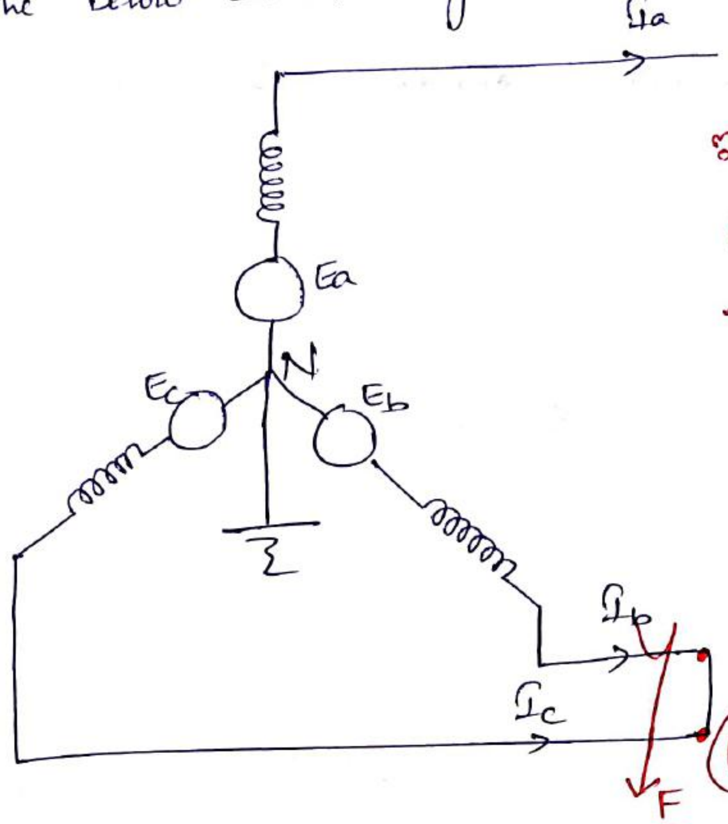
$\Rightarrow E_a - I_{a_1}Z_1 - I_{a_1}Z_2 - I_{a_1}(Z_0 + 3Z_n) = 3I_{a_1}Z_f$

$\Rightarrow I_{a_1} = \frac{E_a}{Z_1 + Z_2 + \underbrace{(Z_{g_0} + 3Z_n)}_{Z_0} + 3Z_f}$

for more information Refer C.L. Wadhwa Page (326)

Line-to-Line fault :-

From the below circuit diagram,



(line-to-line fault)

ఇక్కడ fault ground కి connect అవ్వలేదు కాబట్టి voltages at "b" and "c" will not equal to zero. Other phase current will be equal to zero.

ఇక్కడ two lines short అయ్యాయి, అంటే line-to-line fault occur అయ్యింది. ckt diagram లో చూపించిన current directions base చేసుకుని $I_b+I_c=0$ అని వచ్చింది.

The boundary conditions are

$$\left.\begin{aligned} I_a &= 0 \\ I_b + I_c &= 0 \\ V_b &= V_c \end{aligned}\right\} \text{— (D)}$$

$\because$ fault not grounded, if it is grounded then we can write $V_b = V_c = 0$

Again,

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + \lambda I_b + \lambda^2 I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + I_b \lambda^2 + \lambda I_c\right)$$

on substituting I_a, I_b and I_c from equations (D) in above equations we get,

$$\Rightarrow I_{a_1} = \frac{1}{3}\left(0 + \lambda I_b - \lambda^2 I_b\right) \qquad \left[\because \begin{array}{l} I_b + I_c = 0 \\ I_c = -I_b \end{array}\right]$$ (22)

$$I_{a_2} = \frac{1}{3}\left(0 + \lambda^2 I_b - \lambda I_b\right)$$

$$\therefore I_{a_0} = \frac{1}{3}(0+0) = 0 \qquad \left[\because I_b + I_c = 0 \text{ \& } I_a = 0\right]$$

which means that the zero sequence current will not present in the line-to-line fault.

$$\boxed{\begin{array}{l} V_{a_0} = -I_{a_0} Z_0 \\ V_{a_0} = 0 \end{array}}$$

$$\therefore I_{a_1} = \frac{I_b}{3}(\lambda - \lambda^2)$$

$$I_{a_2} = -\frac{I_b}{3}(\lambda - \lambda^2) \quad \text{(or)} \quad I_{a_2} = \frac{I_b}{3}(\lambda^2 - \lambda)$$

From the above two equations we can say that, the positive component current is equal in magnitude with negative component and opposite in phase with negative sequence component

$$\therefore I_{a_1} = -I_{a_2}$$

That means, the zero sequence component current or impedance is not required in L-L fault.

[Boundary Conditions ని Base చేసుకొని equations రాసుకుంటున్నాం. $V_b = V_c$ కనుక ఫ్రంని కాబట్టి I have considered only V_b & V_c equations]

Now, from equations,

$$V_b = V_{a_0} + \lambda^2 V_{a_1} + \lambda V_{a_2}$$

$$V_c = V_{a_0} + \lambda V_{a_1} + \lambda^2 V_{a_2}$$

Substituting above two equations in eqn (D) we get,

$$(D) \Rightarrow V_b = V_c \Rightarrow \cancel{V_{a_0}} + V_{a_1}\lambda^2 + V_{a_2}\lambda = \cancel{V_{a_0}} + \lambda V_{a_1} + \lambda^2 V_{a_2}$$

$$\Rightarrow V_{a_1}\cancel{(\lambda^2 - \lambda)} = V_{a_2}\cancel{(\lambda - \lambda^2)}$$

$$\Rightarrow \boxed{V_{a_1} = V_{a_2}.}$$

Substituting the sequence network equations in above equation, we get,

$$V_{a1} = V_{a2}$$

$$\Rightarrow E_a - I_{a1} Z_1 = -I_{a2} Z_2 \qquad [\because I_{a1} = -I_{a2}]$$

$$= I_{a1} Z_2$$

$$\Rightarrow E_a - I_{a1} Z_1 = I_{a1} Z_2 \quad \Rightarrow E_a = I_{a1} \cdot (Z_1 + Z_2)$$

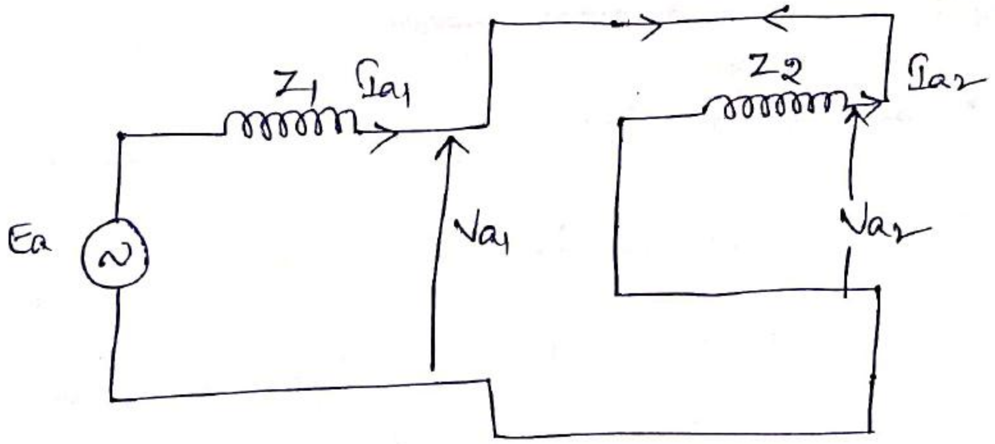
$$\Rightarrow \boxed{I_{a1} = \frac{E_a}{Z_1 + Z_2}}$$

And we have to note that in L-L. fault $I_{a0} = 0$ so, $V_{a0} = 0$

The interconnection of sequence network.

$I_{a1} = -I_{a2}$

$$I_f = +I_b \text{ (or) } -I_c$$
$$= I_{a0}^{0} + \lambda^2 I_{a1} + \lambda I_{a2}$$
$$= \lambda^2 I_{a1} - \lambda I_{a1}$$
$$= I_{a1}(\lambda^2 - \lambda)$$
$$I_f = -\sqrt{3} I_{a1}$$



NOTE: In L-L. fault, $V_{a1} = V_{a2}$,

$I_{a1} = -I_{a2}$ and $I_{a0} = 0$ so

$V_{a0} = 0$

* Line-Line fault through fault impedance :-

Boundary conditions are,

$I_a = 0$

$I_b + I_c = 0$, $V_b = V_c + I_b Z_f$.

~~$= I_a Z_f$~~

Sequence N/w equations are,

$V_{a_1} = E_{a_1} - I_{a_1} Z_1$

$V_{a_2} = - I_{a_2} Z_2$

$V_{a_0} = -I_{a_0}(Z_{g0} + 3Z_n)$

As we know that, $I_{a_1} = -I_{a_2}$ and $I_{a_0} = 0$.

from the boundary condition,

$V_b = V_c + I_b Z_f$

$\Rightarrow \cancel{V_{a_0}} + \lambda^2 V_{a_1} + \lambda V_{a_2} = \cancel{V_{a_0}} + \lambda V_{a_1} + \lambda^2 V_{a_2} + (\lambda^2 I_{a_1} + \lambda I_{a_2}) Z_f$

$\Rightarrow V_{a_1}(\lambda^2 - \lambda) = V_{a_2}(\lambda^2 - \lambda) + (\lambda^2 I_{a_1} - \lambda I_{a_1}) Z_f$ $(\because I_{a_1} = -I_{a_2})$

$\Rightarrow V_{a_1}\cancel{(\lambda^2 - \lambda)} = V_{a_2}\cancel{(\lambda^2 - \lambda)} + \cancel{(\lambda^2 - \lambda)} I_{a_1} Z_f$

$\Rightarrow V_{a_1} = V_{a_2} + I_{a_1} Z_f$

Substituting N/w equations,

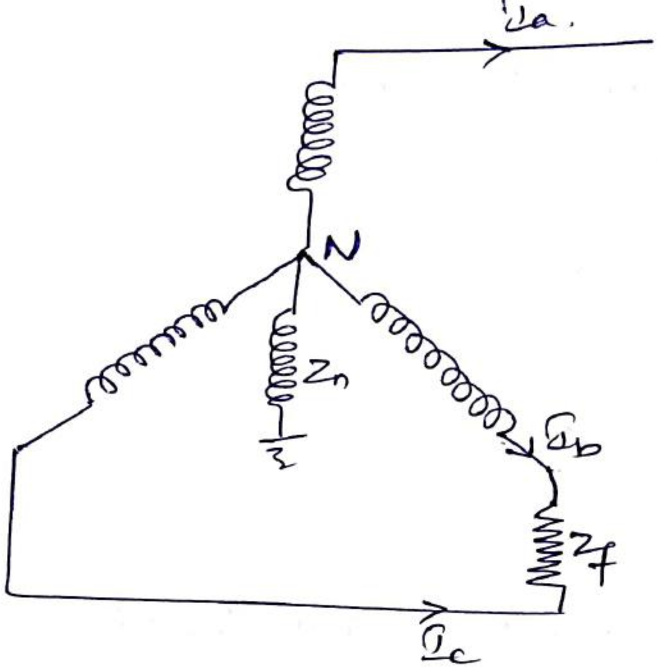
$\boxed{I_{a_1} = -I_{a_2}}$

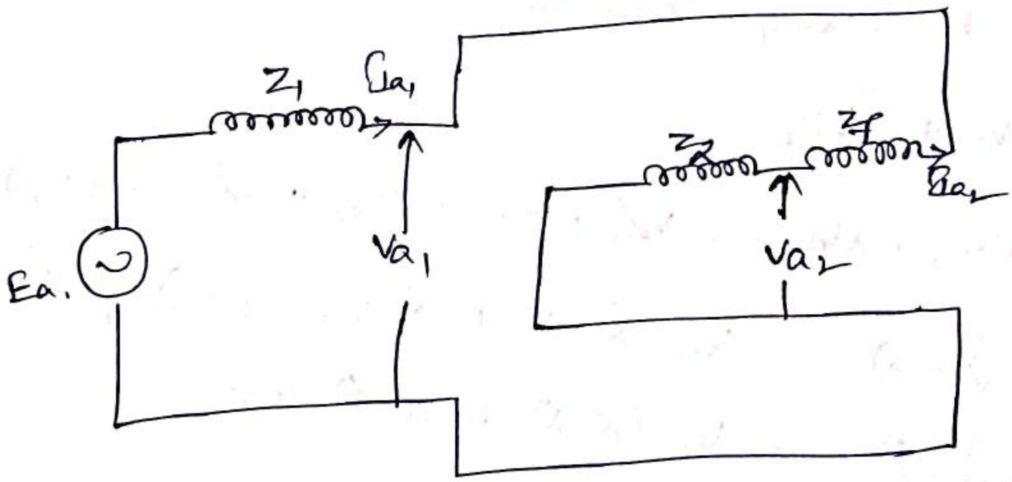
$E_a - I_{a_1} Z_1 = - I_{a_2} Z_2 + I_{a_1} Z_f$

$\Rightarrow E_a - I_{a_1} Z_1 = I_{a_1} Z_2 + I_{a_1} Z_f$

$\Rightarrow I_{a_1}(Z_1 + Z_2 + Z_f) = E_a$.

$$\Rightarrow I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_f}$$





fault Current $I_f = I_b$ (or) $-I_c$ $\quad [\because I_{a0} = 0]$

$\Rightarrow I_f = \lambda^2 I_{a_1} + \lambda I_{a_2}$ $\quad (\because I_{a_1} = -I_{a_2})$

$= \lambda^2 I_{a_1} - \lambda I_{a_1}$

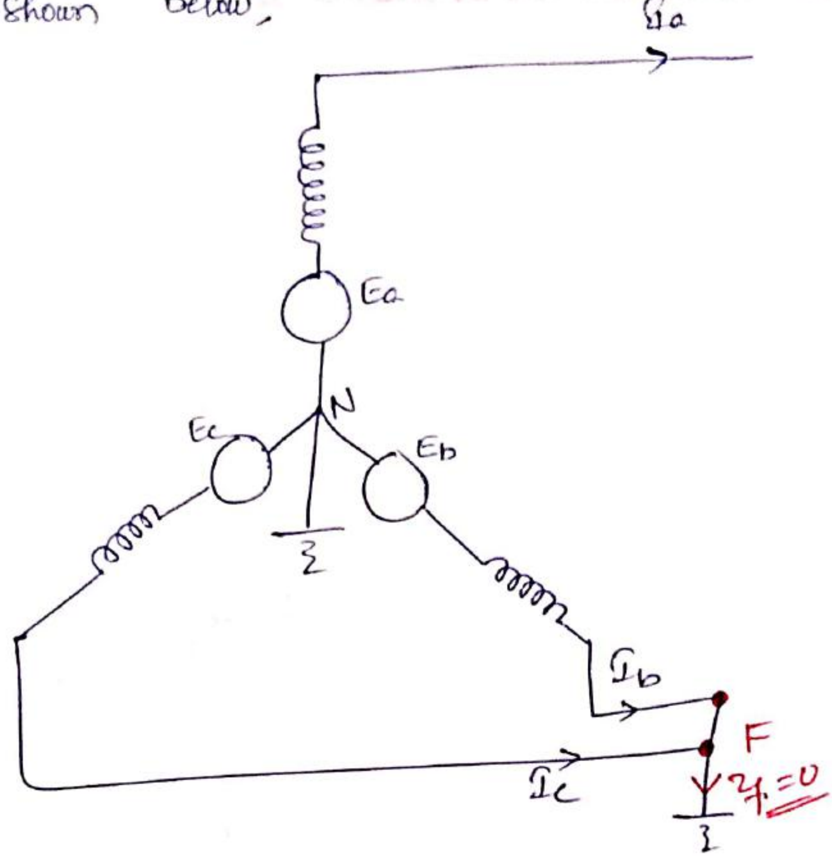
$= I_{a_1}(\lambda^2 - \lambda)$

$\therefore I_f = -\sqrt{3} I_{a_1}$ (or) $\sqrt{3} I_{a_1}$

$$\left[\begin{array}{l} \lambda^2 = -0.5 - j0.866 \\ \lambda = -0.5 + j0.866 \\ \therefore \lambda^2 - \lambda = (-\cancel{0.5} - j0.866) - (\cancel{-0.5} + j0.866) \\ = -j0.1732 = -j\sqrt{3}. \end{array}\right]$$

Double line to ground :-

Here, the fault occurs between two lines and to the ~~fault~~ ground. which is shown below,



$V_b = V_c = (I_b + I_c) Z_f$

$V_b = V_c = 0$

The boundary conditions are

$$\left.\begin{aligned} I_a &= 0 \\ V_b &= V_c = 0. \end{aligned}\right\} \rightarrow (E)$$

As we know that,

$$V_{a0} = \frac{1}{3}(V_{a} + V_b + V_c)$$

$$V_{a1} = \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$

$$V_{a2} = \frac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$

By substituting boundary conditions (E) in above equations we get,

$$V_{a0} = \frac{V_a}{3}$$

$$V_{a1} = \frac{V_a}{3} \quad \text{and} \quad V_{a2} = \frac{V_a}{3}.$$

$[\because V_b = V_c = 0]$

i.e, $\boxed{V_{a1} = V_{a2} = V_{a0}}$

Using the above Relation,

$$V_{a0} = V_{a1}$$

$$\Rightarrow -I_{a0} Z_0 = E_a - I_{a1} Z_1$$

$$\Rightarrow \boxed{I_{a0} = \frac{-(E_a - I_{a1} Z_1)}{Z_0}}$$

Similarly, $V_{a2} = V_{a1}$

$$\Rightarrow -I_{a2} Z_2 = E_a - I_{a1} Z_1$$

$$\therefore \boxed{I_{a2} = \frac{-(E_a - I_{a1} Z_1)}{Z_2}}$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(0 + I_f)$$

$$\boxed{\therefore I_f = 3 I_{a0}}$$

As we know that, $I_a = 0$

$$I_a = I_{a1} + I_{a2} + I_{a0} = 0$$

Substituting I_{a0} and I_{a2} in above equation.

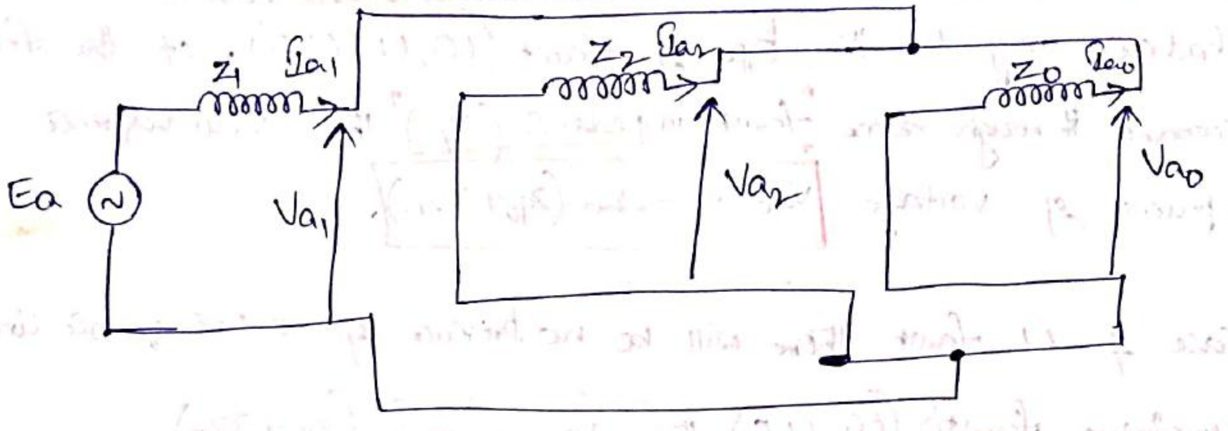
$$0 = I_{a1} - \left(\frac{E_a - I_{a1} Z_1}{Z_0}\right) - \left(\frac{E_a - I_{a1} Z_1}{Z_2}\right)$$

on Symplifying,

$$\Rightarrow \boxed{I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}}$$

From the above Main Equation we can say that to Calculate the L-L-G fault Current all the three Sequence networks are required. The zero & negative Sequence networks are Connected in parallel and in Series with the positive Sequence impedance

The Network is shown below,



Interconnection of Sequence Networks for L-L-G fault

For L-G fault :-

After finding the positive sequence component current I_{a1}

The fault current at phase 'a' is, $\boxed{I_a = 3 I_{a1}}$

For L-L fault :-

Here, also, after finding the value of positive sequence current I_{a1}, calculate the fault current

Actually, L.L fault means, the short circuit condition between the two lines i.e, $I_b + I_c = 0 \Rightarrow I_b = -I_c$

Therefore, to calculate the fault current at phase 'b' and phase 'c' use the basic equations,

$$I_b = \lambda^2 I_{a1} + \lambda I_{a2}$$

$$I_c = \lambda I_{a1} + \lambda^2 I_{a2}$$

then $I_b = -I_c$

[$\because$ In L.L fault the zero sequence current is zero, i.e, $I_{a0} = 0$]

Double line to Ground fault with Z_f :-

Fault impedance Z_f and Neutral impedance are considered.

Boundary Equations are $I_a = 0$

$$\left.\begin{array}{l} I_a = 0 \\ V_b = V_c = (I_b + I_c) Z_f \end{array}\right\} \text{——} (1)$$

Now, the sequence Equations are,

$$V_{a_1} = E_a - I_{a_1} Z_1$$

$$V_{a_2} = - I_{a_2} Z_2$$

$$V_{a_0} = - I_{a_0} (Z_0 + 3Z_n).$$

As we know that, ~~from~~ ~~at~~ ~~fault~~, $I_b + I_c = 3 I_{a_0}$.

from eq. (1) $V_b = V_c = (I_b + I_c) Z_f$ —— (A)

$\Rightarrow$ taking $V_b = V_c$

$$\Rightarrow \lambda^2 V_{a_1} + \lambda V_{a_2} + \cancel{V_{a_0}} = \cancel{V_{a_0}} + \lambda V_{a_1} + \lambda^2 V_{a_2}$$

$$\Rightarrow V_{a_1} = V_{a_2}. \text{——} (2)$$

Now, consider, $V_b = (I_b + I_c) Z_f$ $\quad (\because I_b + I_c = 3 I_{a_0})$

$$\Rightarrow V_b = 3 I_{a_0} Z_f$$

$$\Rightarrow \lambda^2 V_{a_1} + \lambda V_{a_1} + V_{a_0} = 3 I_{a_0} Z_f$$

$$\Rightarrow -V_{a_1} + V_{a_0} = 3 I_{a_0} Z_f$$

$$\Rightarrow \boxed{V_{a_1} = V_{a_0} - 3 I_{a_0} Z_f}$$

$$\left[\begin{array}{l} (\because V_{a_1} = V_{a_2}) \\ V_{a_1}(\lambda^2 + \lambda) = (-0.5 - j0.866) + (-0.5 + j0.866) \\ = -1 (V_{a_1}) \end{array}\right]$$

~~Similarly~~, Substituting the N/w Equations in above Eq.

$$\Rightarrow E_a - I_{a1} Z_1 = -I_{a0}(Z_0 + 3Z_n) - 3 I_{a0} Z_f$$

$$\Rightarrow \boxed{I_{a0} = -\left[\frac{E_a - I_{a1} Z_1}{Z_0 + 3Z_n + 3Z_f}\right]} \quad \text{——} \ (3)$$

$$-I_{a0}(Z_0 + 3Z_n + 3Z_f) = (E_a - I_{a1} Z_1)$$

$$I_{a0} = \frac{-(E_a - I_{a1} Z_1)}{Z_0 + 3Z_f + 3Z_n}$$

Similarly, using the relation, $V_{a1} = V_{a2}$

$$\Rightarrow E_a - I_{a1} Z_1 = -I_{a2} Z_2$$

$$\Rightarrow \boxed{I_{a2} = -\left[\frac{E_a - I_{a1} Z_1}{Z_2}\right]} \quad \text{——} \ (4)$$

Now, substituting the (2) & (3) in boundary condition $I_a = 0$,

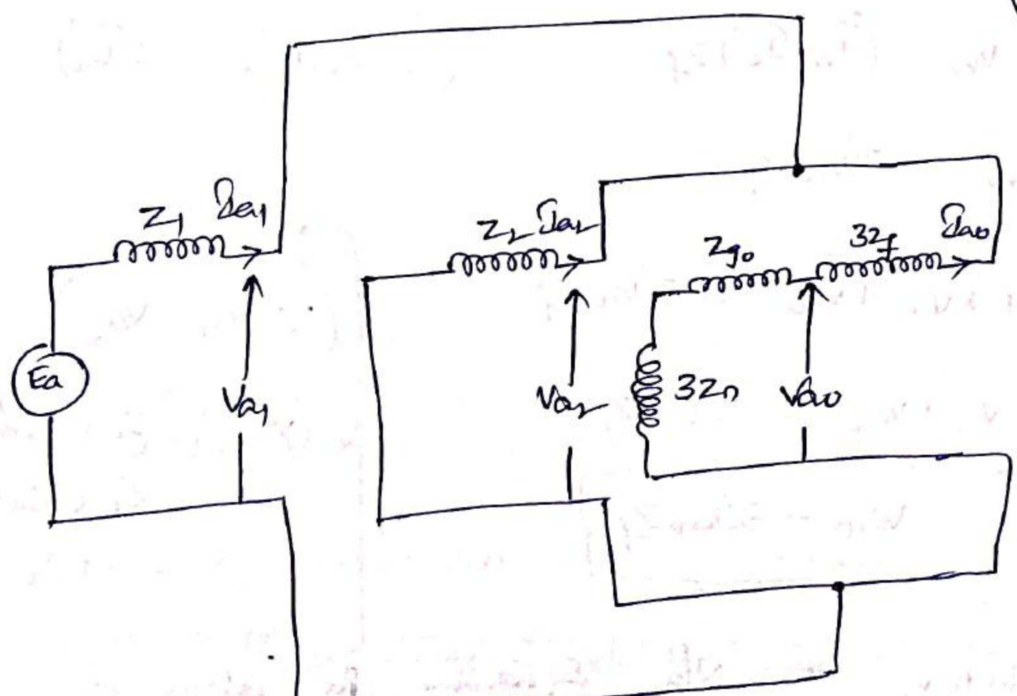
$$\Rightarrow I_a = I_{a1} + I_{a2} + I_{a0} = 0$$

$$\Rightarrow I_{a1} + \left(-\frac{E_a + I_{a1} Z_1}{Z_2}\right) + \left(\frac{-E_a + I_{a1} Z_1}{Z_0 + 3Z_n + 3Z_f}\right) = 0$$

$$\Rightarrow \boxed{I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_n + 3Z_f)}{Z_2 + Z_0 + 3Z_n + 3Z_f}}}$$

$$I_{a0} = \frac{1}{3}(I_{a0} + I_{b0} + I_{c0})$$
$$= \frac{1}{3}(I_a + I_f)$$
$$\Rightarrow I_{a0} = \frac{1}{3}(0 + I_f)$$

$\therefore$ fault current, $I_f = 3 I_{a0}$



* For L-G. fault:-

The fault current $I_a = 3 I_{a1}$

$$\therefore \boxed{I_a = 3 \times \frac{E_a}{Z_1+Z_2+Z_0}}$$

* For L-L. fault:-

For L.L. fault

$$\left[\because I_{a1} = \frac{E_a}{Z_1+Z_2}\right]$$

$$I_1 + I_2 = 0$$

$$\Rightarrow I_1 = -I_2$$

fault current $I_b + I_c = 0$

$$\Rightarrow I_b = -I_c$$

$$= I_0 + \lambda^2 I_1 + \lambda I_2 \qquad [\because I_0 = 0]$$

$$= \lambda^2\left(\frac{E_a}{Z_1+Z_2}\right) + \lambda\left(\frac{-E_a}{Z_1+Z_2}\right)$$

$$= \left(\frac{E_a}{Z_1+Z_2}\right)(\lambda^2 - \lambda)$$

$$\left[\begin{aligned}\lambda^2 - \lambda &= -0.5 - j0.866 + 0.5 - j0.866 \\ &= -j1.732 = -j\sqrt{3}\end{aligned}\right]$$

$$\boxed{\text{fault current} = \frac{\sqrt{3}E_a}{Z_1+Z_2} = -I_c}$$

* For L.L.G fault:-

fault current $I_f = I_Y + I_B$ (or) $I_b + I_c$

$$\left[\begin{aligned}\because I_{a0} &= \frac{1}{3}(I_a + \underbrace{I_b + I_c}) \\ &= \frac{1}{3}(0 + \text{fault current}) \\ \Rightarrow \text{fault current} &= 3 I_{a0}\end{aligned}\right]$$

$$\Rightarrow I_f = 3 I_{a0}$$

$$\boxed{I_f = 3 I_{a0}}$$

$$I_{a0} = \frac{-(E_a - I_{a1}Z_1)}{Z_0}$$

$$\therefore I_{a1} = E_a \Big/ Z_1 + \left(\frac{Z_2 Z_0}{Z_2+Z_0}\right)$$

Problem :-

1. A 25MVA, 13.2 KV Alternator with solidly grounded Neutral has a Sub-transient Reactance of 0.25 pu. The Negative and zero sequence Reactances are 0.35 and 0.1 pu respectively. A single line to ground fault occurs at the terminals of an unloaded alternator. Determine the fault current and line-to-line voltages.

Sol. Normally, the positive sequence impedance depends up on the working of Machine i.e, whether it is working under Sub-transient, transient (or) steady state condition.

According to our problem the alternator is working under sub-transient condition. So, At this condition the positive sequence impedance is $Z_1 = j0.25$ (Resistance has been Neglected in our problem).

$\therefore$ $Z_2 = j0.35$, $Z_0 = j0.1$

The fault is of line to ground type.

$\therefore$ The fault current $I_a = 3 I_{a1}$

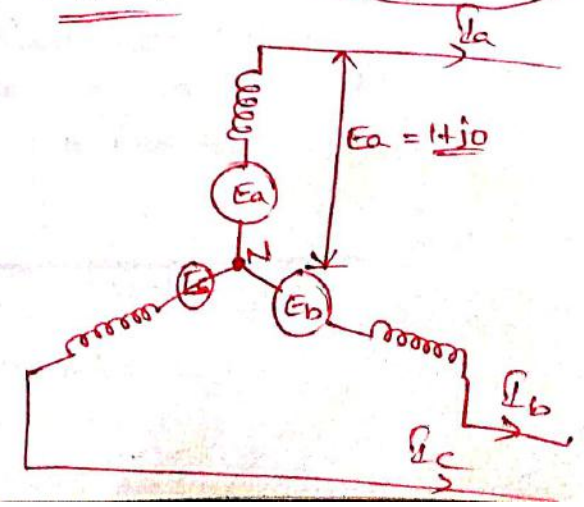
for L-G fault
$I_{a0} = I_{a1} = I_{a2}$ $\therefore$ $I_f = 3I_{a0}$
$I_f = 3I_{a1}$
$I_f = 3I_{a2}$

$$\Rightarrow \boxed{I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0}}$$

Assuming line "a" to neutral voltage be $1 + j0$ pu.

$$\therefore \quad I_{a1} = \frac{1+j0}{j(0.35+0.25+0.1)}$$

$$= -j1.428$$



For a L.G fault

$I_{a_1} = I_{a_2} = I_{a_0} = -j1.428$

Now, the fault current $= 3 I_{a_1}$

$= 3 \times -j1.428$

$= -j4.285$ pu.

fault current in pu. To convert the I_f to amperes, we need to find out base current

Base current (or) full load current $= \dfrac{\text{Base MVA}}{\sqrt{3} \times \text{line kV at point of fault}}$

$= \dfrac{25 \times 10^{\cancel{6}\,3}}{\sqrt{3} \times 13.2 \times \cancel{10^3}} = \dfrac{25 \times 1000}{\sqrt{3} \times 13.2}$

$= 1093.466$ A

Now, the fault current in Amperes $= 4.285 \times 1093.466$

$= 4685.5$ Amperes.

($\because$ Assumed Base values are at Alternator)

To find out the voltages, we have to find out the sequence components of voltages.

$V_{a_1} = E_a - I_{a_1} Z_1$

$= 1 - [(-j1.428)(j0.25)]$

$= 1 - 0.357 = 0.643$ V

$V_{a_2} = -I_{a_2} Z_2$

$= -(-j1.428)(j0.35)$

$= -0.4998$ V.

Similarly, $V_{a_0} = -I_{a_0} Z_0$

$= -(-j1.428)(j0.1) = -0.1428$ V.

from the Boundary Condition, $V_a = 0$

$\Rightarrow$ check whether it satisfies or not

$\Rightarrow$ $V_{a_1} + V_{a_2} + V_{a_0} = 0$

$\Rightarrow$ $0.643 - 0.4998 - 0.1428 \cong 0.$

Now, $V_b = V_{b_0} + V_{b_1} + V_{b_2}$

$V_{b_1} = \lambda^2 V_{a_1} = (-0.5 - j0.866)(0.643)$

$= -0.3215 - j0.5568.$

$V_{b_2} = \lambda V_{a_2} = (0.5 + j0.866)\cancel{(0.643)}\cancel{(0.50)}(-0.4998)$

$= 0.25 - j0.433$

$V_{a_0} = V_{b_0} = V_{c_0} = -0.1428$

$V_{c_1} = \lambda V_{a_1} = (-0.5 + j0.866)\ 0.643$

$= -0.3215 + j0.5568$

$V_{c_2} = \lambda^2 V_{a_2} = (-0.5 - j0.866)(-0.4998)$

$= 0.25 + j0.433$

$\therefore V_b = -0.3215 + j0.5568 + j0.25 + j0.433 - 0.1428$

$= -0.2143 - j0.9898$

$V_c = -0.3215 + j0.5568 + 0.25 + j0.433 - 0.1428$

$= -0.2143 + j0.9898$

Now, the line-to-line voltages are,

$$V_{ab} = V_a - V_b$$
$$= 0 - V_b = -(-0.2143 - j0.9898)$$
$$= 0.2143 + j0.9898$$

[$\because$ from L-G boundary condition $V_a = 0$]

$$V_{bc} = V_b - V_c$$
$$= -\cancel{0.2143} - j0.988 + \cancel{0.2143} + 0.9898$$
$$= -j1.9796$$

$$V_{ac} = V_a - V_c$$
$$= -V_c = -(-0.2143 + j0.9898)$$
$$= 0.2143 - j0.9898$$

The obtained voltages are in pu. To get the actual line to line voltages,

$$V_{ab} = 0.2143 + j0.9898 = \sqrt{(0.2143)^2 + (0.9898)^2} = 1.0127$$

$$V_{bc} = 0 - j1.9796 = \sqrt{(1.9796)^2} = 1.9796$$

$$V_{ac} = 0.2143 - j0.9898 = \sqrt{(0.2143)^2 + (0.9898)^2} = 1.0127$$

Actually, $\boxed{pu = \frac{\text{Actual Value}}{\text{Base Value}}}$ $\qquad V_L = \sqrt{3}\, V_{ph}$

$\Rightarrow \boxed{\text{Actual Value} = \text{pu . Base Value}}$.

[Base value KV = 13.2 KV]

Now, line to line voltages are,

$$V_{ab} = 1.0127 \times \frac{13.2}{\sqrt{3}} = 7.717 \text{ KV}$$

$$V_{ac} = 1.0127 \times \frac{13.2}{\sqrt{3}} = 7.717 \text{ KV}$$

$$V_{bc} = 1.9796 \times \frac{13.2}{\sqrt{3}} = 15.08 \text{ KV}$$

NOTE:- ఇక్కడ denominator లో $\sqrt{3}$ తీసుకుని చేస్తారంటే, assume చేసుకున్న E_a value $1+j0$ as line to Neutral voltage లాగ assume చేసుకున్నాం. కాబట్టి మిగిలిన calculations అన్ని phase voltages పై depend అవుతాయి.

కాబట్టి line-to-line voltages are calculated by taking $\sqrt{3}$ in denominator.

---

(2) Determine the fault current when line-line fault occurs and also determine the line-to-line voltages.

Sol: $E_a = 1+j0$ as line-to Neutral voltage (assumption)

$$I_{a1} = \frac{E_a}{Z_1+Z_2}$$

$\left[\because Z_0 = 0 \Rightarrow I_{a0} = 0 ;\ V_{a0} = 0\right]$

$$= \frac{1+j0}{j0.25+j0.35}$$

[In L-L fault, the Boundary Condition is, $I_{a0} = 0$]

$$= -j1.667$$

for line-to-line fault, $I_{a1} = -I_{a2}$

$\therefore I_{a2} = j1.667$ and $I_{a0} = 0.$

To find out the fault current $I_b = -I_c$

$$\Rightarrow I_b = I_{b1} + I_{b2} \qquad [\because I_{a0} = 0]$$

$$= \lambda^2 I_{a1} + \lambda I_{a2}$$

$$= (-0.5-j0.866)(-j1.667) + (-0.5+j0.866)(+j1.667)$$

$$= j0.833 - 1.4436 - j0.833 - 1.4436$$

$$= -2.8872 \text{ pu}.$$

Now, the Base Current $= \dfrac{\text{Base MVA}}{\sqrt{3} \times \text{Rated kV at point of fault}}$

$= \dfrac{25 \text{ MVA}}{\sqrt{3} \times 13.2 \text{ kV}}$

$= 1093$ amperes.

$\therefore$ fault Current in Amperes $= 1093 \times 2.8872$

$= 3155.71$ A.

To find out line-to-line voltage, we have to find out sequence component of voltages

$V_{a_1} = E_a - I_{a_1} Z_1$

$= (1+j0) - (-1.667)(j0.25)$

$= 0.5833.$

$V_{a_2} = -I_{a_2} Z_2$

$= -(1.667)(j0.35)$

$= 0.5834$

It has been verified that for line-to-line fault ~~voltages~~ the $V_{a_1} = V_{a_2}$

$V_a = V_{a_0} + V_{a_1} + V_{a_2}$

$= 0 + 0.5833 + 0.5834$

$= 1.1666$ pu.

$V_b = \lambda^2 V_{a_1} + \lambda V_{a_2}$

$= (-0.5 - j0.866)(0.5833) + (-0.5 + j0.866)(0.5833)$

$= -0.5833$

and $V_b = V_c \Rightarrow V_c = -0.5833.$

Line-to-line Voltages

$$V_{ab} = V_a - V_b$$
$$= 1.1666 - (-0.5833) = 1.7499 \text{ pu}$$

$$V_{bc} = V_b - V_c$$
$$= 0$$

$$V_{ac} = V_a - V_c$$
$$= 1.1666 - (-0.5833) = 1.7499 \text{ pu}$$

Now, Actual line-to-line Voltages are

$$pu = \frac{\text{Actual}}{\text{Base}}$$

$$\Rightarrow \text{Actual} = pu \times \text{Base value}$$

$$\Rightarrow V_{ab} = 1.7499 \times \frac{13.2}{\sqrt{3}} = 13.33 \text{ kV}$$

$$V_{bc} = 0$$

$$V_{ac} = 1.7499 \times \frac{13.2}{\sqrt{3}} = 13.33 \text{ kV.}$$

(3) Determine the fault current for Double line to ground fault and determine the line to line voltages.

Sol: fault current $= 3 I_{a0}$

$$I_{a0} = -\left(\frac{E_a - I_{a1} Z_1}{Z_0}\right)$$

$$I_{a1} = \frac{E_a}{Z_1 + \left(\frac{Z_2 Z_0}{Z_2 + Z_0}\right)} = \frac{1+j0}{j0.25 + \frac{j0.1 \times j0.35}{j0.1 + j0.35}}$$

$$= -j3.0506 \text{ pu.}$$

$$I_{a0} = -\left[\frac{E_a - I_{a1}Z_1}{Z_0}\right]$$

$$= -\left[\frac{(1+j0) - (-j3.0506)(j0.25)}{j0.1}\right]$$

$$= -\left[\frac{(1+j0) - 0.76265}{j0.1}\right] = j2.375.$$

Now, the fault Current $= I_b + I_c = 3I_{a0}$

$$I_{a0} = \frac{1}{3}(I_b + I_c + \cancel{I_a}^{0})$$

$$I_{a0} = \frac{1}{3}(I_f)$$

$$\Rightarrow I_f = 3I_{a0}$$

$$\Rightarrow I_f = 3 \times j2.375$$

$$= j7.122$$

Since, the Base Current $= 1093$ Amperes.

Actual fault Current in amperes $= 7.122 \times 1093$

$= 7784.3$ amperes.

from the Boundary Condition, $V_b = 0$ and

$V_c = 0$

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

$$\therefore V_{a1} = (E_a - I_{a1}Z_1) = 1 - (-j3.0506)(j0.25)$$

$$\therefore V_{a1} = 0.23737$$

$$V_{a2} = -I_{a2}Z_2$$

$$I_{a2} = -\frac{[E_a - I_{a1}Z_1]}{Z_2} = -\frac{[1 - (-j3.0506)(j0.25)]}{j0.35}$$

$$= +j0.6782$$

$$\therefore V_{a2} = -(+j0.6782)(j0.35)$$

$$\therefore V_{a2} = \cancel{j}0.23737$$

$\therefore V_{a0} = -I_{a0} Z_0$

$= -(j2.375)(j0.1)$ [$\because$ It had been verified for D.L-G fault ; $V_{a1} = V_{a2} = V_{a0}$]

$= \underline{0.2375}$

$\therefore V_a = V_{a0} + V_{a1} + V_{a2}$

$= 0.2374 + 0.2374 + 0.2374$

$= 0.7122$ pu.

Now, line to line voltages. [$\because V_b = V_c = 0$]

$V_{ab} = V_a = 0.7122 \times \frac{13.2k}{\sqrt{3}}$ [$\because$ Base KV $= 13.2$ KV]

$= 5.42$ KV

$V_{bc} = V_b - V_c = \underline{\underline{0}}$

$V_{ac} = V_a - V_c = V_a = 0.7122 \times \frac{13.2k}{\sqrt{3}}$

$= \underline{\underline{5.42 \text{ KV}}}.$

Unit - V

Power System Stability Analysis.

* Stability :-

Stability is the term applied to alternating Current electric power systems, denoting a condition in which various synchronous machines of the system remain in synchronism with each other.

* Types of Stability :-

Power System stability can be classified into

i, Steady state stability and
ii, Transient state stability.

→ Steady state Stability :-

Steady state stability is the operating Condition of the Power System which is characterised by gradual (or) Relatively slow changes with out loss of Synchronism.

→ Steady state stability limit :-

When the load on the system is increased gradually, the maximum power that can be transmitted with out loss of Synchronism is known as steady state stability limit. If an attempt is made to transmit more power than this limit, then the synchronism will lost.

→ Transient State :-

Transient state is the operating condition of power system which is characterised by sudden changes in load.

For a sudden change in power system, power transfer may undergo fast changes whose magnitudes are dependent on the severity of the disturbance. This type of instability is known as Transient instability and is a fast phenomenon usually occuring with in 1 second.

→ Transient State stability limit :-

When the load on the system is increased suddenly, maximum power that can be transmitted without loss of synchronism is termed as Transient State stability limit.

Generally, steady state stability limit is greater than the Transient State stability limit

Note :- steady state stability is classified into two types i, static state stability and ii, Dynamic state stability

→ static stability Refers to steady state stability that prevails with out the aid of Regulating Devices.

→ Dynamic stability Refers to steady state stability prevailing with the help of Regulating devices such as, speed governers, voltage Regulators etc.

* Transfer Reactance :-

It is the Reactance which lies between generating and load points of a transmission line. It connects two voltage sources i.e, Sending end and Recieving end. The Power transferred through the transmission lines is given as.

$$P = \frac{V_1 V_2}{X} \sin\delta$$

where, X = transfer Reactance

* Synchronising power coefficient :-

Definition :- Synchronising power is defined as the varying of the synchronous power on varying in the load angle δ It is so, called as stiffness of coupling (or) Rigidity factor.

Consider a synchronous generator transferring a steady state power P_a at a steady load angle δ_0. Suppose that, due to transient disturbance, the rotor of the generator accelerates, resulting in increasing in load angle by $d\delta$. The operating point of the machine shifts to a new constant power line and the load on the machine increases to $P_a + \delta P$. The steady state input of the system does not change, additional load which is added decreases the speed of the machine and brings it back to the synchronism.

Similarly, if due to a transient disturbance, the rotor of the machine retards resulting a decrease in load angle. Now the operating point of the machine shifts to a new constant power line and the load on the machine decreases to $P_a - \delta P$. Since, the input remains unchanged, the reduction in the load accelerates the Rotor. The machine again comes in synchronism.

The Effectiveness of this correcting action on the change in power transfer for a given change in load angle. And the measure of Effectiveness is known as Synchronising Power Coefficient.

$$P_{syn} = \frac{dP}{d\delta} \text{ watts/Elect. radians.}$$

As we know that, $P = \frac{V_1 V_2}{X} \sin\delta$. — (1)

differentiate above equation,

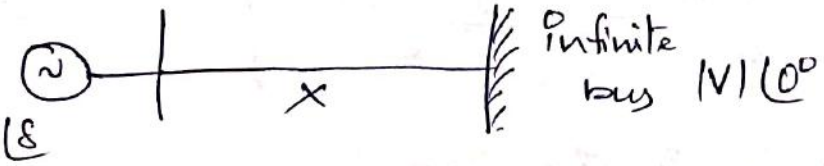
$$\frac{dP}{d\delta} = \frac{V_1 V_2}{X} \cos\delta$$

$\therefore$ Synchronising Power Coefficient (or) Electrical stiffness of a machine $\Big\} = \frac{dP}{d\delta}$

$$= \frac{V_1 V_2}{X} \cos\delta \text{ W/elect rad}$$

* Power Angle Curve :-

Consider a very simple network which consists of a Synchronous generator connected to the infinite bus through a transmission line as shown below.



In general, whenever there is an infinite bus exists must be treated as Reference. So, $\delta = 0$. i.e, $|V|\angle 0^\circ$

Considering the lossless networks and assuming the Resistances of the machines and the transmission lines are neglected.

Now, the Real power transfer is given by,

$$P = \text{Real part of } (E^* I)$$

[$\because S = P + jQ = VI^*$. It is quite inconvinient to analyse the power system with conjugate to I. So,

$$S^* = P - jQ = V^* I$$

Now, $P =$ Real part of $V^* I$]

$$\therefore P = E_G^* I^*$$

$$= E_G^* \left[\frac{E_G - |V|}{jX} \right]$$

$$\left[\because I = \frac{E_G - |V|}{jX} \right]$$

($\because$ Resistances are neglected)

$$P = Re\left[\left(E_g \angle \delta\right)^* \left(\frac{E_g \angle \delta - |V| \angle 0^\circ}{X \angle 90^\circ}\right)\right]$$

$$P = Re\left[E_g \angle -\delta \left(\frac{E_g \angle \delta - |V| \angle 0^\circ}{X \angle 90^\circ}\right)\right]$$

$$P = Re\left[\frac{E_g^2 \angle -90^\circ}{X} - \frac{E_g |V| \angle -\delta - 90^\circ}{X}\right]$$

∴ The Real Power is given by

$$P = \frac{E_g^2}{X} \cos(-90^\circ) - \frac{E_g |V|}{X} \cos\left[-(90^\circ + \delta)\right]$$

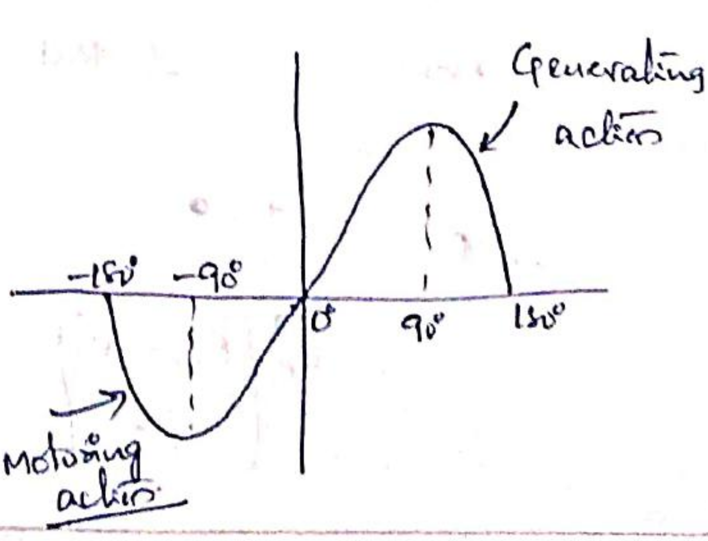
$$\therefore P = 0 - \frac{E_g |V|}{X} \cos(90^\circ + \delta) \qquad \left[\because \cos(-\theta) = \cos\theta\right]$$

$$\therefore \boxed{P = \frac{E_g |V|}{X} \sin\delta}$$

∴ The above equation shows the power transmitted from the generator to the infinite bus.

when E_g, $|V|$ and X are constants, then the Real Power transfer is directly depends on δ.

The Curve plotted between P and δ is known as Power angle Curve.



* Determination of steady state stability limit :-

When the load on the system increased gradually, the maximum power that can be transmitted without loss of synchronism is known as steady state stability limit.

The stability of a system under steady state condition can be described by the swing equation,

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{—(1)}$$

let, the system be operating with steady state power transfer of $P_{eo} = P_m$. Assume, a small increment ΔP in electric power with the input from the prime mover be remains fixed at P_m, causing the torque angle to change to $(\delta_0 + \Delta\delta)$

$$\therefore (1) \Rightarrow M\frac{d^2\Delta\delta}{dt^2} = P_m - (P_{eo} + \Delta P) \qquad (\because P_{eo} = P_m)$$

$$= \cancel{P_m} - \cancel{P_m} - \Delta P$$

$$\therefore M\frac{d^2\Delta\delta}{dt^2} = -\Delta P$$

$$= -\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0} \Delta\delta$$

$$\Rightarrow M\frac{d^2\Delta\delta}{dt^2} + \left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0} \Delta\delta = 0$$

$$\Rightarrow \left[Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0}\right]\Delta\delta = 0 \qquad \left[\because \frac{d}{dt} = p\right]$$

Now, the System stability due to Small changes Can be determined with the help of Characteristic Equation.

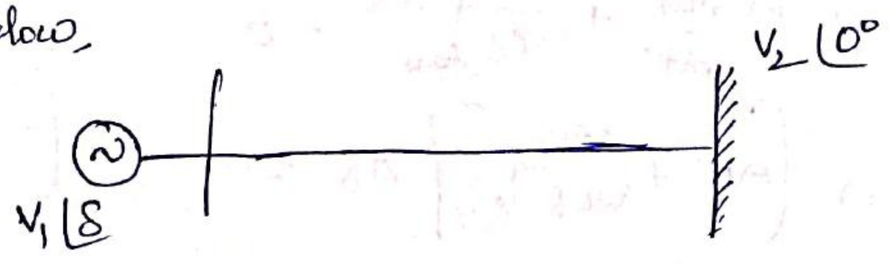
$$Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0} = 0$$

$$\Rightarrow p = \pm \left[\frac{-\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0}}{M} \right]^{1/2} \quad \text{—— (2)}$$

Now, as long as $\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0}$ is positive the roots are purely imaginary and conjugate and the System is oscillatory in nature. And, the System Can be stable for a Small increment in power i.e. $\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0} > 0$.

Similarly, when $\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0}$ is negative, the roots are real and unequal. Therefore, the System is unstable for $\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0} < 0$, where $\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta=0}$ is known as Synchronizing Coefficient.

Let us now study the steady state stability of the System when the Resistances and Capacitances are included. For that, consider a sample System given below.



In general Transmission lines are normally operated with a balanced 3ϕ load but ~~for~~ the analysis can be done on per phase basis. A transmission line on per phase basis can be regarded as two port Networks, where the Sending end Voltage (V_s) and Current (I_s) are Related to Recieving end voltage (V_R) and Current (I_R) through ABCD Constants,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \text{--- (3)}$$

$$\Rightarrow V_S = AV_R + B I_R \quad \text{--- (4)}$$

$$I_S = CV_R + D I_R. \quad \text{--- (5)}$$

$$(4) \Rightarrow I_R = \frac{V_S}{B} - \frac{A}{B} V_R$$

$$(5) \Rightarrow I_S = CV_R + DI_R$$

$$= CV_R + D\left(\frac{1}{B} V_S - \frac{A}{B} V_R\right)$$

$$= CV_R + \frac{D}{B} V_S - \frac{AD}{B} V_R$$

$$= V_R \left[C - \frac{AD}{B}\right] + \frac{D}{B} V_S$$

$$= \frac{D}{B} V_S + V_R \left[\frac{BC - AD}{B}\right] \qquad \left[\because AD - BC = 1\right]$$

$$= \frac{D}{B} V_S - \frac{V_R}{B}$$

$$\therefore \left.\begin{aligned} I_R &= \frac{1}{B} V_S - \frac{A}{B} V_R \\ I_S &= \frac{D}{B} V_S - \frac{1}{B} V_R \end{aligned}\right\} \quad \text{--- (A)}$$

let, A, B, D are the constants of transmission line,

$A = |A| \angle\alpha$, $B = |B| \angle\beta$, $D = |D| \angle\alpha$ $[\because A = D]$.

$\therefore$ Ⓐ $\Rightarrow I_R = \left|\frac{1}{B}\right| |V_S| \angle\delta-\beta - \left|\frac{A}{B}\right| |V_R| \angle\alpha-\beta$

$I_S = \left|\frac{D}{B}\right| |V_S| \angle\alpha+\delta-\beta - \left|\frac{1}{B}\right| |V_R| \angle-\beta$ $\left[\because \begin{array}{l} V_1\angle\delta = V_S\angle\delta \\ V_2\angle 0 = V_R\angle 0 \end{array}\right]$

As we know that,

$S_S = V_S I_S^*$ and

$S_R = V_R I_R^*$

$\Rightarrow S_R = V_R\angle 0^\circ \left[\left|\frac{1}{B}\right| |V_S| \angle\beta-\delta - \left|\frac{A}{B}\right| |V_R| \angle\beta-\alpha\right]$

$\therefore S_R = \frac{|V_S||V_R|}{|B|} \angle\beta-\delta - \left|\frac{A}{B}\right| |V_R|^2 \angle\beta-\alpha$.

$$\left.\begin{array}{l} \therefore \text{Real part } P_R = \frac{|V_S||V_R|}{|B|}\cos(\beta-\delta) - \left|\frac{A}{B}\right| |V_R|^2 \cos(\beta-\alpha) \\ \text{Imaginary part } Q_R = \frac{|V_S||V_R|}{|B|}\sin(\beta-\delta) - \left|\frac{A}{B}\right| |V_R|^2 \sin(\beta-\alpha) \end{array}\right\} \text{— Ⓑ}$$

$\therefore$ The maximum power recieved at recieving end at steady state condition is,

$$P_R = \frac{|V_S||V_R|}{|B|}\cos(\beta-\delta) - \left|\frac{A}{B}\right| |V_R|^2 \cos(\beta-\alpha)$$

when the system is critically stable i.e, $\left(\frac{\partial P_e}{\partial \delta}\right) = 0$

then, $\beta = \delta$

$$\therefore \boxed{P_{R\,max} = \frac{|V_S||V_R|}{|B|} - \left|\frac{A}{B}\right| |V_R|^2 \cos(\beta-\alpha)} \text{ — Ⓐ}$$

Eq. ④ is the indication of steady state stability power limit.

*Methods to improve steady state stability :-

The maximum power transfer capability is defined as,

$$P_{max} = \frac{|E|\,|V|}{X} \sin\delta.$$

when it is assumed, the max. power to be transmitted without loss of stability (steady state) occurs for $\delta = 90°$.

$$\therefore P_{max} = \frac{|E|\,|V|}{X}.$$

$\therefore P_{max}$ can be increased by

i, Either by increasing $|E|$ (or) $|V|$ or both

ii, Reducing X.

i, Methods for increasing system voltage :-

Higher Excitation voltage and fast response excitation system substantially increase the steady state stability limit. Rising the line voltage also increases the transmission system transfer capability.

ii, Methods for Reducing the Reactance :-

1. Use of double circuit line :-

Even if one of the line fails, it can be switched off and hence the continuity of supply can be maintained. But, the provision of additional line cannot be justified by stability consideration alone.

2. Use of Bundled Conductors :-

Bundling of Conductors can reduces the Reactance

3. Series Compensation of line Reactance :-

Inductive Reactance of a line can be Reduced by Connecting static Capacitors in Series with the line

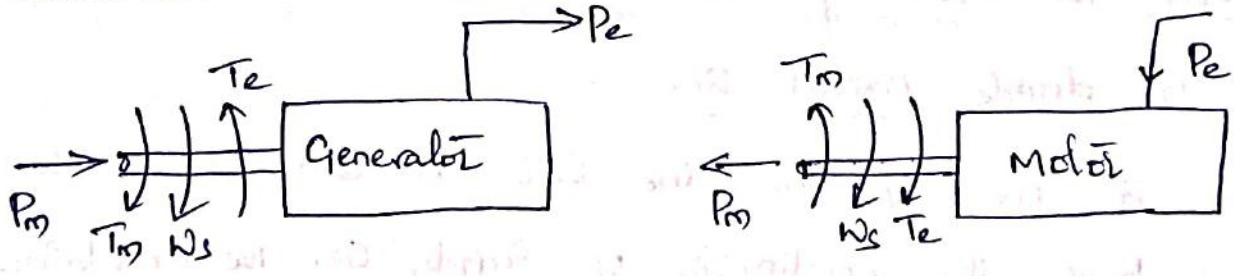
4. Use of Synchronous phase Modifiers :-

Synchronous Phase Modifiers may be installed in intermediate Substations to increase the Power limit.

* Derivation of Swing Equation :-

The behaviour of Synchronous machine during transient period is described by Swing Equation.

The below fig. shows the torque, speed and flow of mechanical and Electrical powers in an Synchronous machine.



As we know that, Torque exerted on a Rotating body is given by the Product of movement of Inertia of a body and angular acceleration.

$T_a = J\alpha$ $\left[\because \alpha = \frac{d^2\theta}{dt^2}\right]$

$$\therefore T_a = J\frac{d^2\theta}{dt^2} \quad \text{--- (1)}$$

where, θ = angular position of the rotor

t = time in seconds.

It is convinient to measure θ with respect to a reference axis that is rotating at synchronous speed.

If δ is the angular displacement of the rotor in elct. deg. and ω_s is the synchronous speed in elct. radians, then θ can be expressed as.

$$\theta = \omega t + \delta \quad \text{--- (2)}$$

$$\Rightarrow \frac{d\theta}{dt} = \omega + \frac{d\delta}{dt}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \quad \text{--- (3)}$$

In General swing equation discusses about the stability of a system

Substituting (3) in (1)

$$\Rightarrow T_a = J\frac{d^2\delta}{dt^2}$$

In an synchronous generator, the accelerating torque (T_a) is equal to the difference of Mechanical torque (T_m) and Electromagnetic Torque (T_e).

$$\therefore T_a = T_m - T_e = J\frac{d^2\theta}{dt^2} = J\frac{d^2\delta}{dt^2}$$

$$\Rightarrow T_a = T_m - T_e = J\frac{d^2\delta}{dt^2}$$

multiplying ω on both sides,

$$T_a \cdot \omega = \omega T_m - \omega T_e = J\omega \frac{d^2\delta}{dt^2}$$

$$\Rightarrow P_a = P_m - P_e = J\omega \frac{d^2\delta}{dt^2}$$

$$\Rightarrow \boxed{P_a = P_m - P_e = M \frac{d^2\delta}{dt^2}}$$

Swing equation is mainly used to study whether the system is stable (or) not.

The above equation is known as Swing Equation.

*Constants used in stability Analysis:-

The kinetic energy of Rotor of synchronous Machine,

$$K.E = \frac{1}{2} J \omega_{sm}^2 \times 10^{-6} \text{ MJ}$$

where, J = Moment of inertia of Rotor

ω_{sm} = Synchronous speed in Mech. rad/sec

But, $\omega_s = \left(\frac{P}{2}\right)\omega_{sm}$ = rotor speed in elect. rad/sec

where, P = Number of Poles.

$$\therefore K.E = \frac{1}{2}\left[J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right]\omega_s$$

$$\Rightarrow \boxed{K.E = \frac{1}{2} M\omega_s}$$

where $M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$.

Now, Inertia Constant (H) can be defined as,

$$H = \frac{K.E}{\text{Rated MVA } (G)}$$

$$\Rightarrow \boxed{K.E = \text{Stored Energy} = GH.}$$

$$\therefore GH = \frac{1}{2} M\omega_s$$

$$\Rightarrow M = \frac{2GH}{\omega} \quad (\because \omega = 2\pi f)$$

$$= \frac{\cancel{2}GH}{\cancel{2}\pi f}$$

$$\therefore \boxed{M = \frac{GH}{\pi f}} \quad MJ / \text{elect. rad.}$$

$$\Rightarrow \boxed{M = \frac{GH}{180 f}} \quad MJ / \text{elect. degree}$$

where, G = Rated MVA (or) Base MVA

H = Inertia Constant.

Problem:-

1. A 200MVA, 11kV, 50Hz, 4-Pole generator has an Inertia Constant of 6 MJ/MVA.

i, Find the stored Energy in Rotor at Synchronous Speed.

ii, The Machine is operating at a load of 120MW, when the load is suddenly increased to 160MW. Find the Rotor Retardation. Neglect losses.

iii, The Retardation Calculated above is maintained for 5 cycles. Find the change in power angle and Rotor speed in RPM at the End of this period.

Sol i, Stored Energy $= GH$ $\quad$ [given $G = 200$MVA, $H = 6$ MJ/MVA]

$$= 200 \times 6$$

$$= 1200 \text{ MJ}.$$

ii, Accelerating power $P_a = 120 - 160$

$= -40 \text{ MW}.$

Angular momentum $M = \frac{GH}{180f}$

$= \frac{1200}{180 \times 50}$

$= \frac{2}{15}$ MJ sec/ elect. deg.

Now, $M\alpha = P_a$

$\Rightarrow \frac{2}{15}\alpha = P_a$

$\Rightarrow \alpha = -40 \times \frac{15}{2}$

$= -300$ electrical deg/sec^2.

iii, for 5 cycles, Equivalent time $(t) = \frac{5 \text{ cycles}}{50 \text{ Hz}}$

$= 0.1$ sec

change in $\delta = \frac{1}{2}\alpha t^2$

$= \frac{1}{2}(-300)(0.1)^2$

$= -1.5$ electrical deg.

Since, $P = 4$; 1 Revolution $= 4 \times 180°$

$= 720°.$

$\alpha = -300$ elect. deg. /sec^2

$= -300 \times \left(\frac{60}{720}\right)$

$= -25$ RPM/sec

Now, Synchronous Speed $(N_s) = \frac{120f}{P}$

$$= \frac{120 \times 50}{4}$$

$$= 1500 \text{ RPM}$$

$\therefore$ Rotor Speed at the end of 5 cycles,

$$= 1500 + (-25 \times 0.1)$$

$$= 1497.5 \text{ RPM}$$

* Equal Area Criterion :-

In a System, where one machine is Swinging with respect to an infinite bus, it is possible to study transient stability by means of a simple criterion without going for a Numerical solution of Swing Equation is known as Equal Area criterion.

Now, Consider the Swing Equation.

$$M \frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

$$\Rightarrow \frac{d^2\delta}{dt^2} = \frac{1}{M} P_a. \quad \text{——— (1)}$$

Now, if the System is unstable, δ Continues to increase indefinitely with time and Machine losses its Synchronism.

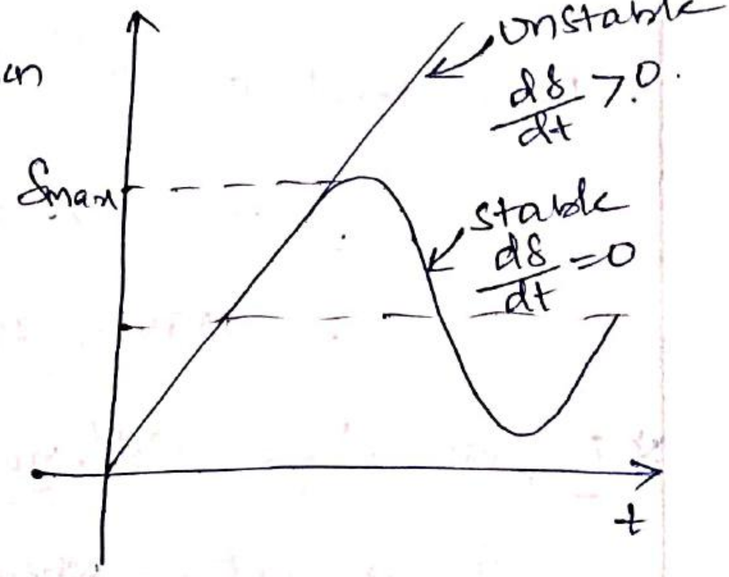
on the other hand, if the system is stable. $\delta(t)$ performs oscillations whose amplitude decreases in practice because of damping terms. These two situations are shown in the below fig.

Now, the stability criterion can be stated as,

the system is stable when

$$\frac{d\delta}{dt} = 0, \quad \text{and}$$

is unstable when $\frac{d\delta}{dt} > 0$.



The stability criterion for power systems can be converted into a simple and easily applicable form for a single machine connected to an infinite bus.

Multiplying both sides by $2\frac{d\delta}{dt}$ to Eq ① we get,

$$\Rightarrow 2\frac{d\delta}{dt}\left(\frac{d^2\delta}{dt^2}\right) = \frac{2P_a}{M}\frac{d\delta}{dt}$$

$$\Rightarrow 2\frac{d\delta}{dt}\left[\frac{d}{dt}\left(\frac{d\delta}{dt}\right)\right] = \frac{2P_a}{M}\frac{d\delta}{dt}$$

integrating on both sides we gets

$$\Rightarrow 2\int \frac{d\delta}{dt}\left(\frac{d}{dt}\left(\frac{d\delta}{dt}\right)\right)dt = 2\int \frac{P_a}{M}\frac{d\delta}{\cancel{dt}}\cdot\cancel{dt}$$

$$\left[\because \int f(x)\, f'(x) = \frac{[f(x)]^2}{2}\right] \qquad \left[\because f(x) = \frac{d\delta}{dt} \quad f'(x) = \frac{d}{dt}\left(\frac{d\delta}{dt}\right)\right].$$

$$\Rightarrow \left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} Pa.d\delta$$

$$\Rightarrow \frac{d\delta}{dt} = \left[\frac{2}{M}\int_{\delta_0}^{\delta} Pa.d\delta\right]^{1/2}$$

$$\Rightarrow 0 = \left[\frac{2}{M}\int_{\delta_0}^{\delta} Pa.d\delta\right]^{1/2}$$

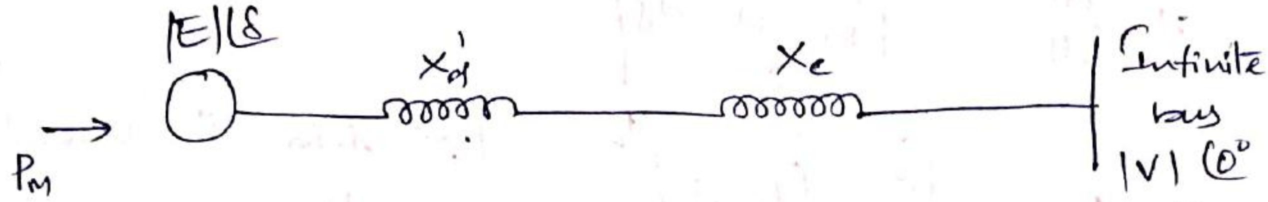
$\left(\because \text{Condition for stability, } \frac{d\delta}{dt} = 0\right)$

$$\Rightarrow \boxed{\int_{\delta_0}^{\delta} Pa.d\delta = 0} \quad \text{—— (A)}$$

Now, the Condition for stability can therefore be stated as the system is stable when the accelerating power (Pa) is zero. That means the mechanical i/p power (Pm) and the Electrical o/p power (Pe) are Equal.

In other words, Positive (accelerating) area under Pa - δ Curve must Equal to Negative (decelerating) area. Hence, the name Equal Area Criterion.

* Application of Equal Area Criterion for Stability when there is a Sudden Change in Mechanical i/p :-



The above figure shows the transient model of a single machine connected to an infinite bus.

Now, the Electrical power transferred is given by,

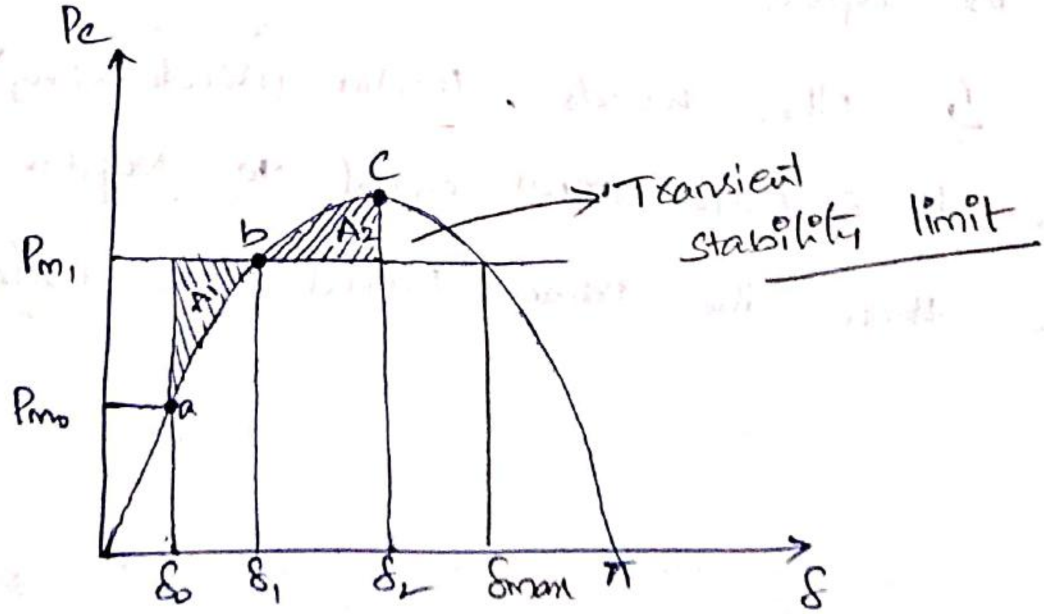
$$P = \frac{|E|\,|V|}{X_d' + X_e} \sin\delta$$

Mechanical i/p increase అంటే ఏంటి?
load increase అని అర్థం.
load increase అయితే Generation increase
అవ్వాలి, అంటే mechanical i/p increase
అవుతుంది load demand & generation
& increase చేస్తాం.

under steady state condition,

$$P_{m0} = P_{e0} = P_{max} \sin\delta_0 .$$

This is indicated by the point "a" in the below fig.



Now, let the Mechanical input to the rotor be Suddenly increased to P_{m_1}. The accelerating Power $P_a = P_{m_1} - P_e$ Causes the rotor Speed to increase and so does the Rotor angle. At angle δ_1, $P_a = P_{m_1} - P_e = 0$, but the Rotor angle Continous to increase due to the moment of inertia of a machine. P_a, now becomes negative (deccelerating). ~~but~~ the rotor Speed begins to reduce but the angle Continous to increase till the angle δ_2.

At point "C" the deccelerating Area A_2 Equals the Accelerating Area A_1 i.e, $\int_{\delta_0}^{\delta_2} P_a \, d\delta = 0$.

Since the Rotor is decelerating, the speed Reduces below ω_s and the rotor angle begins to reduce.

Now, the System Settles to a new steady Steady State,

$$P_{m_1} = P_e = P_{max} \sin\delta_1$$

from the above fig A_1 and A_2 are given as,

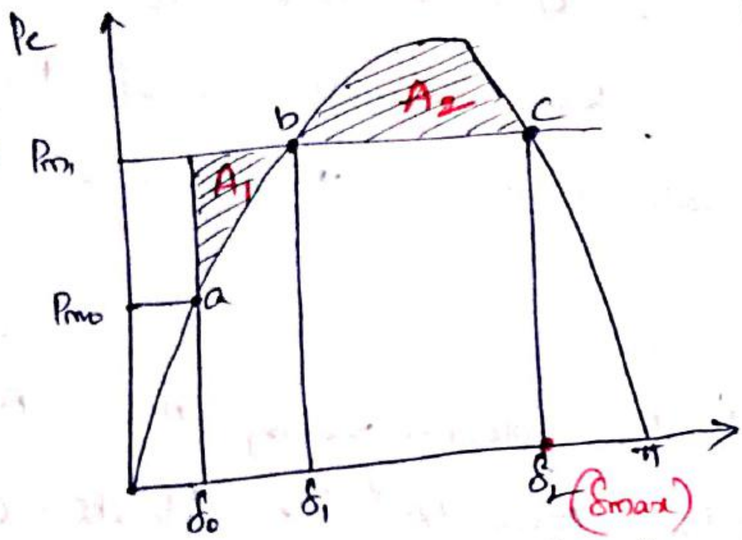
$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m_1} - P_e) \, d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m_1}) \, d\delta$$

Now for the System to be stable, it is necessary to find δ_2 Such that $A_1 = A_2$.

As P_{m}, is further increased, the limiting condition is finally reached when A_1 Equals the area above P_{m}, as shown below.

System stable గా [illegible] $A_1 = A_2$ [illegible] $\delta_2 = \delta_{max}$ [illegible] $A_1 = A_2$ Condition Satisfy [illegible].



Now, any further increase in P_{m}, the area available for A_2 is less than A_1 and Consequently the system become unstable

A_2 [illegible] Area δ_{max} [illegible] A_1 [illegible] Equal [illegible] system stable
A_2 [illegible] Area δ_{max} [illegible] A_1 [illegible] Equal [illegible] System unstable
[illegible] P_{m} [illegible] P_{m}, increase [illegible] δ_2 δ_{max} [illegible] Reach [illegible] P_{m}, line [illegible] increase [illegible] δ_2 [illegible] increase [illegible] P_{m}, line [illegible] touch [illegible], [illegible] A_2 Area [illegible].
A_1 (Accelerating Area) below P_{m} line. A_2 (decelerating) above P_{m} line
Decelerating Area [illegible] δ_2 [illegible] P_{m} line [illegible] touch [illegible].

Note :- Transient Stability analysis involves 3 types of Problems.

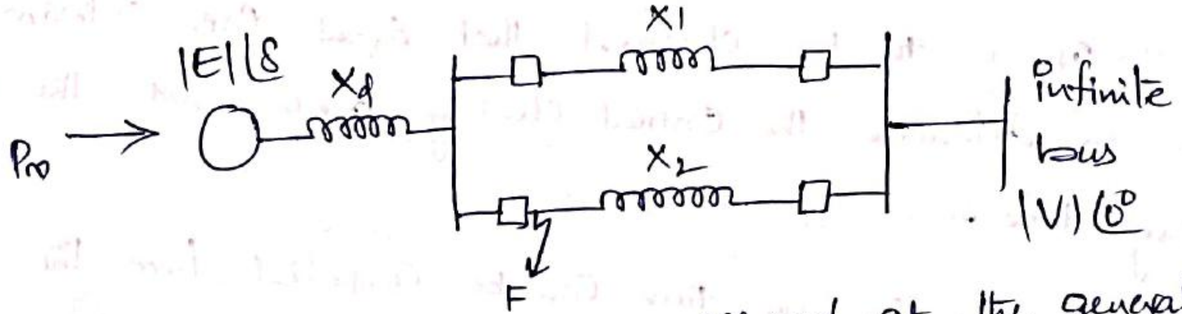
i, if a Mechanical i/p increased how about Stability.

ii, if a fault occured at one End of Tr. line how will be the System Stability.

iii, if a fault occurs at the middle of the Tr. line, how the System Stability will be analysed.

In all the three Conditions, the System Stability Can be examined with the help of Equal Area Criterion.

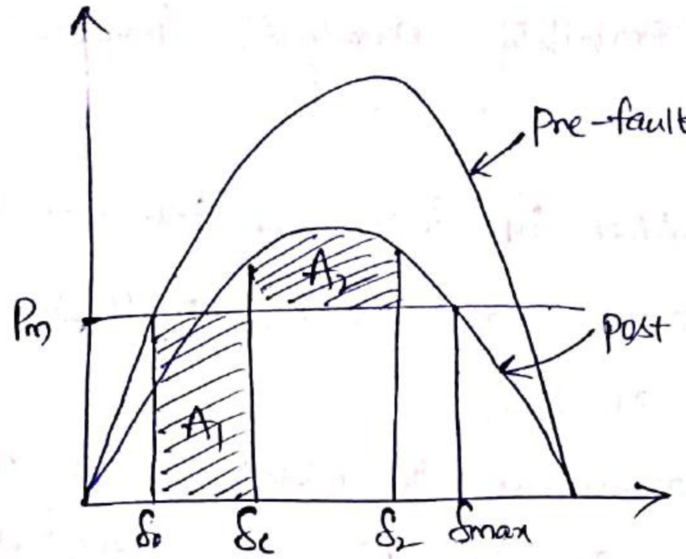
* Fault occurs at one End of Transmission line :-



let us assume the disturbance occured at the generator end of line-2 as shown in the figure. Assume the disturbance to be a 3ϕ fault.

upon the occurance of fault, the generator gets isolated from the System. The Power flow during this period is zero.

The Rotor therefore accelerates and the angle δ increases. The circuit breakers at the two Ends of the line (faulted line) open at time t_c, (clearing time), disconnecting the faulted line. The Power flow is now Restored via the healthy line.



δ_c = clearing angle
[the angle at which the C/B clears the fault at time t_c]

[t_c = clearing time of C.B., when fault occurs]

The Rotor now starts decelerates from δ_c as shown in the fig. Now the System is stable if decelerating Area is equal to Accelerating area A_1, before δ Reaches the Maximum value of δ_{max}.

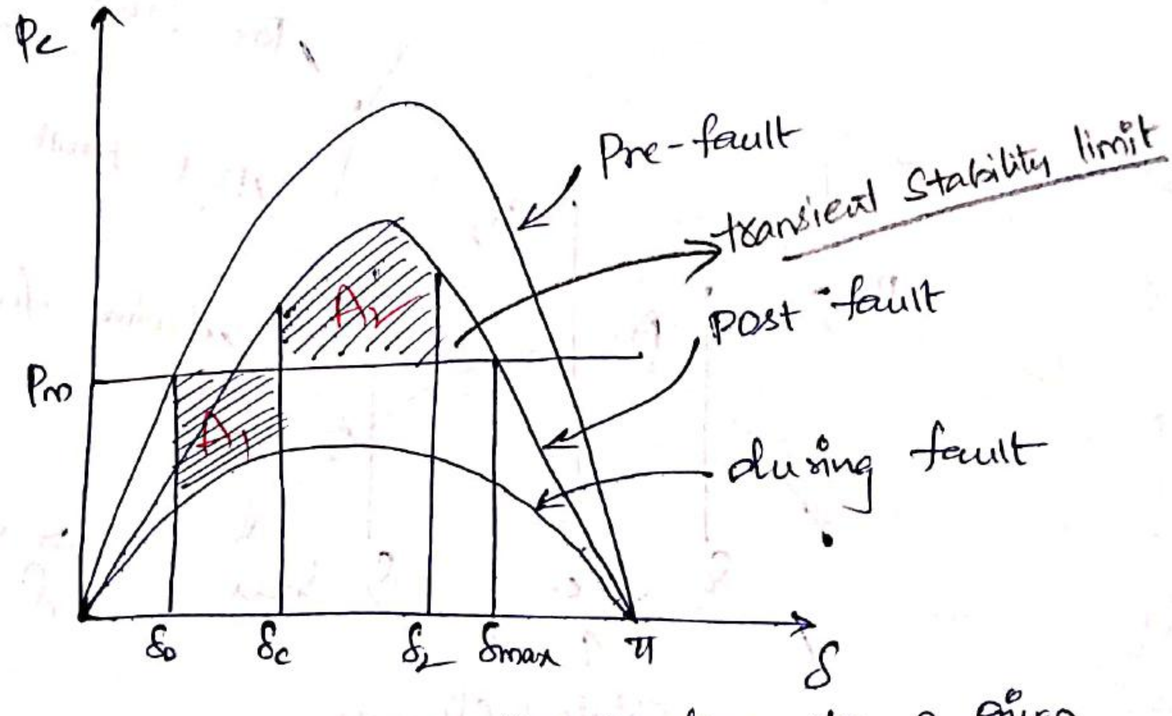
It is to be observed that Equal Area Criterion helps to determine the critical clearing angle not the critical clearing time.

Critical clearing time can be computed from the Swing Equation.

* Fault occurs at the middle of Tr. line (or) far away from the line Ends:-

When the fault occurs away from the line Ends, there is some power flow during the fault though considerably reduced. When the fault occurs at the line Ends as of previous Case, the power flow during fault is zero.

The power angle curve during pre-fault; during fault and post fault conditions are shown in the below figure



Accelerating Area A_1 corresponding to a given clearing angle δ_c is less in this case, than in case 'a'. (previous case), giving better chance for stable operation.

Transient stability limit:- It gives the information, that system can withstand capacity. (ఇక్కడ occur అయిన fault [illegible], దాని severity [illegible] fault occur అయినా system తట్టుకోగలదు)

Stable system operation is possible to find any area A_2 equal to A_1 for $\delta_2 < \delta_{max}$. As the clearing angle δ_c is increased, area A_1 increases and to find $A_2 = A_1$, δ_2 increases till it has a value of δ_{max}, the maximum allowable angle for stability. This case of critical clearing angle is shown in below figure.

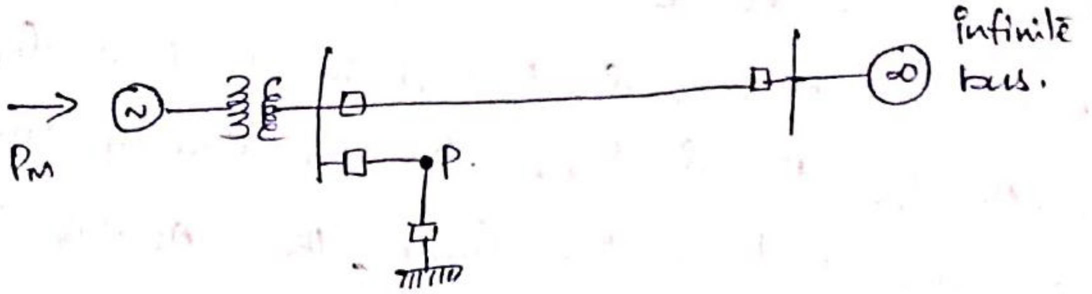
$\delta_2 = \delta_{max}$ అయినప్పుడు transient stability limit అని అంటారు. ఒకవేళ ఇప్పుడు fault occur అయితే, ఇంకా ఎక్కువ severity ఉన్న fault occur అయితే system తట్టుకోలేదు, unstable అవుతుంది.

Transient stability limit ని increase చేయాలంటే clearing time and clearing angle తగ్గించాలి. అప్పుడు δ_2 కి δ_{max} కి మధ్య gap పెరుగుతుంది. దాని వల్ల system stable గా ఉంటుంది.

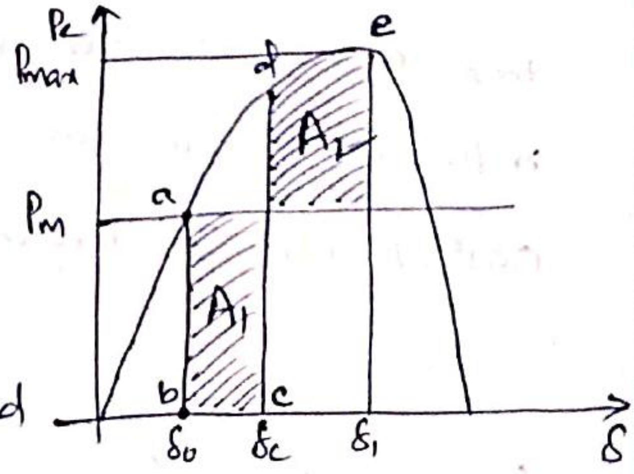
ఏ angle వద్ద C.B's operate అవుతాయో, అప్పుడు $\delta_2 = \delta_{max}$ అవుతుంది. ఆ time ను critical clearing time (t_{cr}) అంటారు. t_{cr} కంటే ఎక్కువ సమయం C.B తీసుకుంటే δ_2, δ_{max} కంటే దాటిపోతుంది, అప్పుడు $A_1 \neq A_2$ అవుతుంది. So, C.B. t_{cr} లోపలే operate అయితే అప్పుడు $\delta_2 < \delta_{max}$ గా ఉంటుంది. అప్పుడు transient stability limit increase అవుతుంది.

* Concept of Critical clearing time and angle :-

let the system shown in below figure be operating with Mechanical input P_m at a steady angle δ_0, as shown in P_e-δ Curve.



If a 3ϕ fault occurs at Point 'p' then, the output of the generator instantly reduces to '0'. i.e, $P_e = 0$. and the steady state point drops to 'b'.



Now, the acceleration area begins to increase and so does the rotor angle.

At time t_c corresponding to angle δ_c the faulted line is cleared by opening of the line circuit breaker.

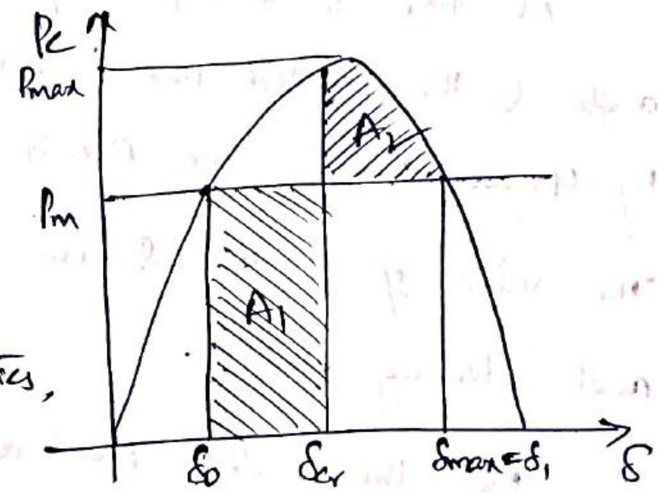
The value of t_c and δ_c are respectively known as clearing time and clearing angle.

Now, the system once again becomes healthy and transmits $P_e = P_{max} \sin\delta$. i.e, the point shifts to 'd' from 'c' on the $P_e - \delta$ Curve

If an angle δ_1 can be found such that Accelerating area A_1 is Equal to Decclerating Area A_2, then the System is said to be stable.

Now, As the clearing of faulty line is delayed, A_1 increases and so does δ_1, to find the condition for stability $A_1 = A_2$ till $\delta_1 = \delta_{max}$. If the clearing time (or) clearing angle is larger than δ_{max}, then $A_2 < A_1$ simply the System would be unstable.

Therefore, the Maximum allowable value of clearing time and angle for the system to be stable are respectively known as critical clearing time and critical clearing angle



for a simple case ($P_e = 0$, during fault), the δ_c and t_c are calculated below,

from the above characteristics,

$$\delta_{max} = \pi - \delta_0 \quad (1)$$

and $$P_m = P_{max} \sin \delta_0 \quad (2)$$

Now,

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_e)\, d\delta = P_m(\delta_{cr} - \delta_0) \quad (\because P_e = 0) \quad \text{--- (3)}$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max}\sin\delta - P_m)\, d\delta$$

$$= P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr}) \quad \text{--- (4)}$$

Now, for the system to be stable, $A_1 = A_2$.

$$\Rightarrow P_m(\delta_{cr} - \delta_0) = P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m\delta_{max} + P_m\delta_{cr}$$

$$\Rightarrow \cancel{P_m\delta_{cr}} - P_m\delta_0 = P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m\delta_{max} + \cancel{P_m\delta_{cr}}$$

$$\Rightarrow P_m(\delta_{max} - \delta_0) = P_{max}(\cos\delta_{cr} - \cos\delta_{max})$$

$$\Rightarrow \cos\delta_{cr} - \cos\delta_{max} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0)$$

$$\Rightarrow \cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}. \quad \text{--- (5)}$$

Now, Substituting (1) and (2) in (5) we get

$$\Rightarrow \cos\delta_{cr} = \frac{\cancel{P_{max}}\sin\delta_0}{\cancel{P_{max}}}(\pi - \delta_0 - \delta_0) + \cos(\pi - \delta_0)$$

$$\therefore \Rightarrow \cos\delta_{cr} = \sin\delta_0(\pi - 2\delta_0) - \cos\delta_0.$$

$\therefore$ clearing (critical) angle

$$\Rightarrow \boxed{\delta_{cr} = \cos^{-1}((\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0)} \quad \text{--- (6)}$$

During the period of fault the swing equation is:

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m \qquad (\because P_e = 0)$$

Integrating twice on both sides we get

$$\delta = \frac{\pi f t^2}{2H} P_m + \delta_0 .$$

(or)

$$\boxed{\delta_{cr} = \frac{\pi f t_{cr}^2}{2H} P_m + \delta_0 .} \quad (7)$$

$$\Rightarrow \boxed{t_{cr} = \left[\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}\right]^{1/2}} \quad (8)$$

These formulas are applicable when a fault occurs at one end of Tr. line assuming at Generator end $\therefore P_e = 0$

$\therefore$ Equation (6) for critical clearing angle and equation (8) for critical clearing time.

Note :- 1. These formulas will not be applicable to all the types of fault conditions.

2. when the fault occurs at middle of Tr. line (or) at the far away of ends of Tr. line the t_{cr} and δ_{cr} are to be calculated.